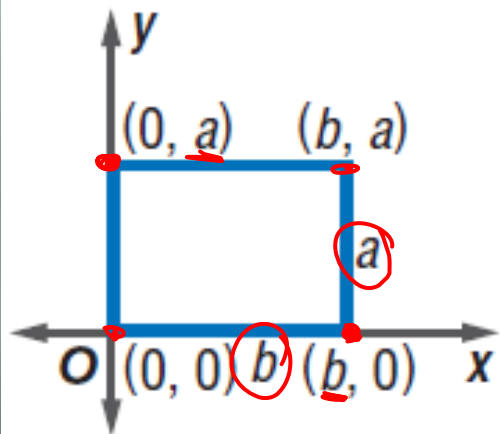


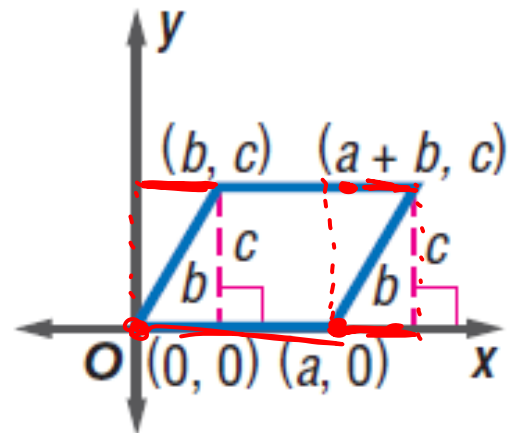
COORDINATE PROOFS WITH QUADRILATERALS

Positioning

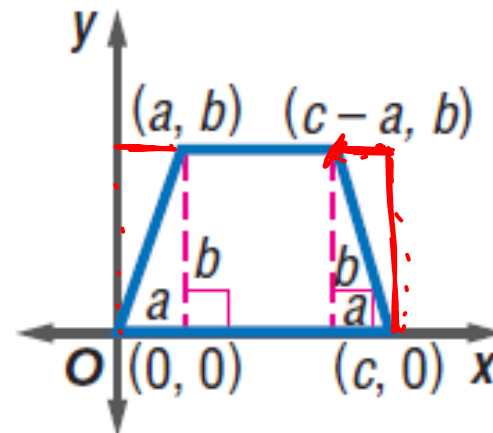
- The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.



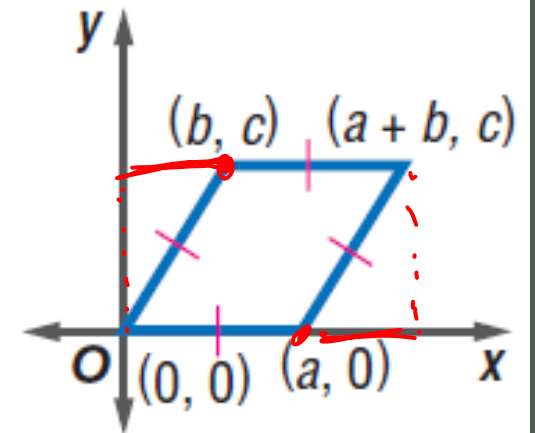
rectangle



parallelogram



isosceles trapezoid

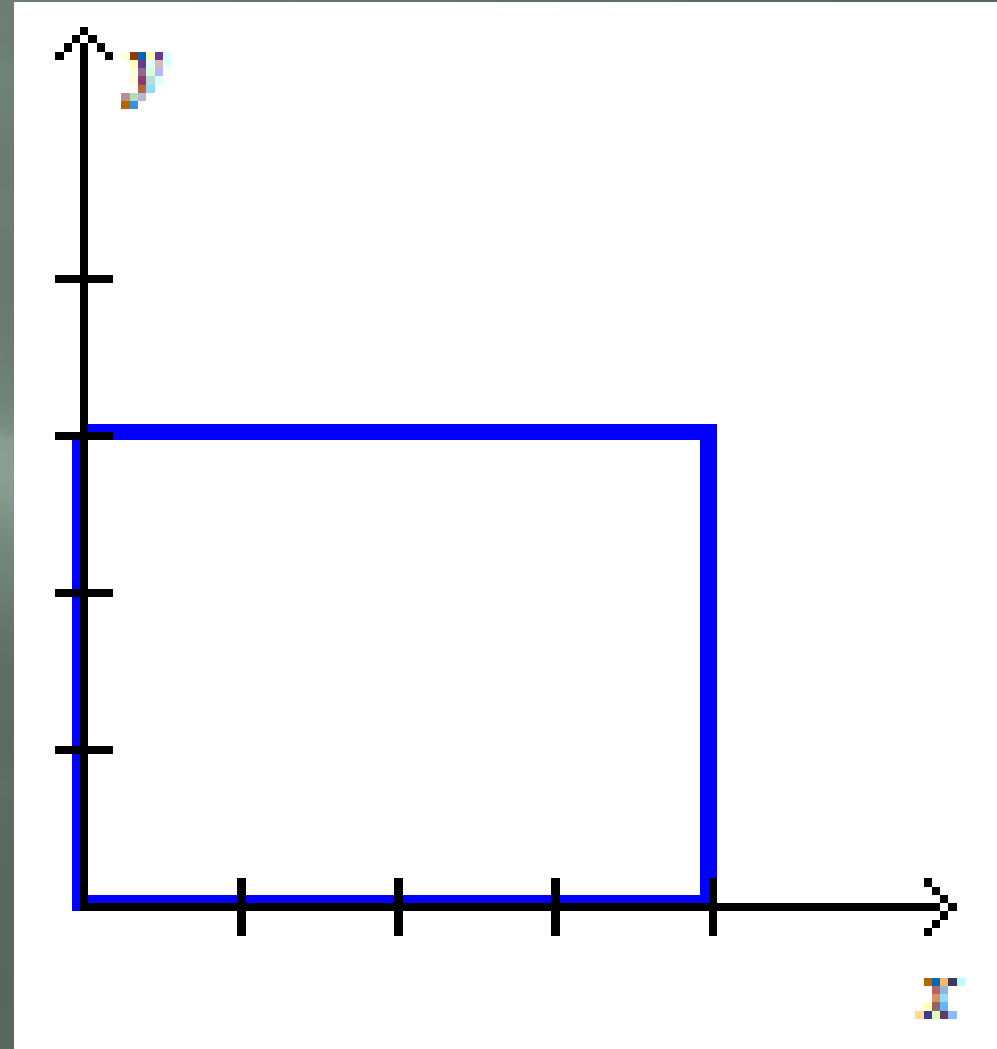


rhombus

Positioning

Let A , B , C , and D be vertices of a quadrilateral with a length of a and a height of b .

Place the quadrilateral with vertex A at the origin, AB along the positive x -axis, and AD along the y -axis. Label the vertices A , B , C , and D .

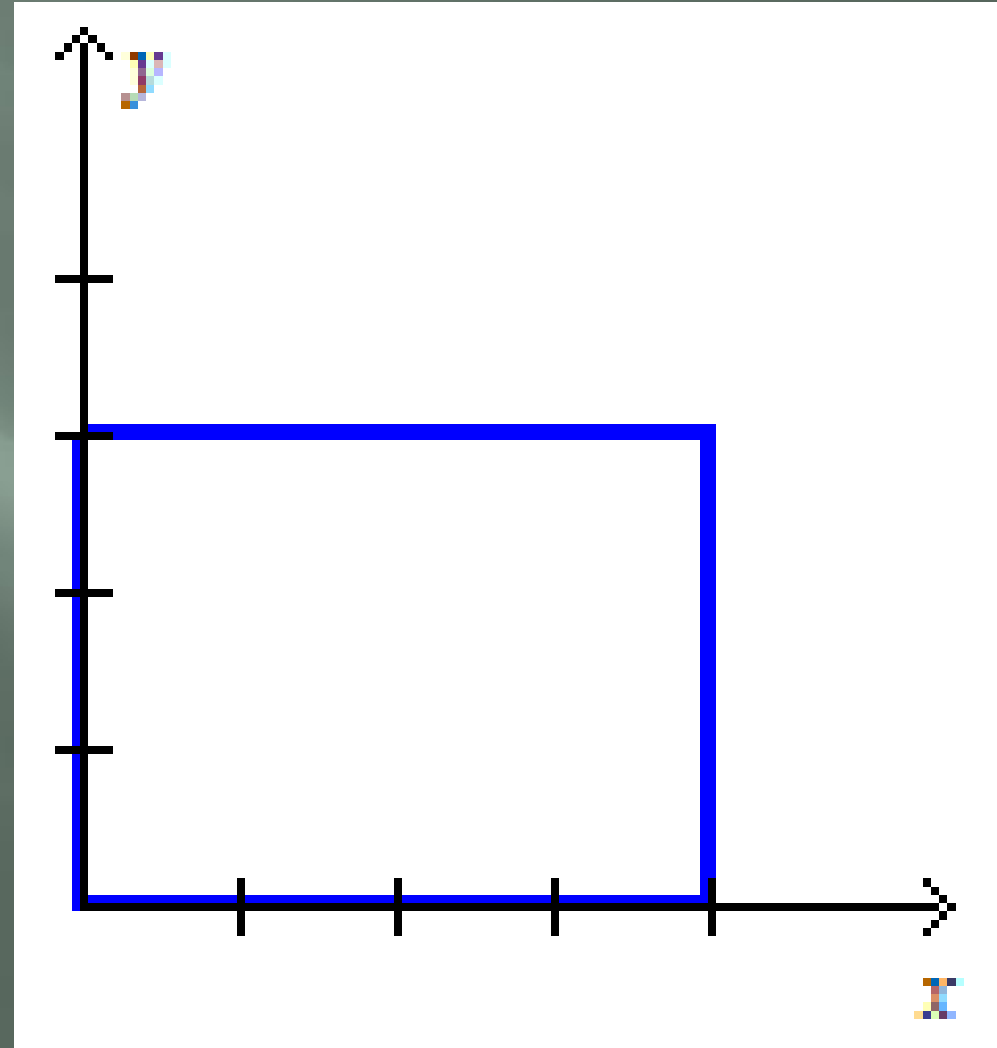


Positioning

The y -coordinate of B is 0 because the vertex is on the x -axis. Since the side length is a , the x -coordinate is a .

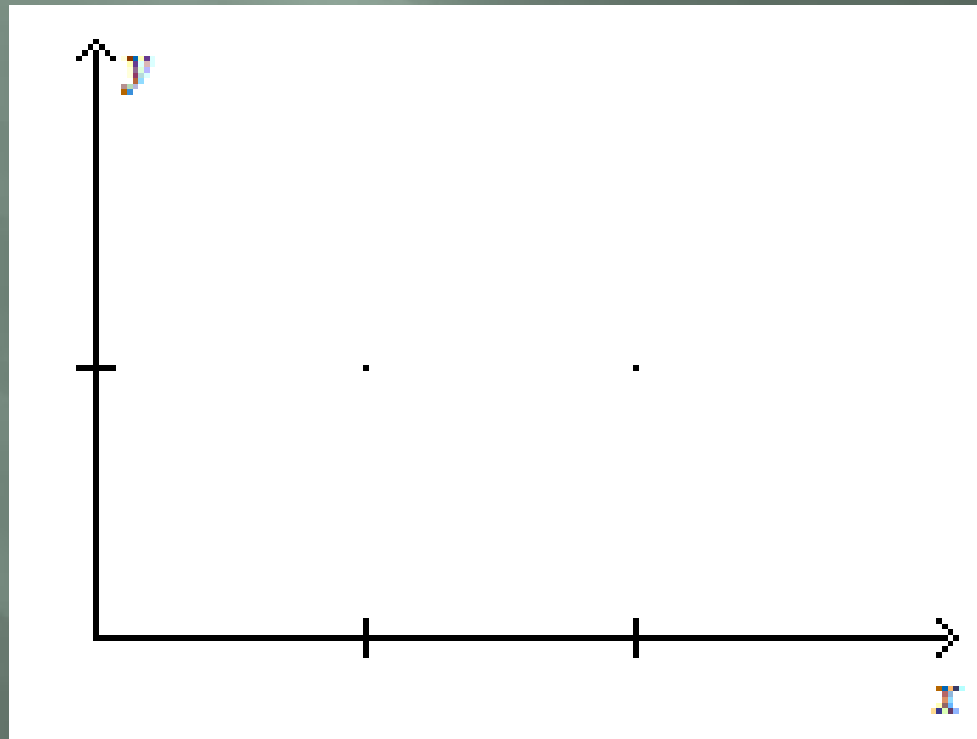
D is on the y -axis so the x -coordinate is 0. The y -coordinate is $0 + b$ or b .

The x -coordinate of C is also a . The y -coordinate is $0 + b$ or b because the side BC is b units long.



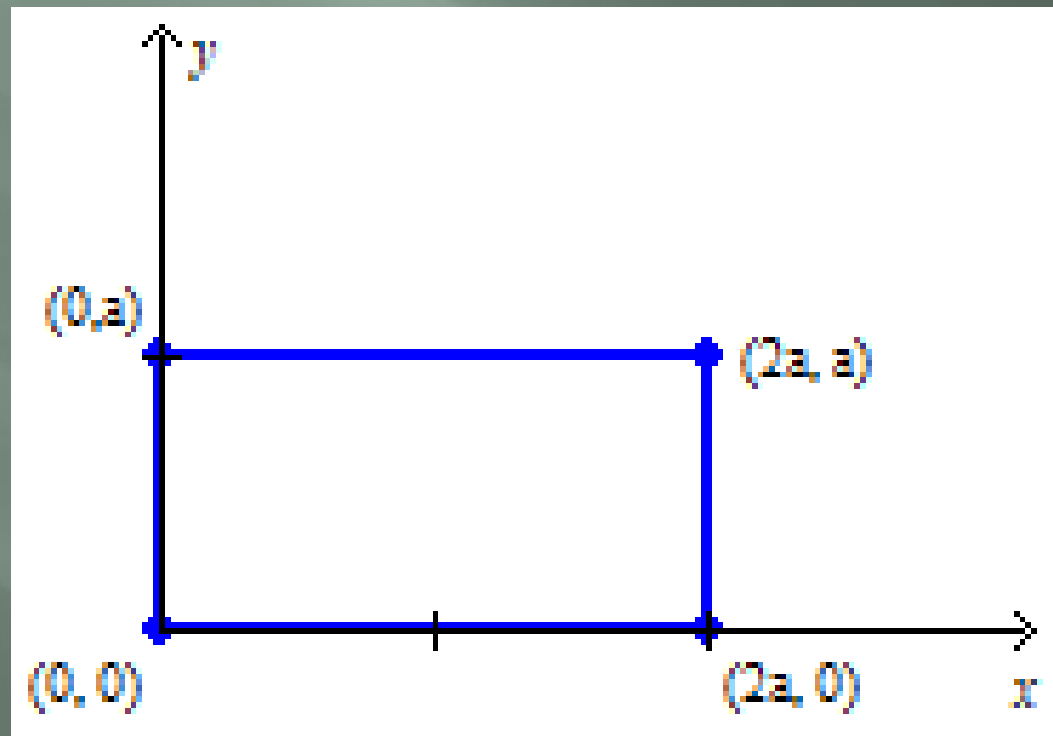
Examples

- Position and label a rectangle with a length of $2a$ units and a width of a units.



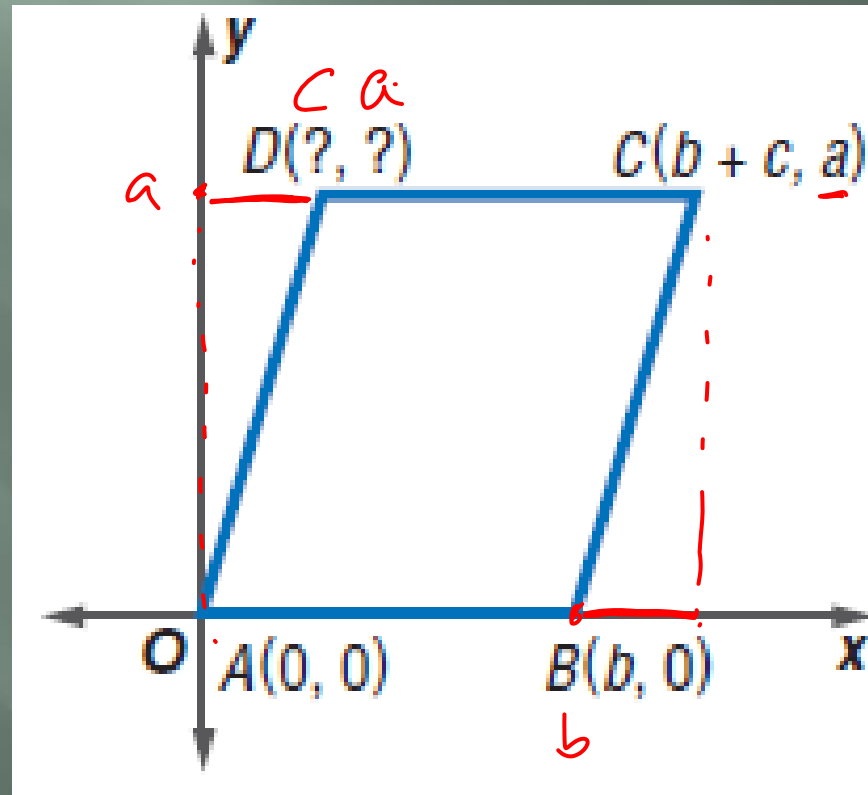
Examples

- Position and label a rectangle with a length of $2a$ units and a width of a units.



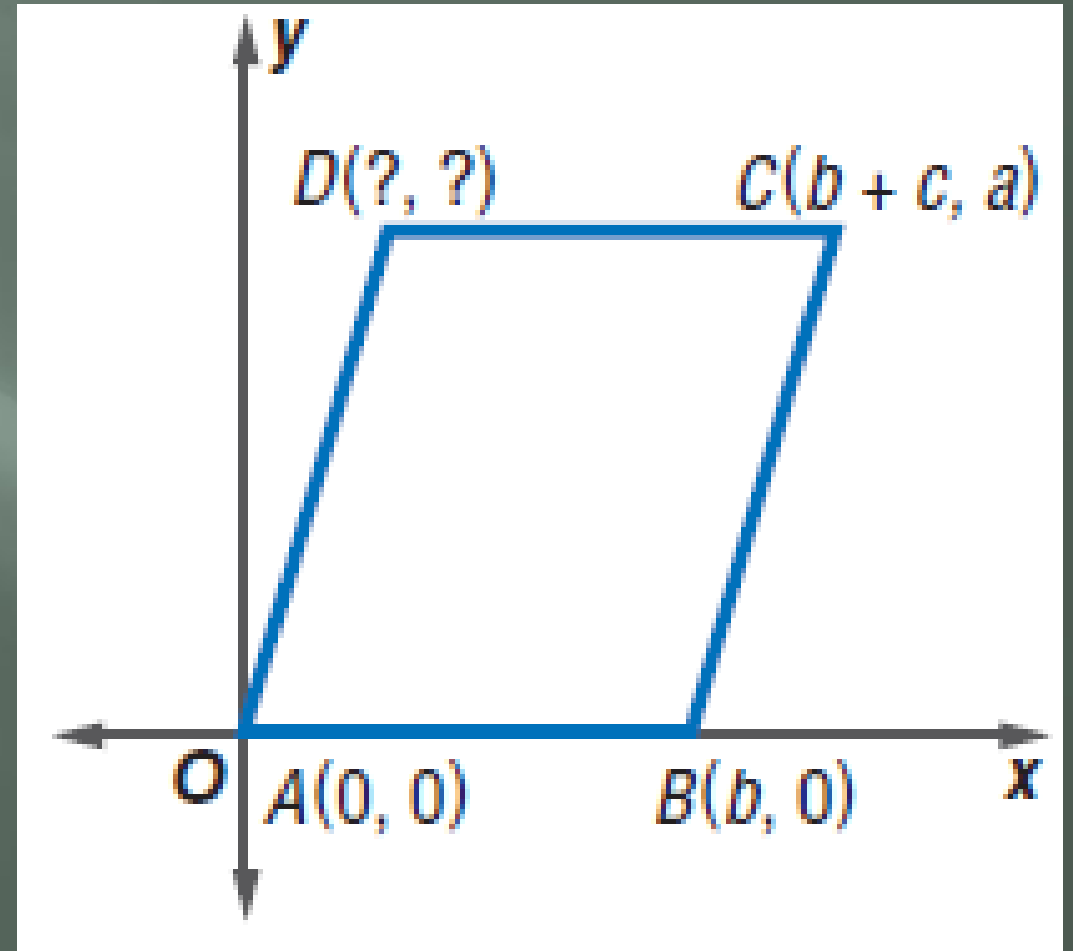
Examples

- ▣ Name the missing coordinates for the parallelogram.



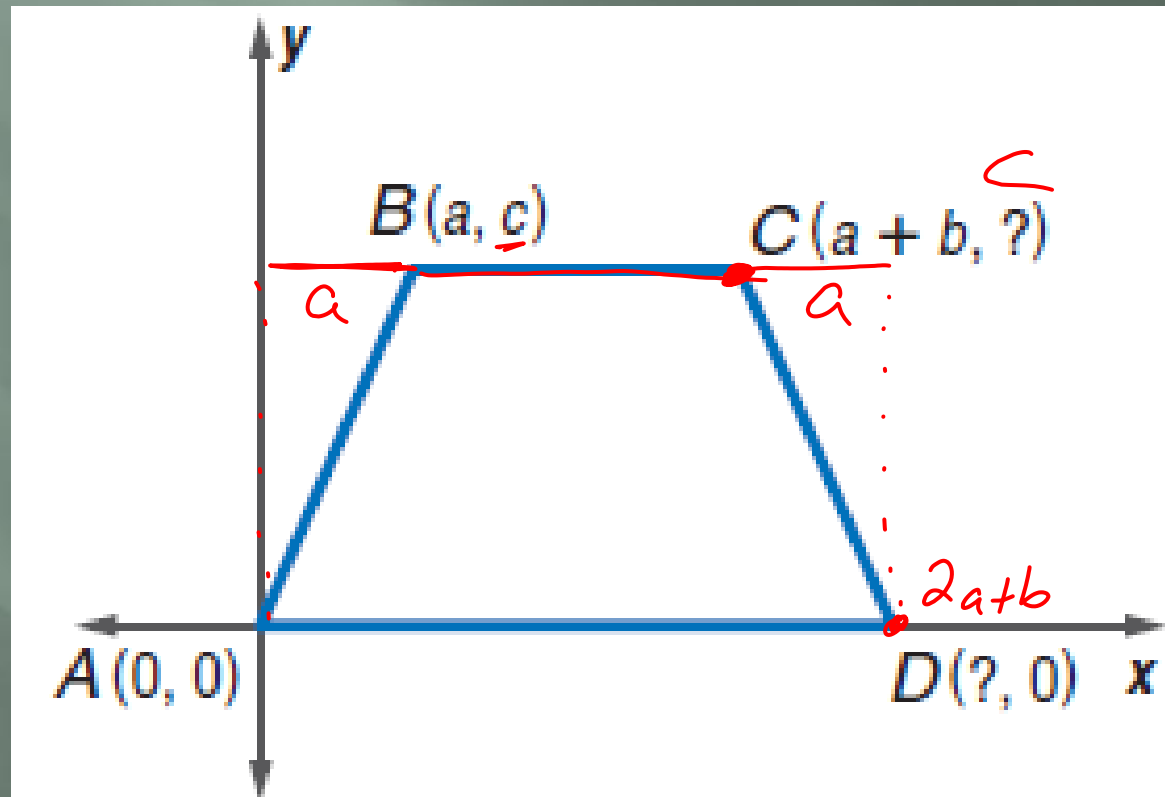
Examples

- ▣ Name the missing coordinates for the parallelogram.
- ▣ Opposite sides of a parallelogram are congruent and parallel. So, the y -coordinate of D is a .
- ▣ The length of AB is b , and the length of DC is b .
- ▣ So, the x -coordinate of D is $(b + c) - b$ or c .
- ▣ The coordinates of D are (c, a) .



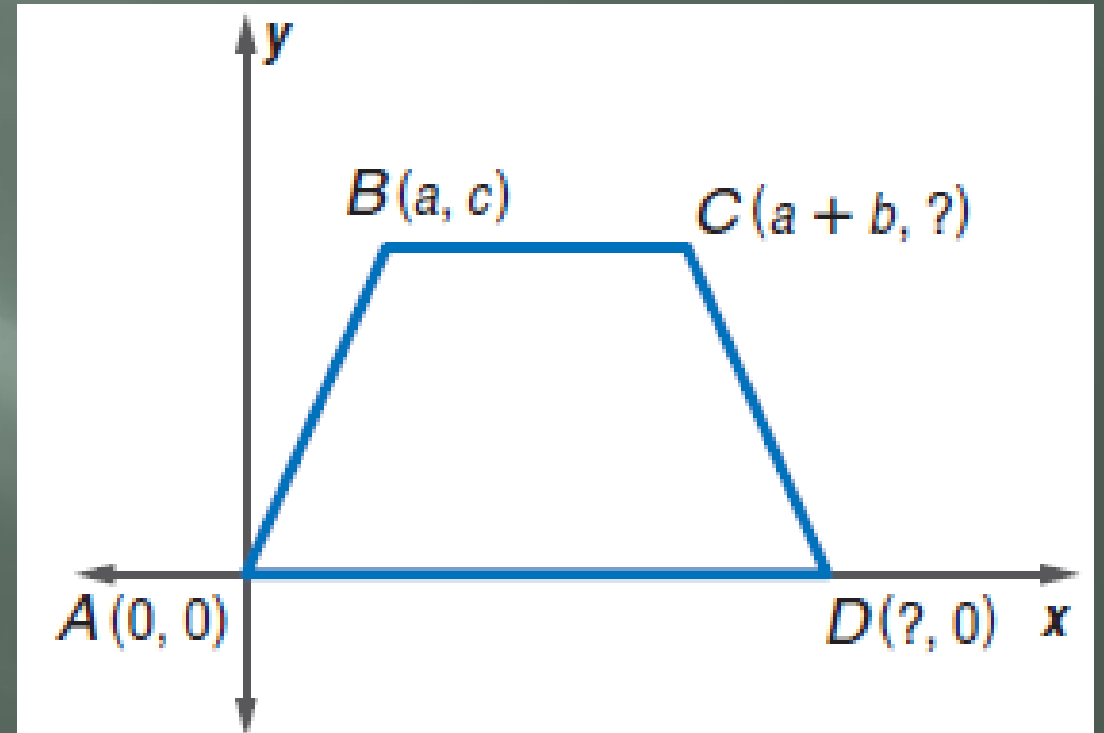
Examples

- ▣ Name the missing coordinates for the isosceles trapezoid.



Examples

- Name the missing coordinates for the isosceles trapezoid.
- Bases of a trapezoid are parallel, so C must have a y -value of c .
- The distance in the x -direction from A to B is a . Since the trapezoid is isosceles, the distance in the x -direction from C to D should also be a . The length of the top base is $(a + b) - a$ or b , so the length from A to D is $a + b + a$, or $2a + b$.
- The coordinates of C are $(a + b, c)$.
- The coordinates of D are $(2a + b, 0)$.



Coordinate Proof

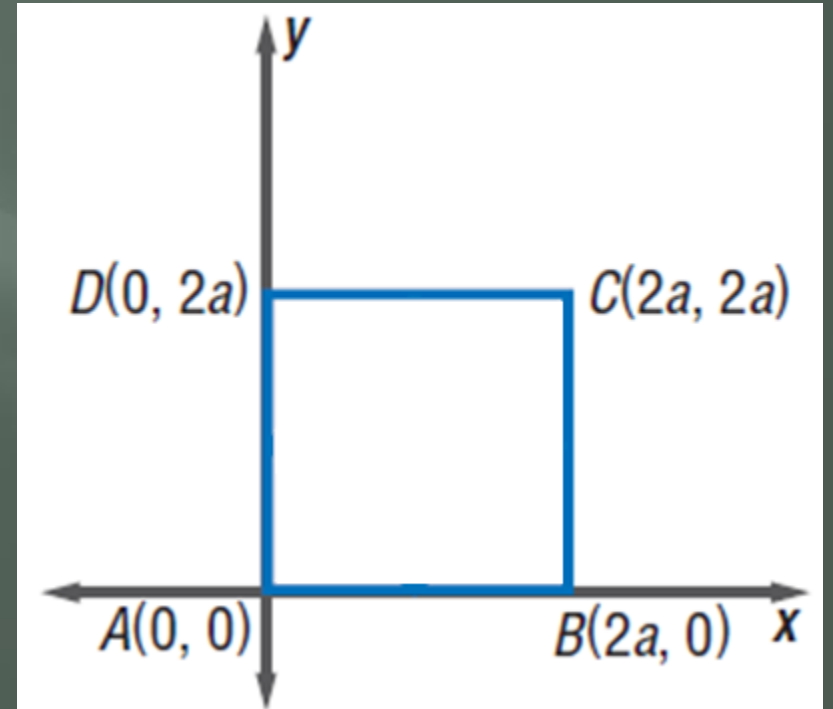
Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

Examples

- ▣ Place a square on a coordinate plane. Label the midpoints of the sides, M , N , P , and Q . Write a coordinate proof to prove that $MNPQ$ is a square.

Examples

- ▣ Place a square on a coordinate plane. Label the midpoints of the sides, M , N , P , and Q . Write a coordinate proof to prove that $MNPQ$ is a square.
- ▣ The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.



Examples

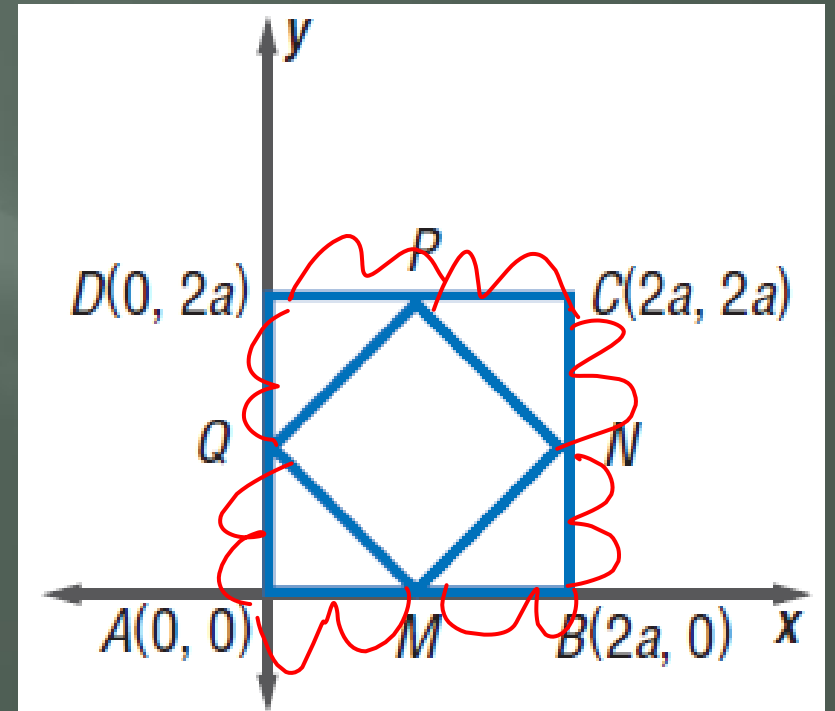
- Place a square on a coordinate plane. Label the midpoints of the sides, M , N , P , and Q . Write a coordinate proof to prove that $MNPQ$ is a square.
- By the Midpoint Formula, the coordinates of M , N , P , and Q are as follows.

$$M\left(\frac{2a + 0}{2}, \frac{0 + 0}{2}\right) = (a, 0)$$

$$N\left(\frac{2a + 2a}{2}, \frac{2a + 0}{2}\right) = (2a, a)$$

$$P\left(\frac{0 + 2a}{2}, \frac{2a + 2a}{2}\right) = (a, 2a)$$

$$Q\left(\frac{0 + 0}{2}, \frac{0 + 2a}{2}\right) = (0, a)$$



Examples

- Place a square on a coordinate plane. Label the midpoints of the sides, M , N , P , and Q . Write a coordinate proof to prove that $MNPQ$ is a square.

- Find the slopes of \overline{QP} , \overline{MN} , \overline{QM} , and \overline{PN} .

$$\begin{array}{ll} \text{slope of } \overline{QP} = \frac{2a - a}{a - 0} \text{ or } 1 & \text{slope of } \overline{MN} = \frac{a - 0}{2a - a} \text{ or } 1 \\ \text{slope of } \overline{QM} = \frac{0 - a}{a - 0} \text{ or } -1 & \text{slope of } \overline{PN} = \frac{a - 2a}{2a - a} \text{ or } -1 \end{array}$$

- Each pair of opposite sides have the same slope, so they are parallel.
- Consecutive sides form right angles because their slopes are negative reciprocals.

Examples

- Place a square on a coordinate plane. Label the midpoints of the sides, M , N , P , and Q . Write a coordinate proof to prove that $MNPQ$ is a square.

- Use the distance formula to find the lengths of \overline{QP} and \overline{QM}

$$\begin{aligned} QP &= \sqrt{(0 - a)^2 + (a - 2a)^2} & QM &= \sqrt{(0 - a)^2 + (a - 0)^2} \\ &= \sqrt{a^2 + a^2} & &= \sqrt{a^2 + a^2} \\ &= \sqrt{2a^2} \text{ or } a\sqrt{2} & &= \sqrt{2a^2} \text{ or } a\sqrt{2} \end{aligned}$$

- $MNPQ$ is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.