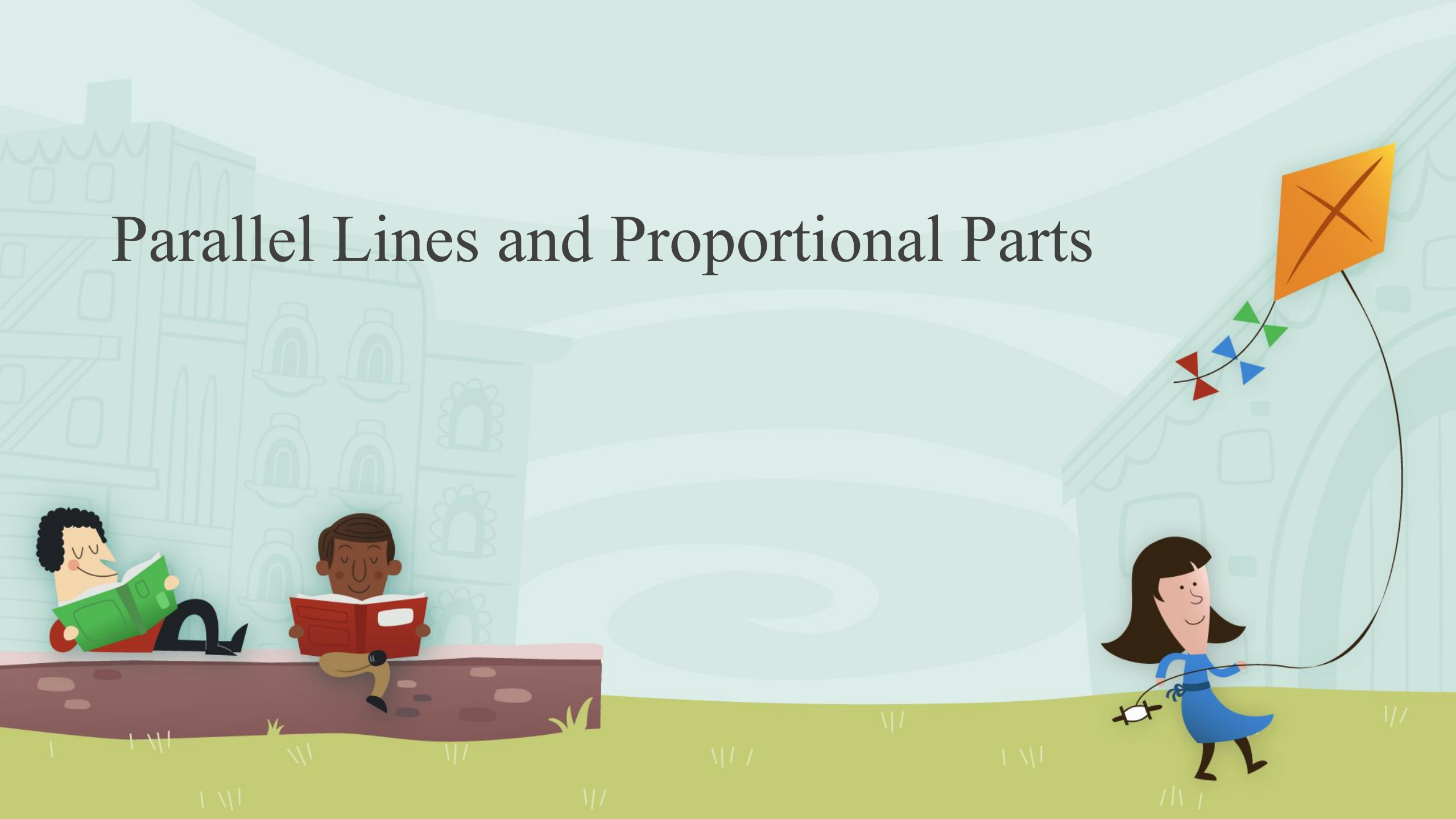
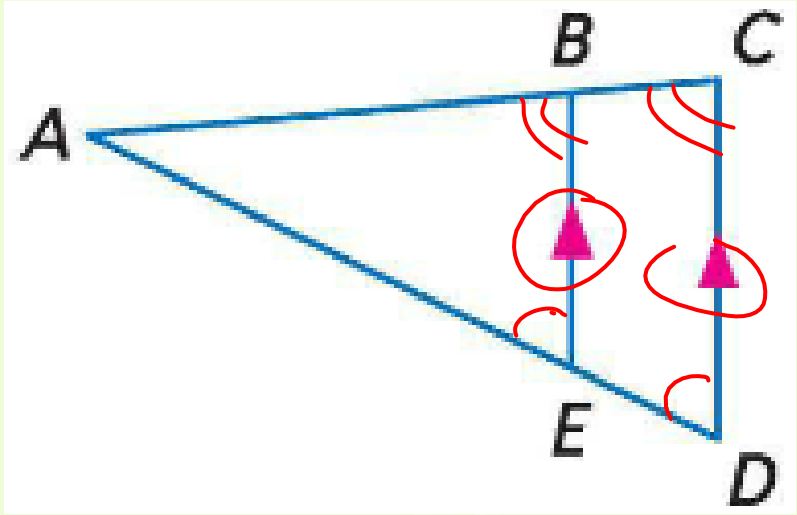
Parallel Lines and Proportional Parts



Triangle Proportionality Theorem

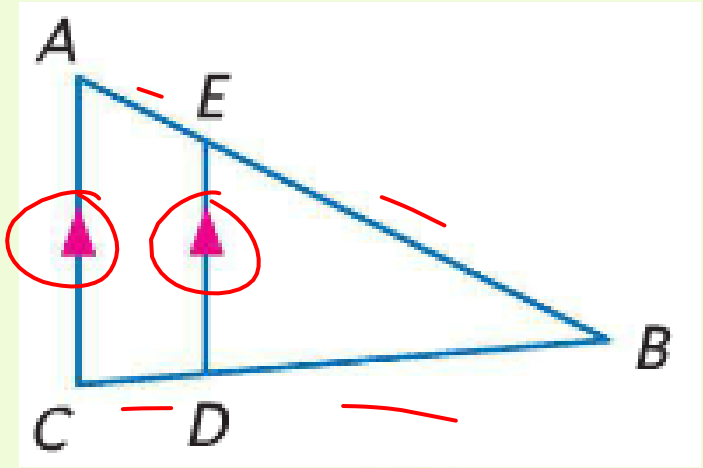
- If a line is parallel to one side of a triangle and intersects the other two sides, then it divides the sides into segments of proportional lengths.



If $\overline{BE} \parallel \overline{CD}$, then $\frac{AB}{BC} = \frac{AE}{ED}$.

Converse of Triangle Proportionality Theorem

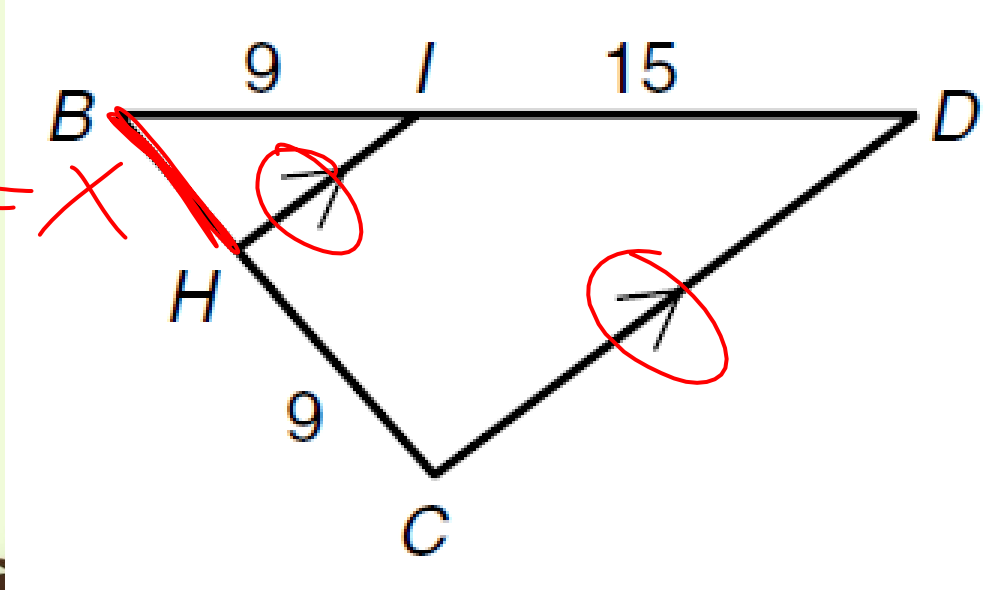
- If a line intersects two sides of a triangle and separates the sides into proportional corresponding segments, then the line is parallel to the third side of the triangle.



If $\frac{AE}{EB} = \frac{CD}{DB}$, then $\overline{AC} \parallel \overline{ED}$.

Examples

- Find the length of BH.



$5.4 = x$

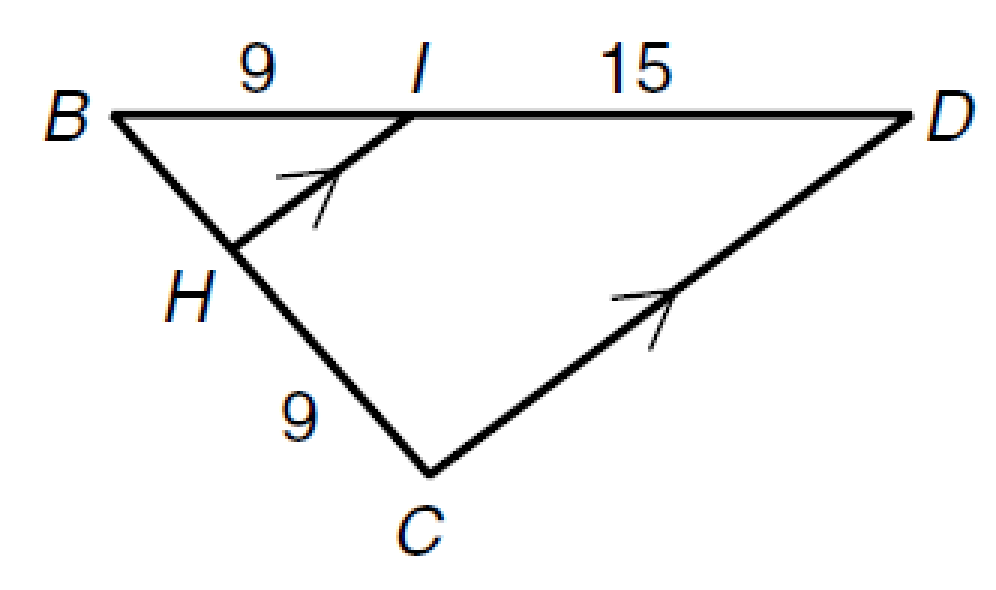
$\frac{x}{9} = \frac{9}{15}$

$\frac{15x}{15} = \frac{81}{15}$

$x = 5.4$

Examples

- Find the length of BH.



$\frac{x}{9} = \frac{9}{15}$

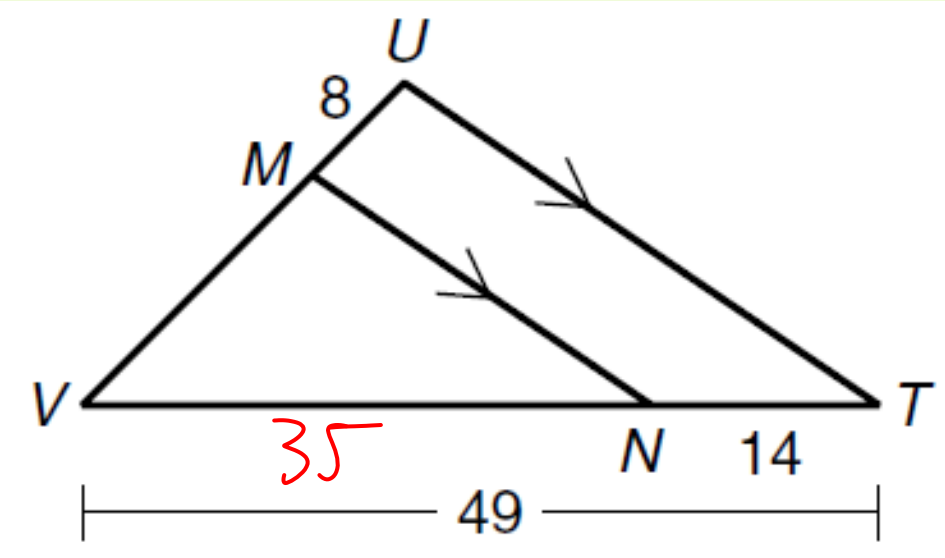
$15x = 81$

$x = \frac{81}{15}$

$x = 5.4$

Examples

- Find the length of MV.



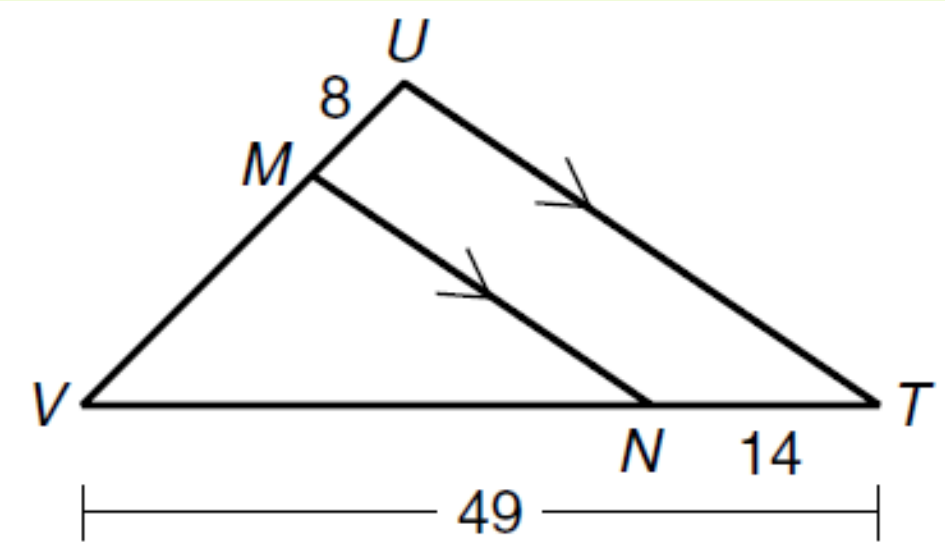
$$\frac{x}{35} = \frac{8}{14}$$

$$\frac{14x}{14} = \frac{280}{14}$$

$$x = 20$$

Examples

- Find the length of MV.



$$\frac{x}{8} = \frac{35}{14}$$

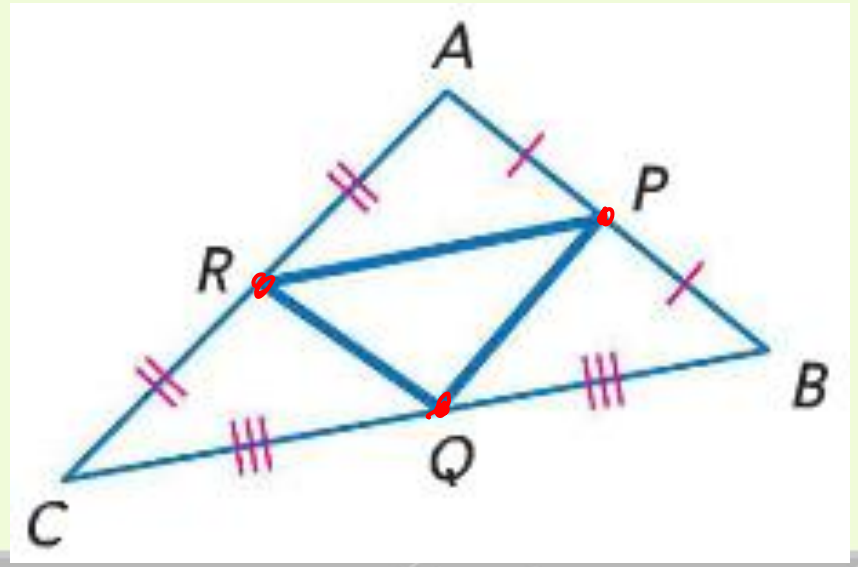
$14x = 280$

$$x = \frac{280}{14}$$

$x = 20$

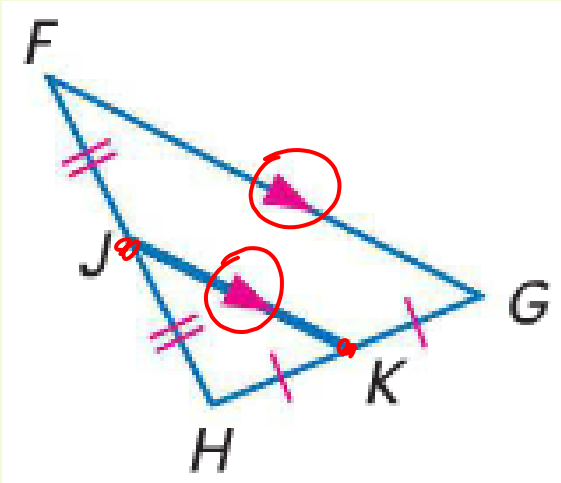
Midsegment

- A midsegment of a triangle is a segment with endpoints that are the midpoints of two sides of the triangle.
- Every triangle has three midsegments.



Triangle Midsegment Theorem

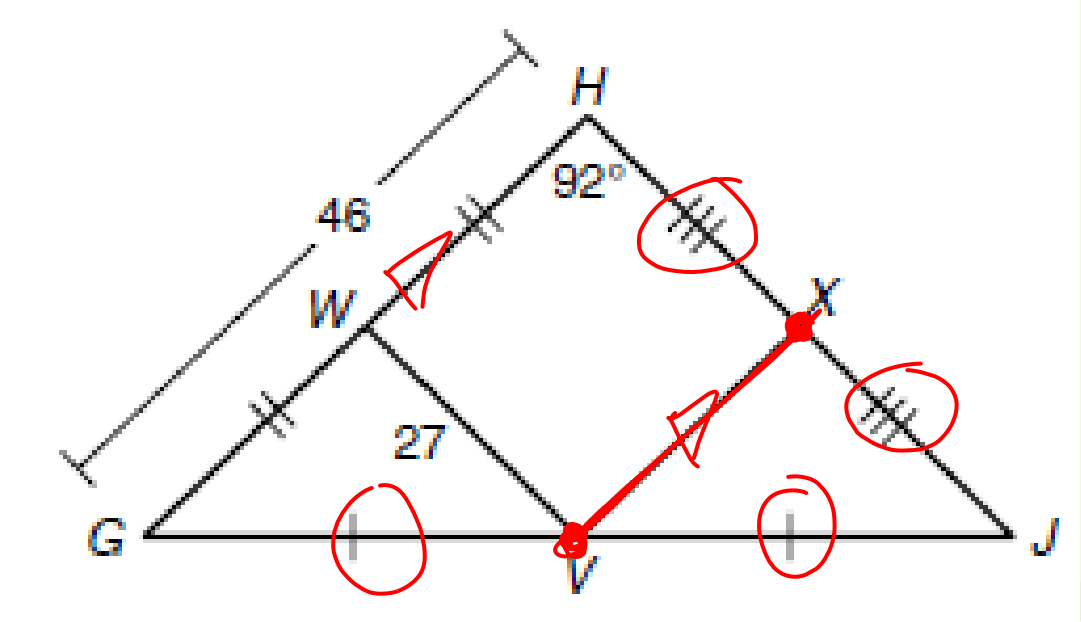
- A midsegment of a triangle is parallel to one side of the triangle, and its length is one half the length of that side.



If J and K are midpoints of $\overline{FH}$ and $\overline{HG}$, respectively, then $\overline{JK} \parallel \overline{FG}$ and $JK = \frac{1}{2}FG$.

Examples

- Find the measure of VX.

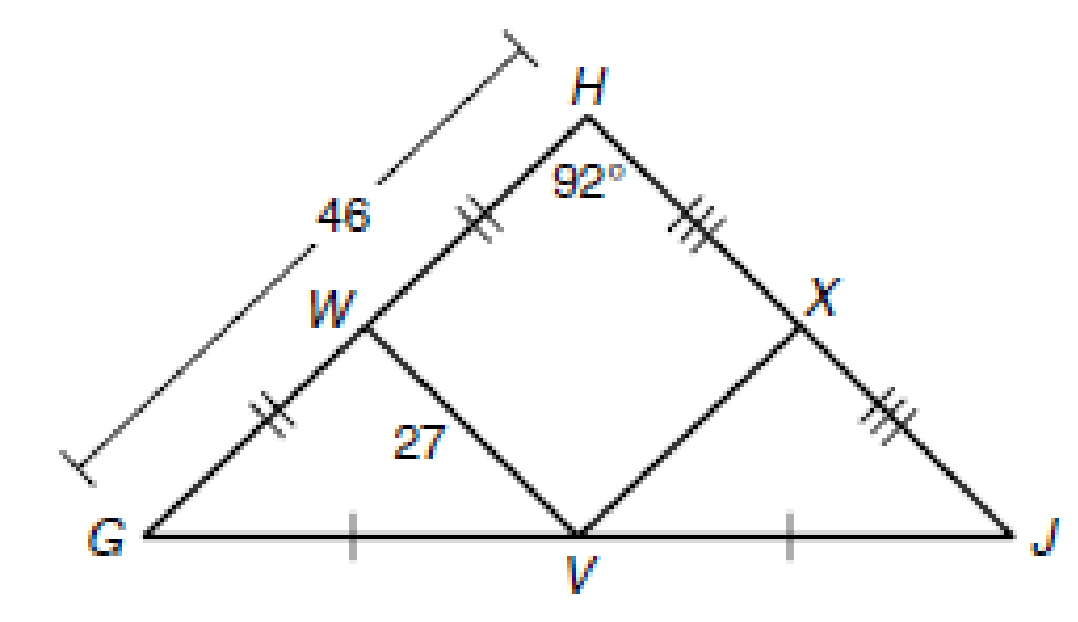


VX = ~~46~~

= 23

Examples

- Find the measure of VX.



$VX = \frac{1}{2}GH$

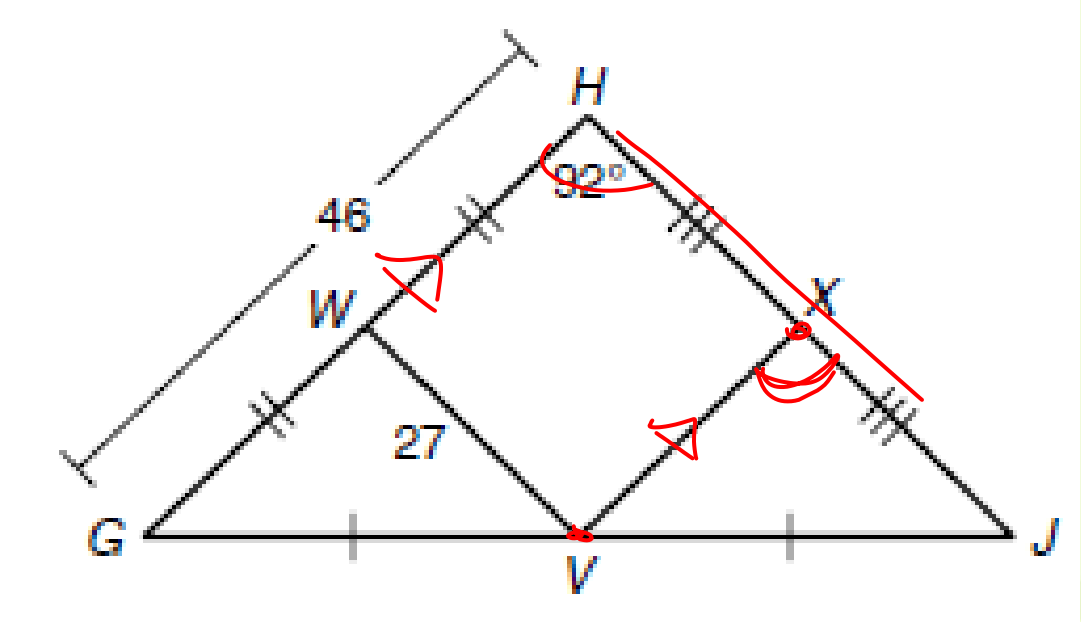
$VX = \frac{1}{2}(46)$

$VX = 23$

$\angle VXJ = 92$

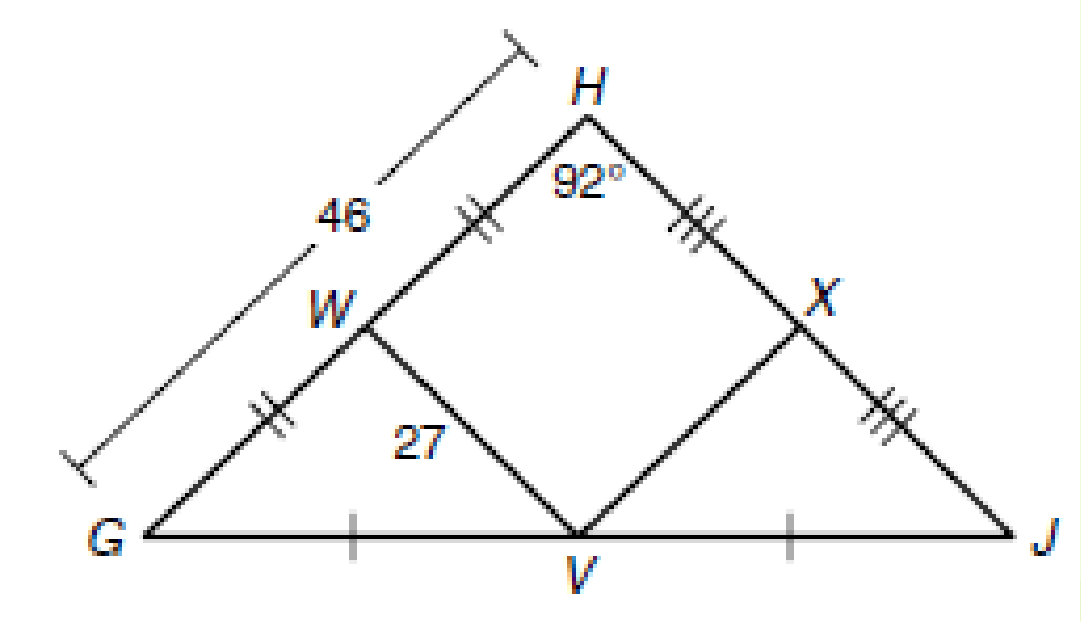
Examples

- Find the measure of m$\angle VXJ$.



Examples

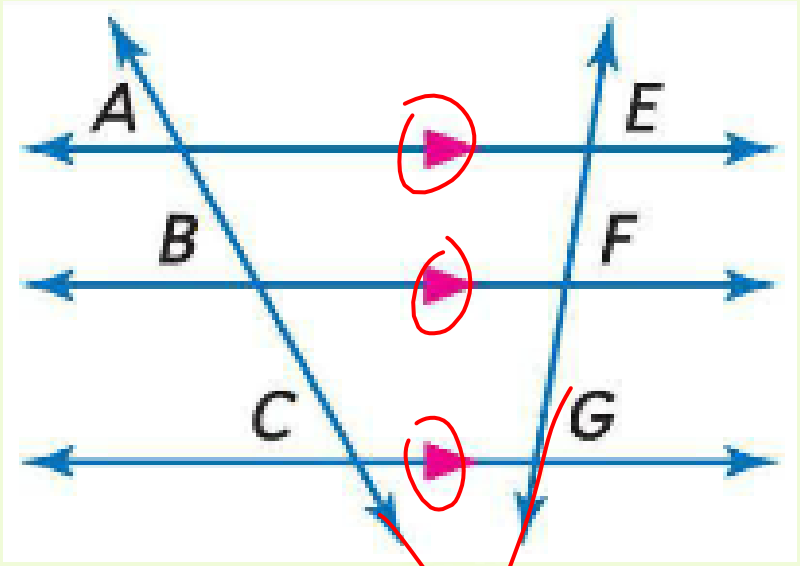
- Find the measure of m$\angle VXJ$.



Since midsegments are parallel to the sides of the triangle, then m$\angle VXJ$ has to be equal to m$\angle WHX$ since they are corresponding angles.

Proportional Parts of Parallel Lines

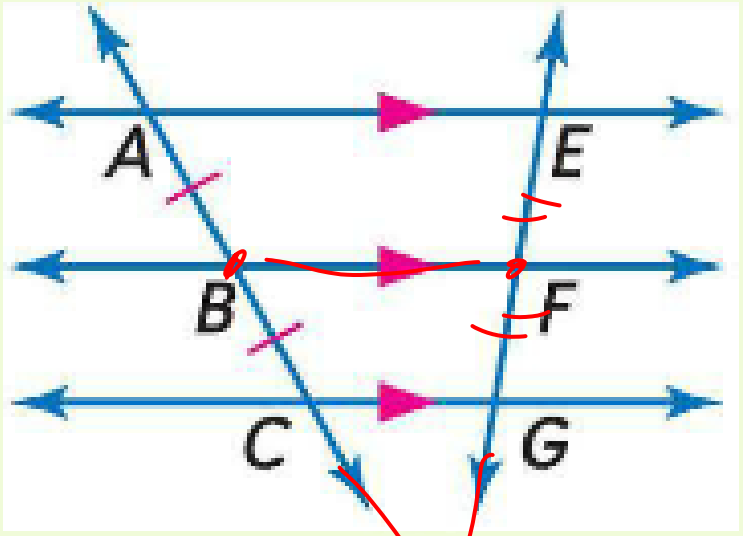
- If three or more parallel lines intersect two transversals, then they cut off the transversals proportionally.



If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, then $\frac{AB}{BC} = \frac{EF}{FG}$.

Congruent Parts of Parallel Lines

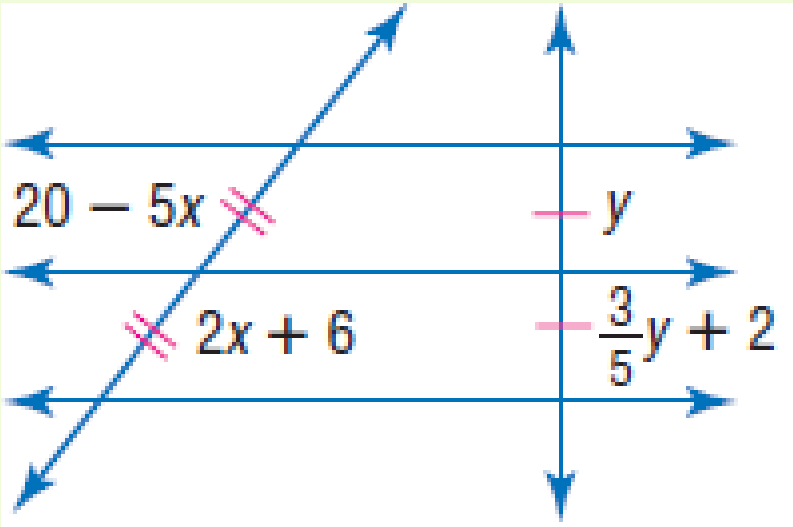
- If three or more parallel lines cut off congruent segments on one transversal, then they cut off congruent segments on every transversal.



If $\overline{AE} \parallel \overline{BF} \parallel \overline{CG}$, and $\overline{AB} \cong \overline{BC}$, then $\overline{EF} \cong \overline{FG}$.

Examples

- Find x and y.



$20 - 5x = 2x + 6$

$-6 \quad +5x \quad +5x \quad +6$

$\frac{14}{7} = \frac{7x}{7}$

$2 = x$

$y = \frac{3}{5}y + 2$ 5.

$5y = 3y + 10$

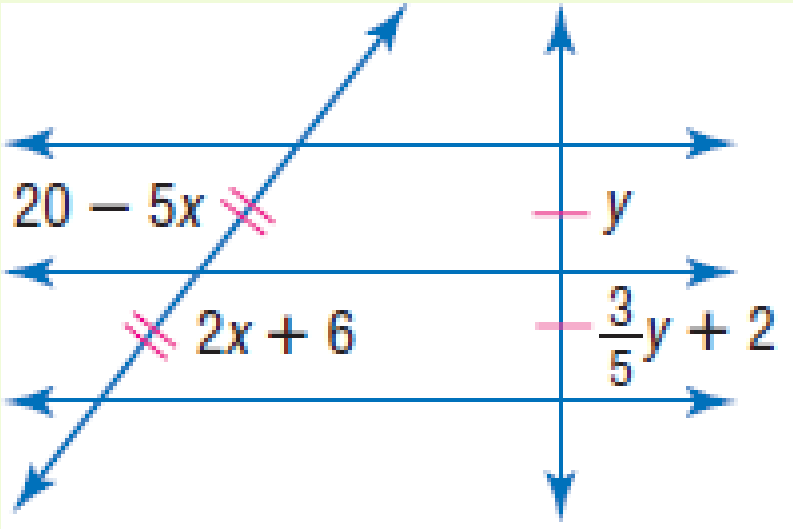
$-3y \quad -3y$

$\frac{2y}{2} = \frac{10}{2}$

$y = 5$

Examples

- Find x and y.



Since the segments are congruent, simply set them equal to each other.

$20 - 5x = 2x + 6$

$14 = 7x$

$2 = x$

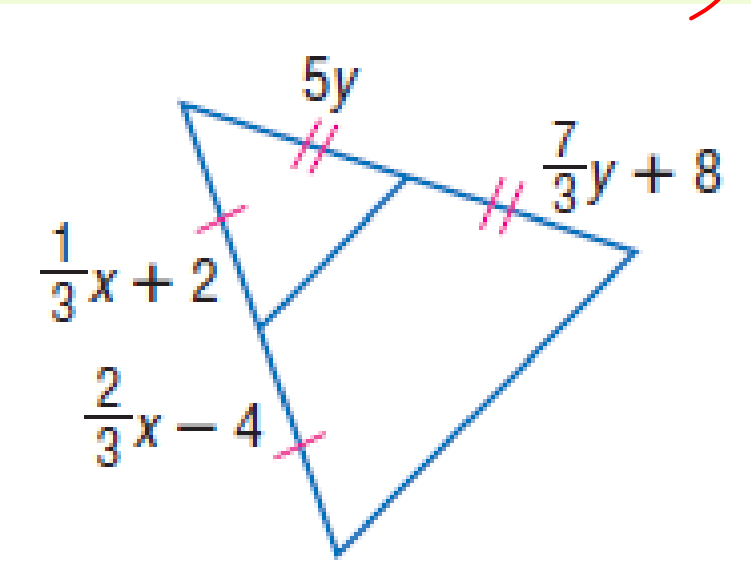
$y = \frac{3}{5}y + 2$

$\frac{2}{5}y = 2$

$y = 5$

Examples

- Find x and y.



$x = 18$ ✓ $\quad y =$

~~6~~ $\quad 3$

$3\left[\frac{1}{3}x + 2 = \frac{2}{3}x - 4\right]$

$x + 6 = 2x - 12$

$18 = x$

$\left[5y = \frac{7}{3}y + 8\right]3$

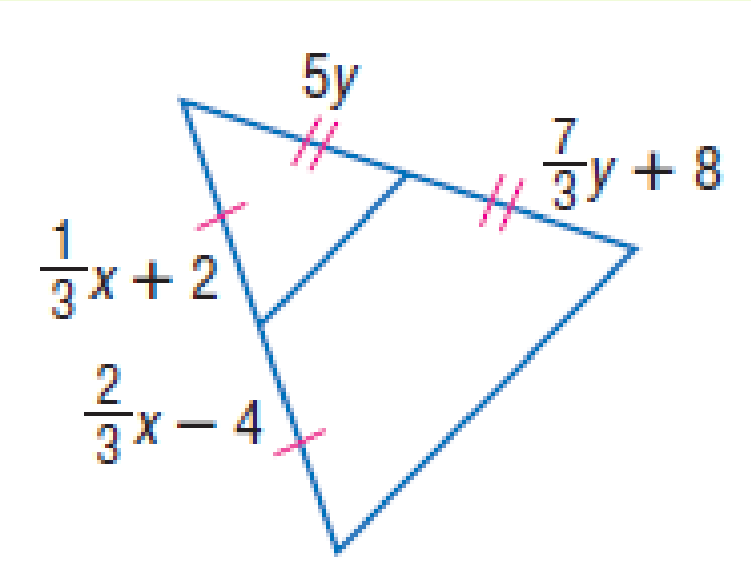
$15y = 7y + 24$

$8y = 24$

$y = 3$

Examples

Find x and y.



Since the segments are congruent, simply set them equal to each other.

$\frac{1}{3}x + 2 = \frac{2}{3}x - 4$

$6 = \frac{1}{3}x$

$18 = x$

$5y = \frac{7}{3}y + 8$

$\frac{8}{3}y = 8$

$y = 3$