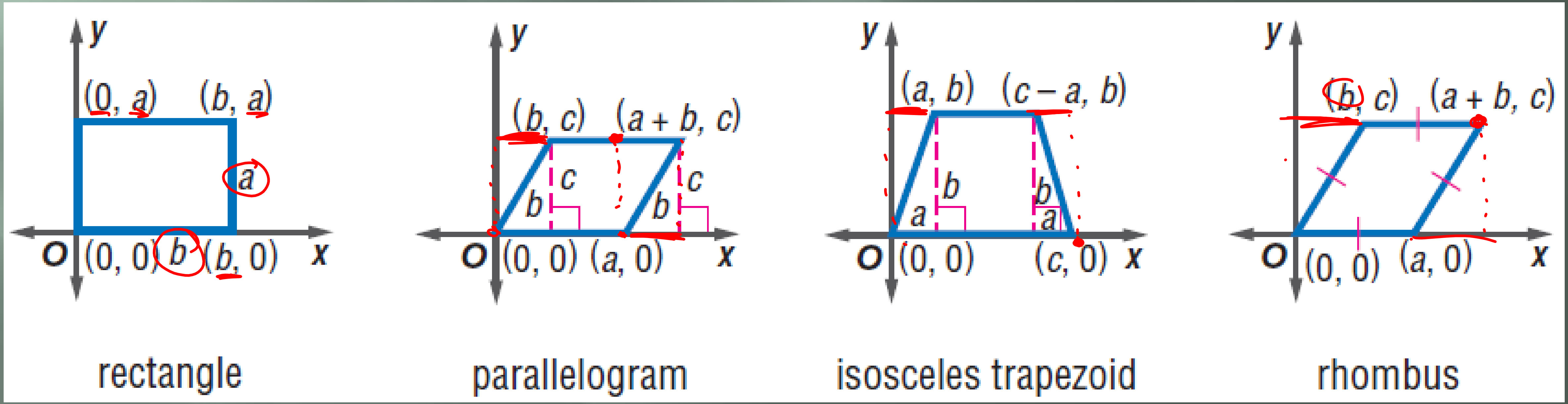
COORDINATE PROOFS WITH QUADRILATERALS

Positioning

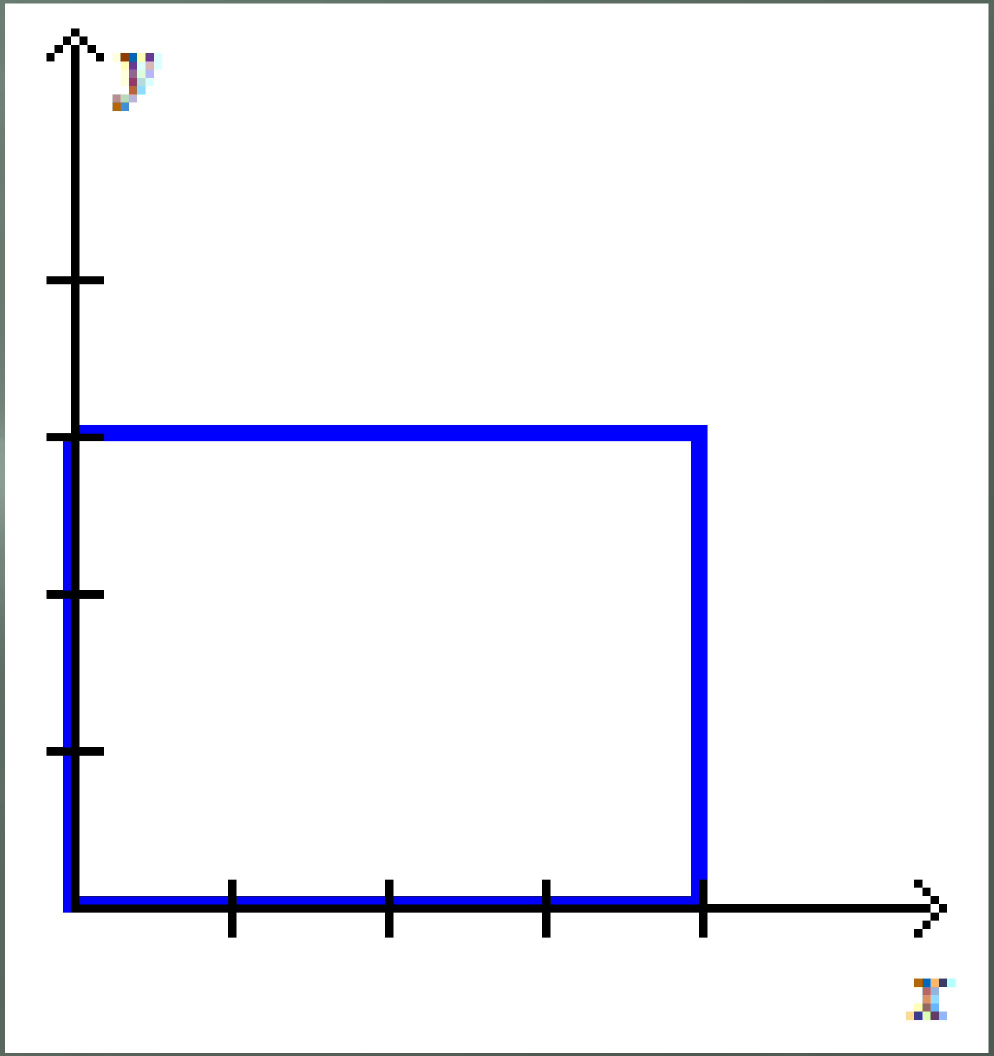
- The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.



Positioning

Let *A*, *B*, *C*, and *D* be vertices of a quadrilateral with a length of *a* and a height of *b*.

Place the quadrilateral with vertex *A* at the origin, *AB* along the positive *x*-axis, and *AD* along the *y*-axis. Label the vertices *A*, *B*, *C*, and *D*.

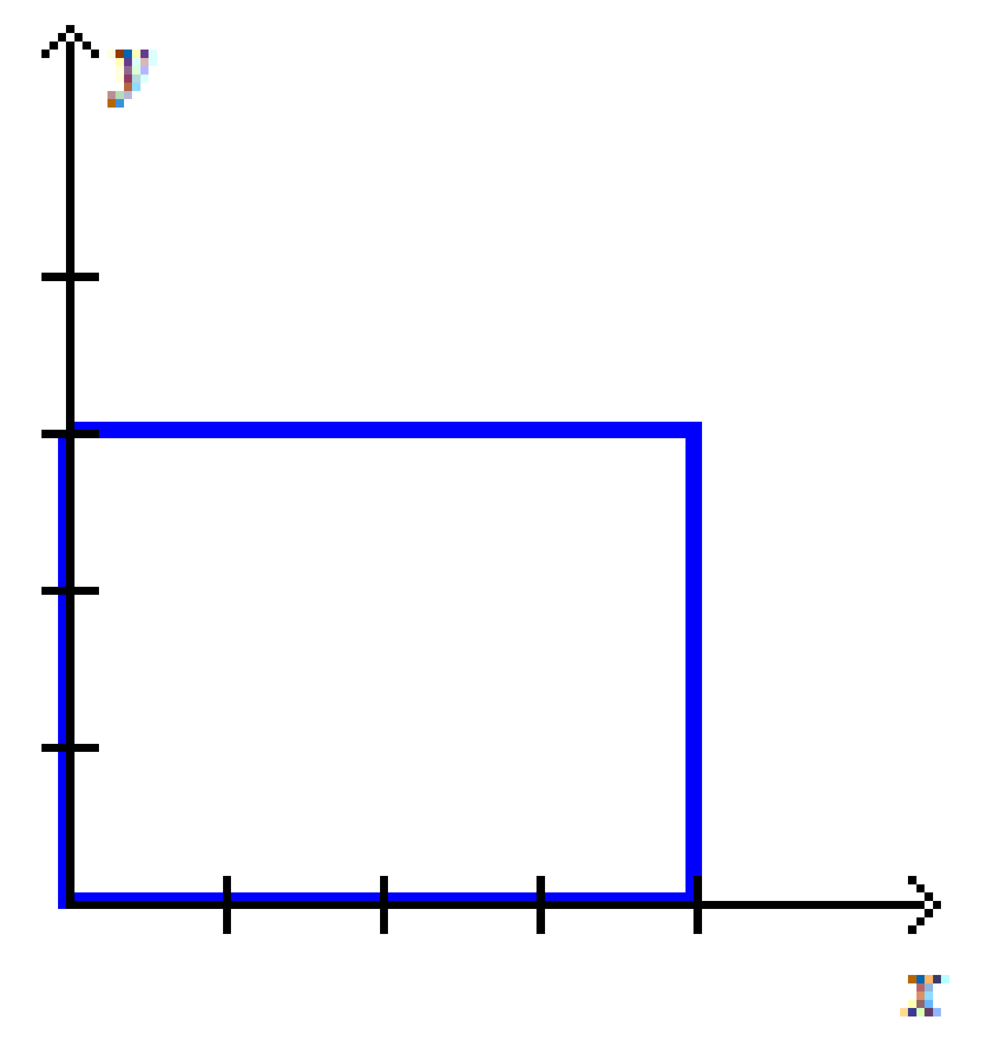


Positioning

The *y*-coordinate of *B* is 0 because the vertex is on the *x*-axis. Since the side length is *a*, the *x*-coordinate is *a*.

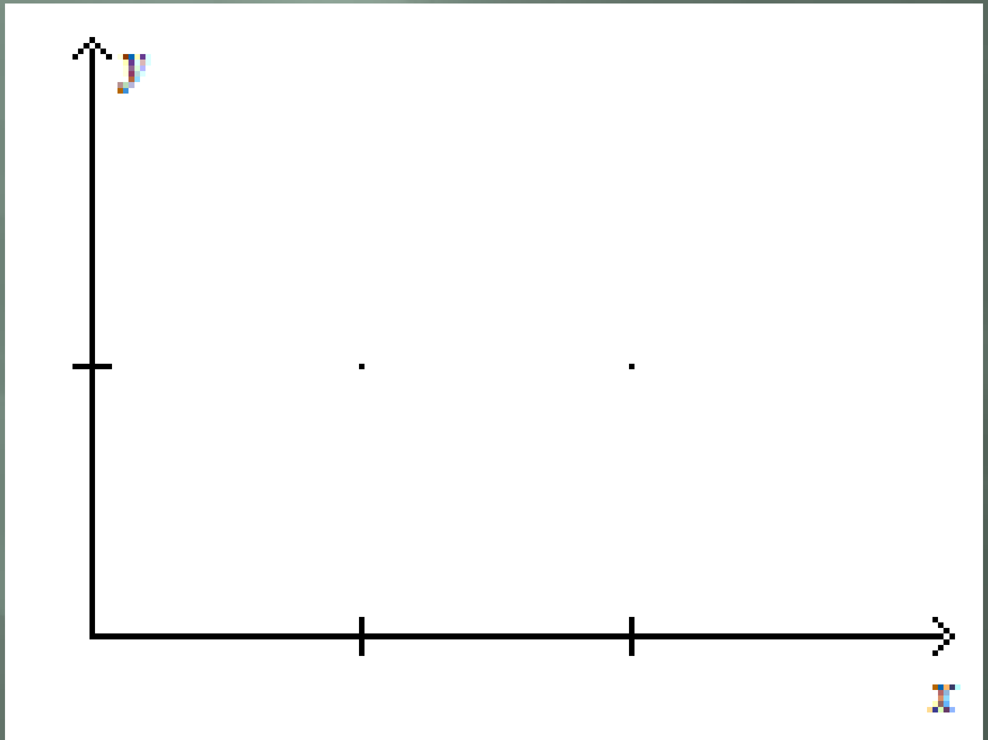
D is on the *y*-axis so the *x*-coordinate is 0. The *y*-coordinate is 0 + *b* or *b*.

The *x*-coordinate of *C* is also *a*. The *y*-coordinate is 0 + *b* or *b* because the side *BC* is *b* units long.



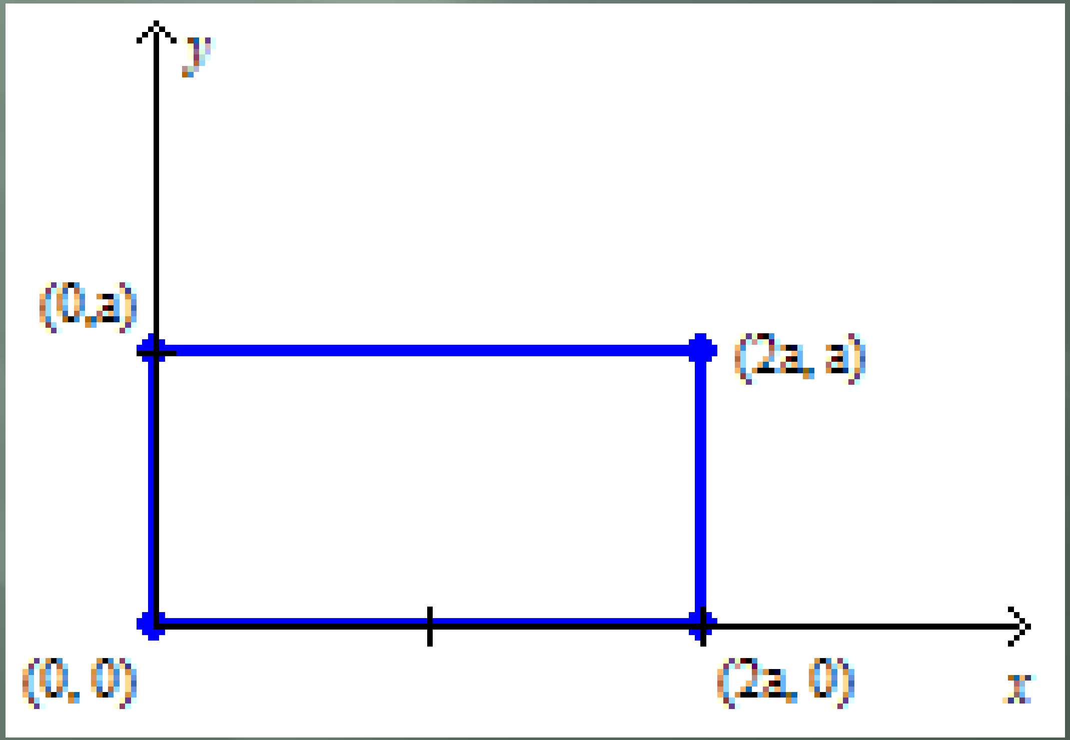
Examples

- Position and label a rectangle with a length of 2*a* units and a width of *a* units.



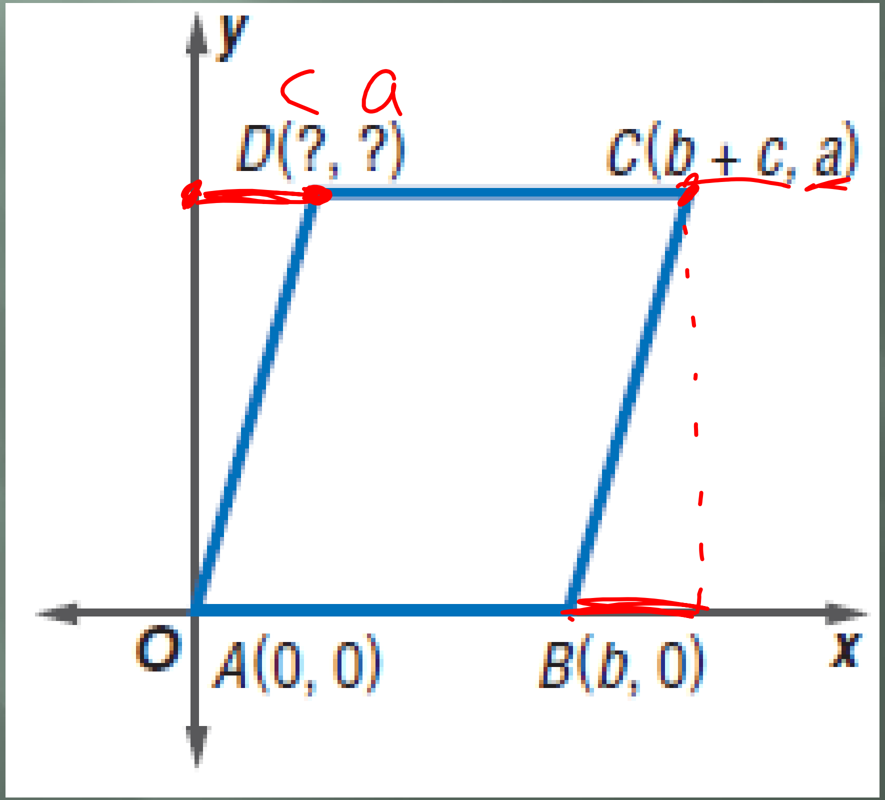
Examples

- Position and label a rectangle with a length of 2*a* units and a width of *a* units.



Examples

- Name the missing coordinates for the parallelogram.

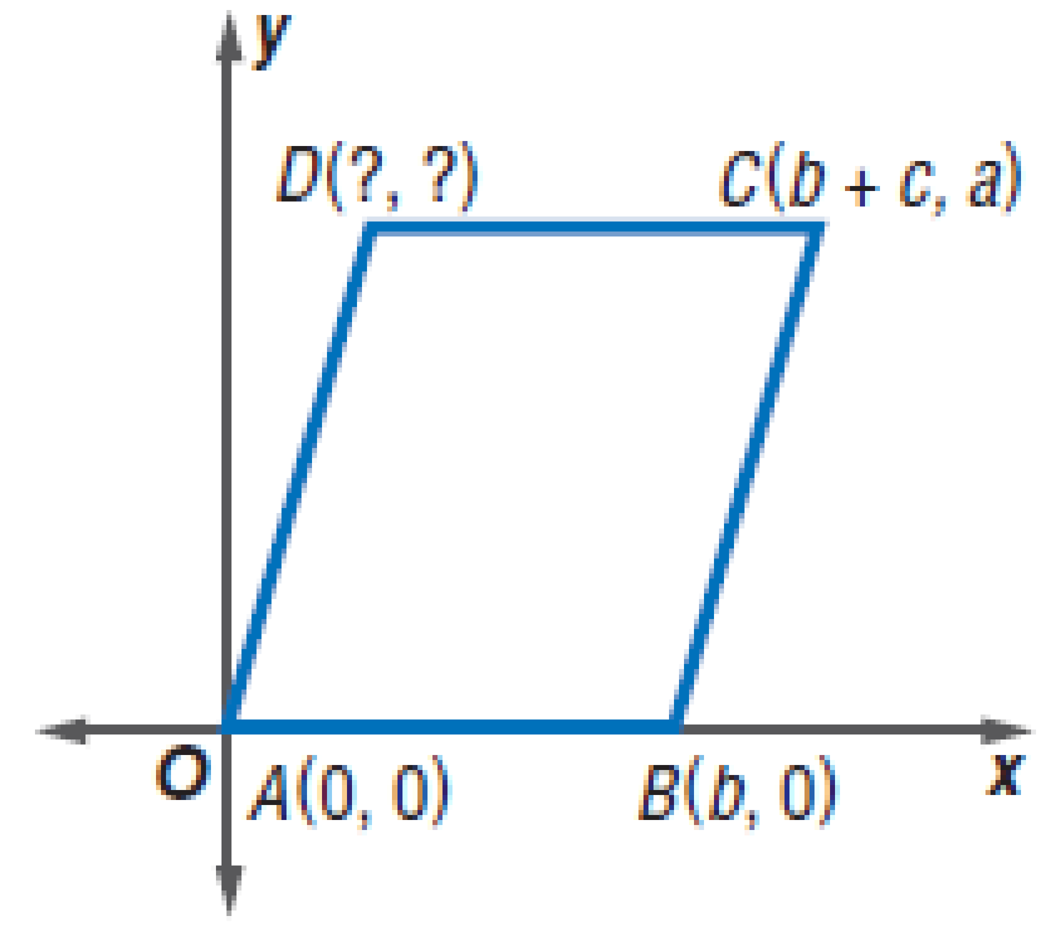


Examples

- Name the missing coordinates for the parallelogram.

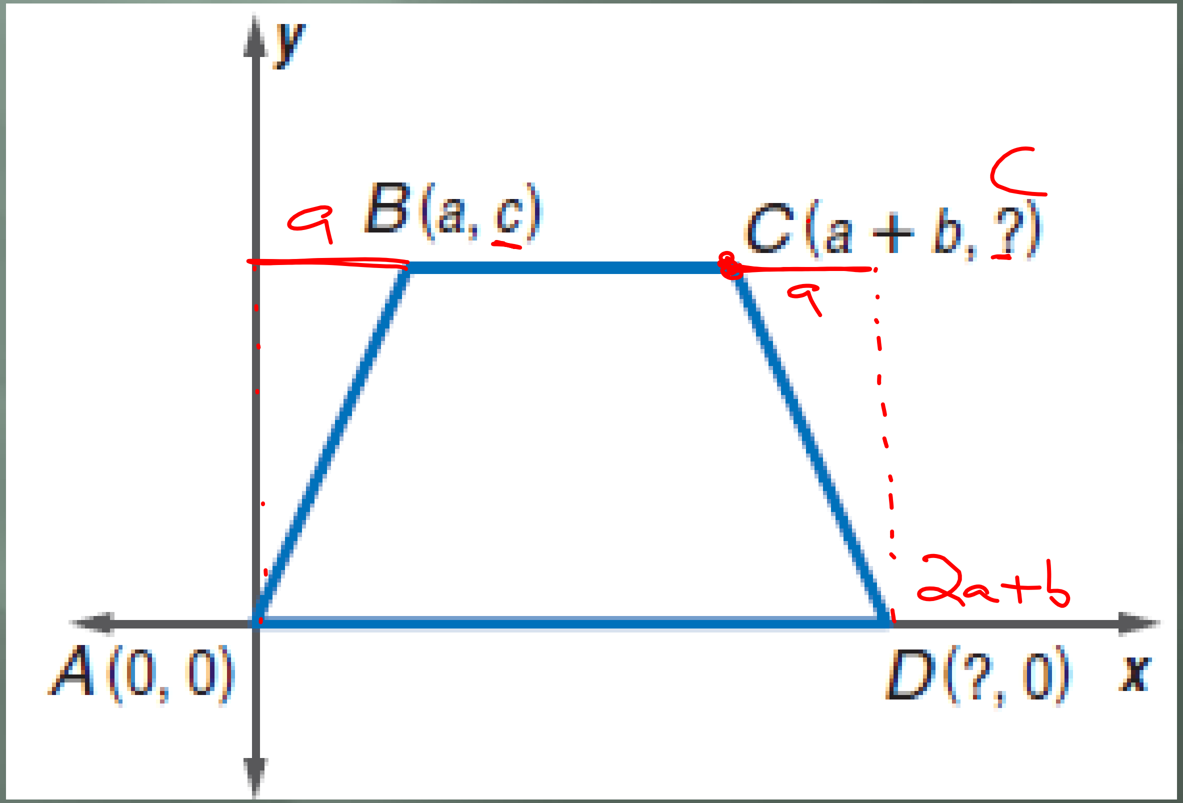
- Opposite sides of a parallelogram are congruent and parallel. So, the y-coordinate of D is a.
- The length of AB is b, and the length of DC is b.
- So, the x-coordinate of D is $(b + c) - b$ or c.

- The coordinates of D are (c, a).



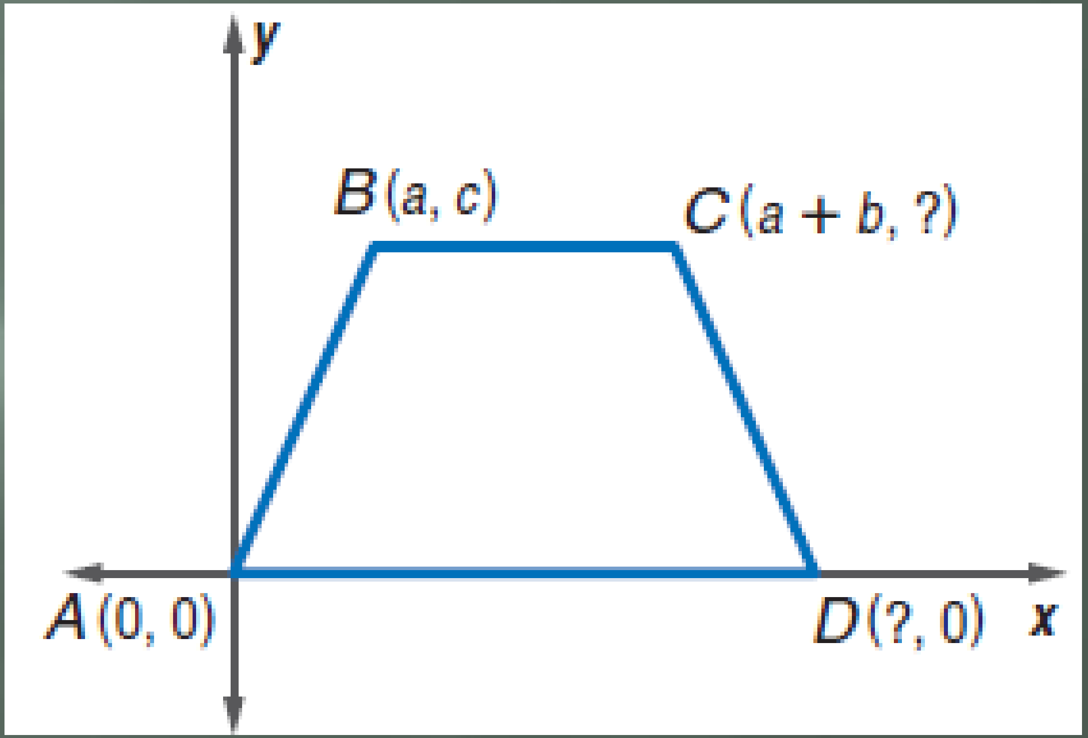
Examples

- Name the missing coordinates for the isosceles trapezoid.



Examples

- Name the missing coordinates for the isosceles trapezoid.
- Bases of a trapezoid are parallel, so *C* must have a y-value of *c*.
- The distance in the x-direction from *A* to *B* is *a*. Since the trapezoid is isosceles, the distance in the x-direction from *C* to *D* should also be a. The length of the top base is (*a* + *b*) – *a* or *b*, so the length from *A* to *D* is *a* + *b* + *a*, or 2a + b.
- The coordinates of *C* are (*a* + *b*, *c*).
- The coordinates of *D* are (2*a* + *b*, 0).



Coordinate Proof

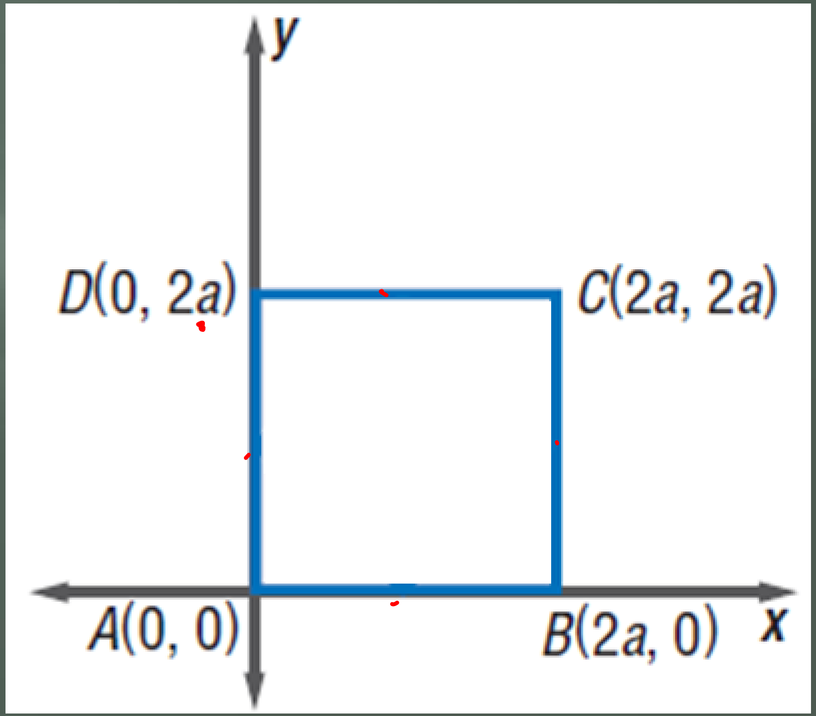
Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

Examples

- **Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.**

Examples

- **Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.**
- The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.



Examples

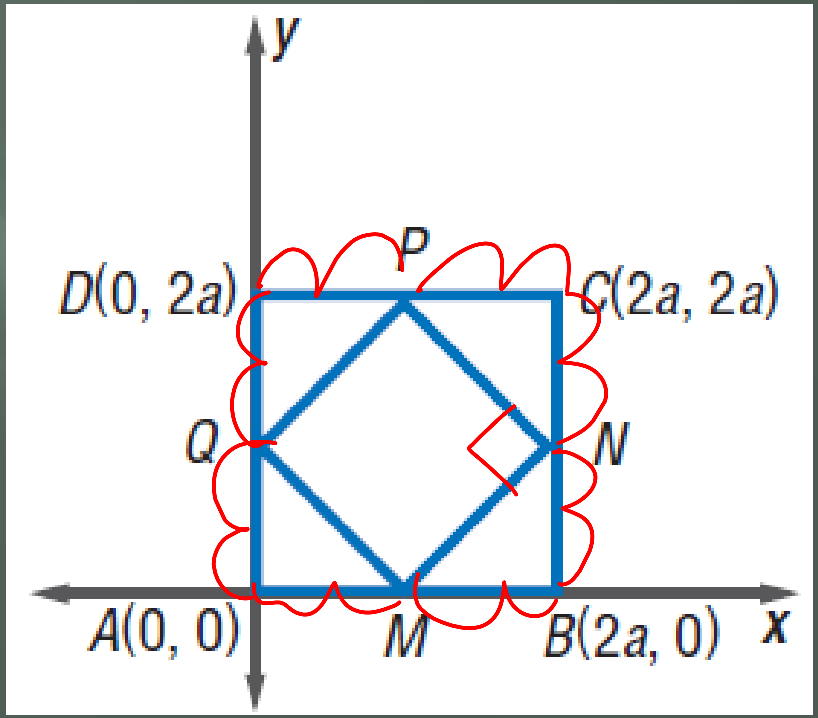
- **Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.**
- By the Midpoint Formula, the coordinates of *M*, *N*, *P*, and *Q* are as follows.

$$M\left(\frac{2a+0}{2}, \frac{0+0}{2}\right) = (a, 0)$$

$$N\left(\frac{2a+2a}{2}, \frac{2a+0}{2}\right) = (2a, a)$$

$$P\left(\frac{0+2a}{2}, \frac{2a+2a}{2}\right) = (a, 2a)$$

$$Q\left(\frac{0+0}{2}, \frac{0+2a}{2}\right) = (0, a)$$



Examples

- **Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.**

- Find the slopes of
- $\overline{QP}$, $\overline{MN}$, $\overline{QM}$, and $\overline{PN}$.

$$\text{slope of } \overline{QP} = \frac{2a - a}{a - 0} \text{ or } 1 \qquad \text{slope of } \overline{MN} = \frac{a - 0}{2a - a} \text{ or } 1$$

$$\text{slope of } \overline{QM} = \frac{0 - a}{a - 0} \text{ or } -1 \qquad \text{slope of } \overline{PN} = \frac{a - 2a}{2a - a} \text{ or } -1$$

- Each pair of opposite sides have the same slope, so they are parallel.
- Consecutive sides form right angles because their slopes are negative reciprocals.

Examples

- **Place a square on a coordinate plane. Label the midpoints of the sides, *M*, *N*, *P*, and *Q*. Write a coordinate proof to prove that *MNPQ* is a square.**

- Use the distance formula
- to find the lengths of
- $\overline{QP}$ and $\overline{QM}$

$$QP = \sqrt{(0-a)^2 + (a-2a)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

$$QM = \sqrt{(0-a)^2 + (a-0)^2} = \sqrt{a^2 + a^2} = \sqrt{2a^2} \text{ or } a\sqrt{2}$$

- *MNPQ* is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.