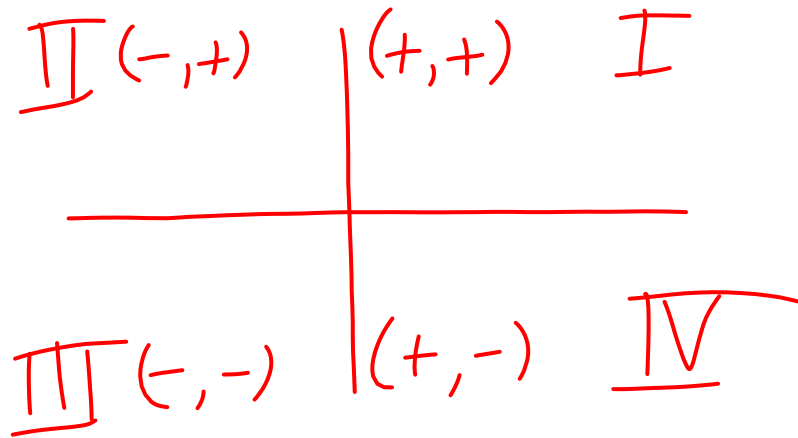


Coordinate Proofs with Triangles

Coordinate Proof



- ▶ **Coordinate proof** uses figures in the coordinate plane and algebra to prove geometric concepts.
- ▶ The first step in a coordinate proof is placing the figure on the coordinate plane.

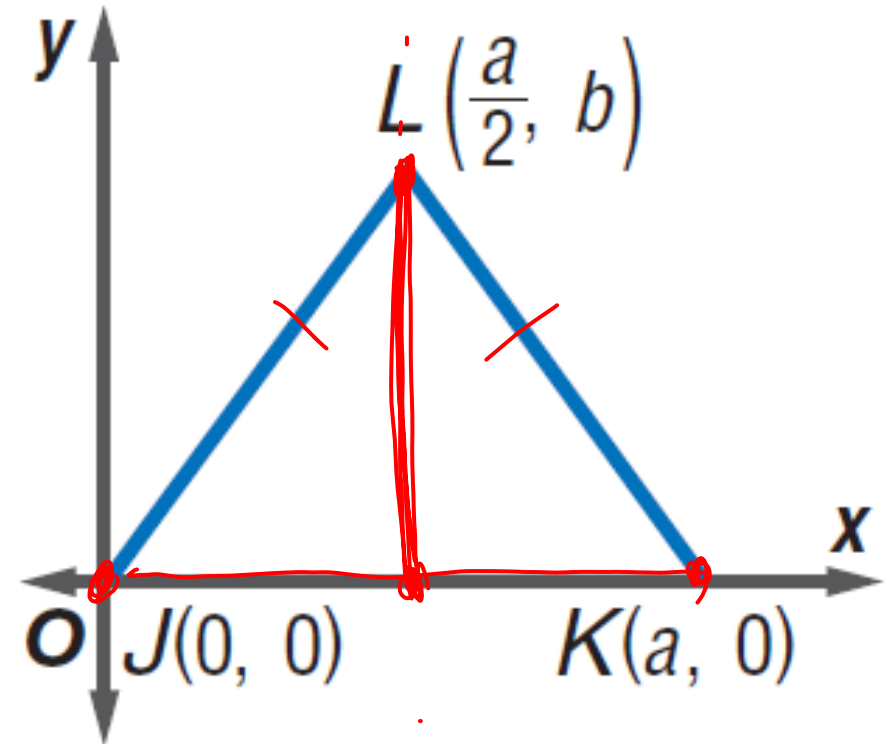
KEY CONCEPT

Placing Figures on the Coordinate Plane

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

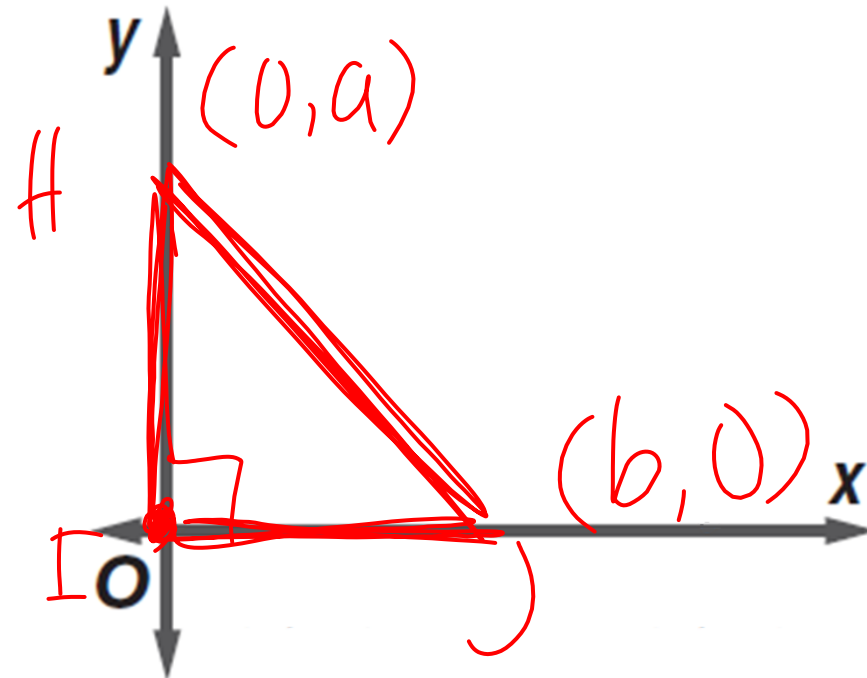
Examples

- ▶ Position and label isosceles triangle JKL on a coordinate plane so that base JK is a units long.
- ▶ Use the origin as vertex J of the triangle.
- ▶ Place the base of the triangle along the positive x -axis.
- ▶ Position the triangle in the first quadrant.
- ▶ Since K is on the x -axis, its y -coordinate is 0.
- ▶ Its x -coordinate is a because the base is a units long.
- ▶ JKL is isosceles, so the x -coordinate of L is halfway between 0 and a or $\frac{a}{2}$.
- ▶ We cannot write the y -coordinate in terms of a , so call it b .



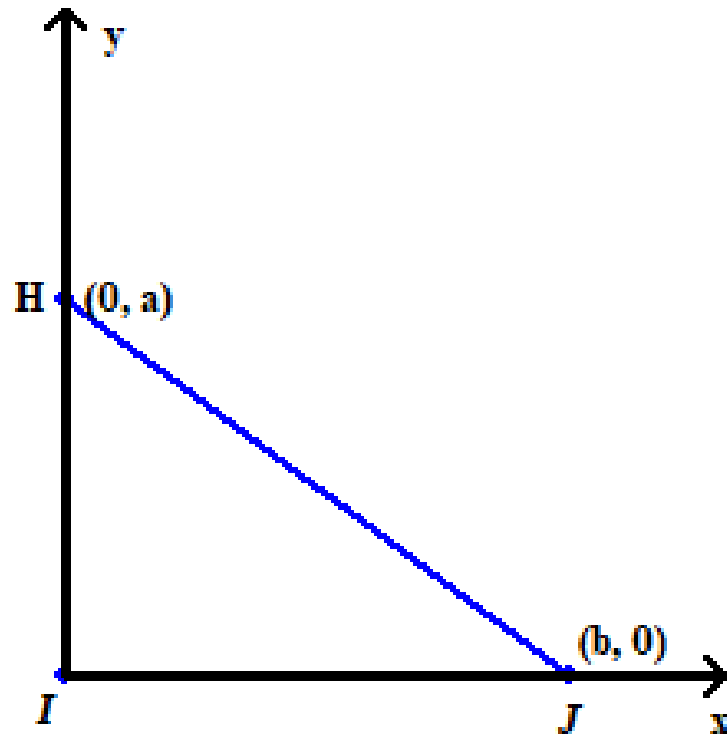
Examples

- ▶ Position and label right triangle HIJ with legs HI and IJ on a coordinate plane so that HI is a units long and IJ is b units long.



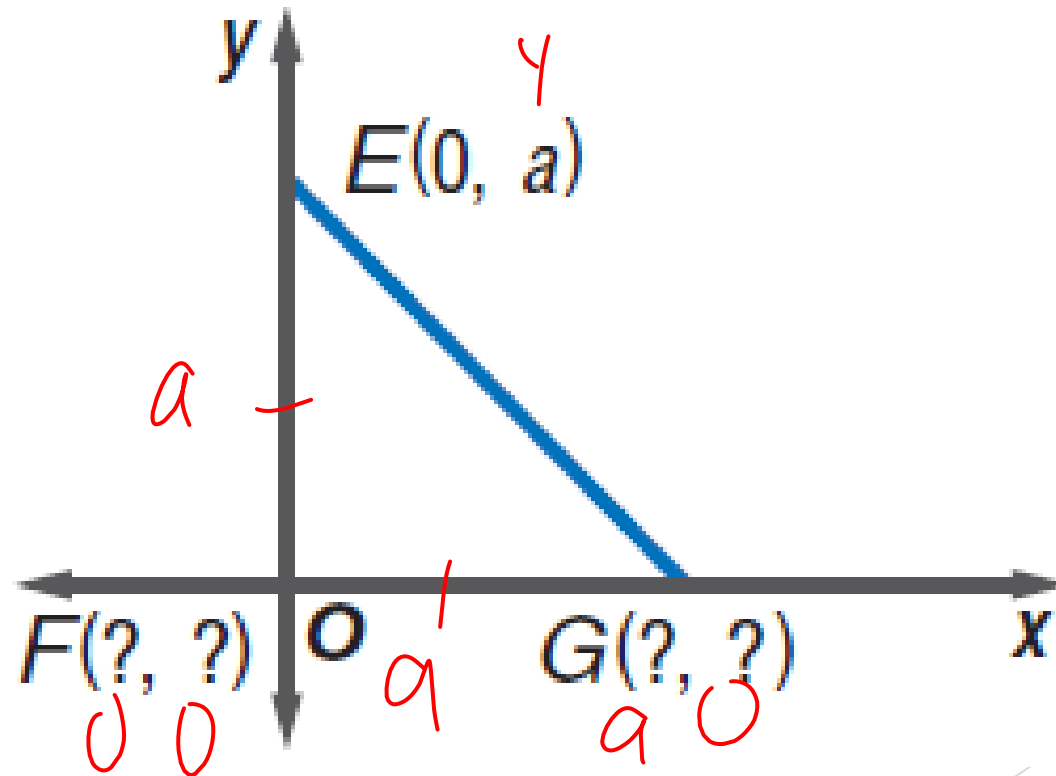
Examples

- ▶ Position and label right triangle HIJ with legs HI and IJ on a coordinate plane so that HI is a units long and IJ is b units long.



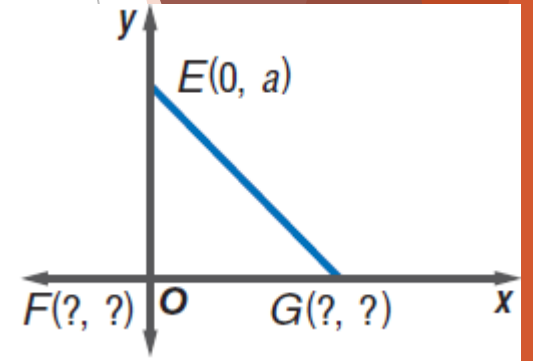
Examples

- ▶ Name the missing coordinates of isosceles right triangle EFG .



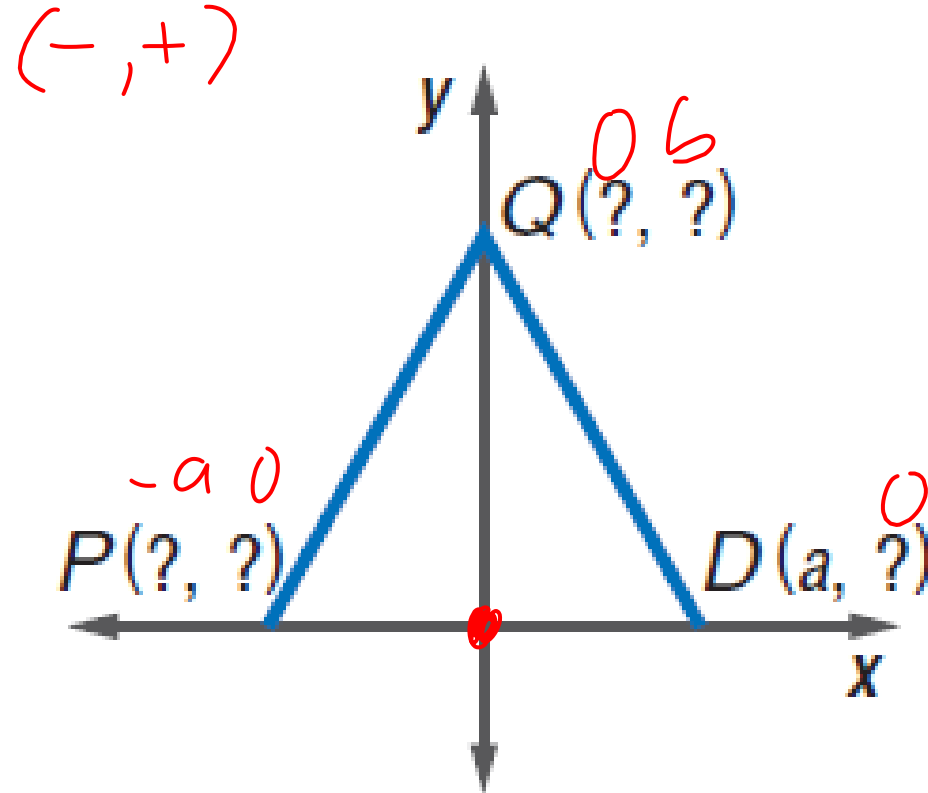
Examples

- ▶ Name the missing coordinates of isosceles right triangle EFG .
- ▶ Vertex F is positioned at the origin; its coordinates are $(0, 0)$.
- ▶ Vertex E is on the y -axis, and vertex G is on the x -axis.
- ▶ So EFG is a right angle. Since EFG is isosceles, $EF \cong GF$.
- ▶ EF is a units and GF must be the same. So, the coordinates of G are $(a, 0)$.



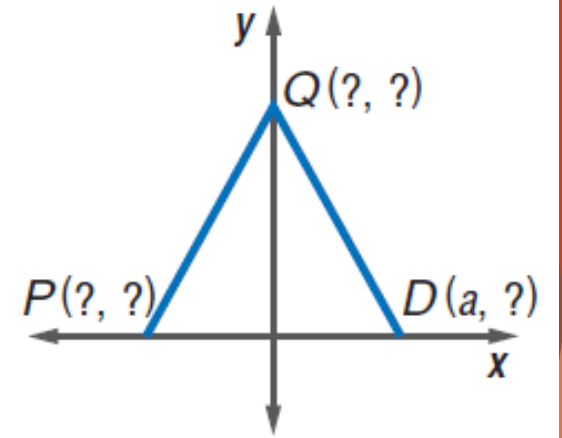
Examples

- ▶ Name the missing coordinates of isosceles triangle PDQ .



Examples

- ▶ Name the missing coordinates of isosceles triangle PDQ .
- ▶ Vertices P and D are positioned on the x -axis; their y -coordinates are 0.
- ▶ Since PDQ is isosceles, the distance from the origin to P is the same as the distance from the origin to D . So, the x -coordinate of P is $-a$. So the coordinates of P are $(-a, 0)$ and D are $(a, 0)$.
- ▶ Vertex Q is on the y -axis, so its x -coordinate is 0. We cannot write the y -coordinate in terms of a , so call it $b \rightarrow Q(0, b)$

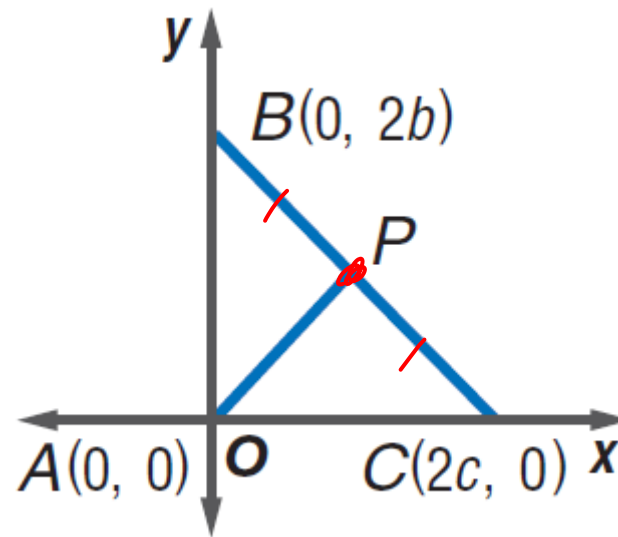


Writing Coordinate Proofs

- ▶ After a figure is placed on the coordinate plane and labeled, we can use coordinate proof to verify properties and to prove theorems.

Examples

- ▶ Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.
- ▶ Place the right angle at the origin and label it A . Use coordinates that are multiples of 2 because the Midpoint Formula takes half the sum of the coordinates.
- ▶ **Given:** right $\triangle ABC$ with right $\angle BAC$
- ▶ P is the midpoint of BC .
- ▶ **Prove:** $AP = \frac{1}{2}BC$



Examples

- ▶ Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.

▶ **Proof:**

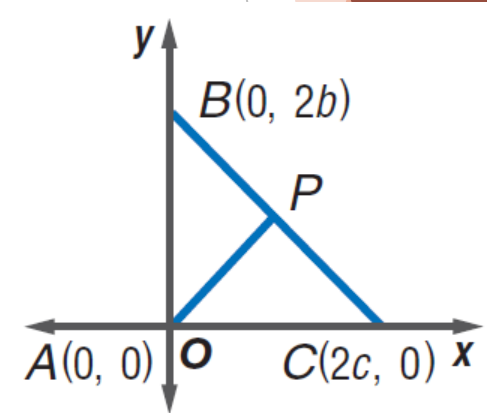
- ▶ By the Midpoint Formula, the coordinates of P are $(\frac{0+2c}{2}, \frac{2b+0}{2})$ or (c, b) .

- ▶ Use the Distance Formula to find AP and BC .

$$\begin{aligned} \text{▶ } AP &= \sqrt{(c - 0)^2 + (b - 0)^2} & BC &= \sqrt{(2c - 0)^2 + (0 - 2b)^2} \\ &= \sqrt{c^2 + b^2} & &= \sqrt{4c^2 + 4b^2} \text{ or } 2\sqrt{c^2 + b^2} \end{aligned}$$

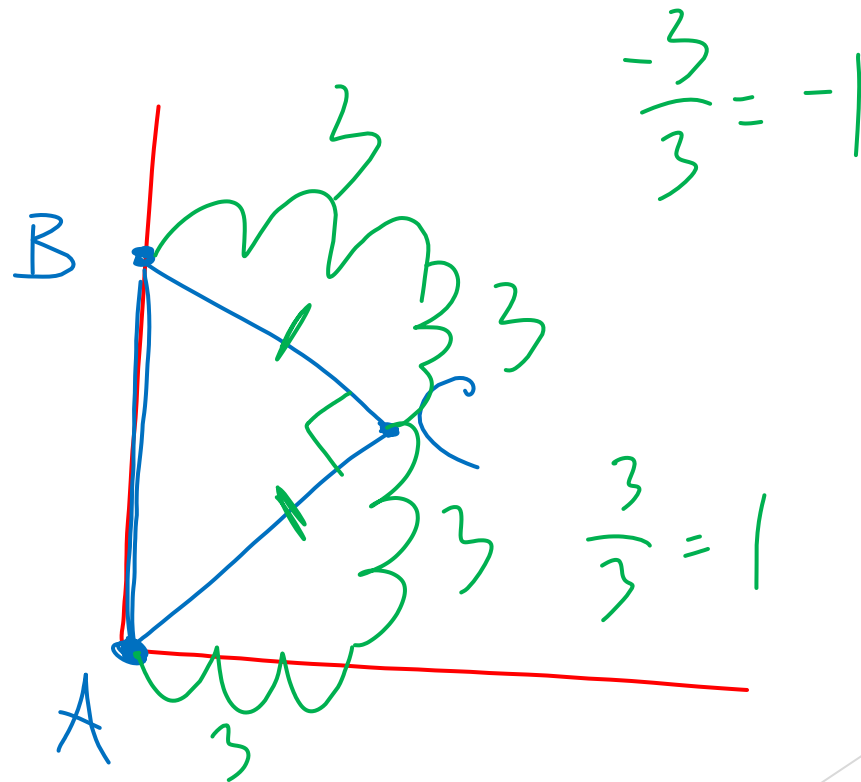
$$\frac{1}{2}BC = \sqrt{c^2 + b^2}$$

- ▶ Therefore, $AP = \frac{1}{2}BC$



Examples

- Use coordinate geometry to classify a triangle with vertices located at the following coordinates: $A(0, 0)$, $B(0, 6)$, and $C(3, 3)$.



~~Isosceles~~
right isosceles
~~Acute isosceles~~

Examples

- ▶ Use coordinate geometry to classify a triangle with vertices located at the following coordinates: $A(0, 0)$, $B(0, 6)$, and $C(3, 3)$.
- ▶ Proof:
- ▶ By the Distance Formula, $AB = 6$, BC moves 3×3 , and AC moves 3×3
- ▶ Since BC and AC are congruent, the triangle is isosceles.

