

# Ratios and Proportions

# Ratio

- A ratio is a comparison of two quantities using division. A ratio of quantities  $a$  and  $b$  can be expressed as  $a$  to  $b$ ,  $a:b$ , or  $a/b$ , where  $b \neq 0$ .
- Ratios are usually expressed in simplest form.

# Examples

- The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio.

$$\frac{546}{26} \cdot \frac{26}{26} = \frac{21}{1}$$

# Examples

- The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio.
- $\frac{\textit{number of football players}}{\textit{number of high schools}} = \frac{546}{26}$  or  $\frac{21}{1}$

# Extended Ratios

- Extended ratios can be used to compare three or more quantities.
- $a:b:c$  means that the ratio of the first two quantities is  $a:b$ , the ratio of the last two quantities is  $b:c$ , and the ratio of the first and last quantities is  $a:c$ .

# Examples

- In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.

$$40 \quad 60 \quad 90$$
$$4x + 6x + 9x = 190$$

$$\frac{19x}{19} = \frac{190}{19}$$

$$x = 10$$

# Examples

- In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.
- $4x + 6x + 9x = 190$
- $19x = 190$
- $x = 10$
- $9(10) = 90$  in

# Proportions

- A proportion is an equation that says two ratios are equal
- Equivalent fractions set equal to each other form a proportion
- $\frac{2}{3}$  and  $\frac{6}{9}$  are equivalent fractions,  $\frac{2}{3} = \frac{6}{9}$  is a proportion

# Proportions

$$\frac{a}{b} = \frac{c}{d}$$

Diagram showing a proportion with horizontal lines under 'a' and 'c', and vertical lines under 'b' and 'd'. A red 'x' is drawn between the two fractions, indicating cross-multiplication.

$$\frac{a}{b} = \frac{c}{d}$$

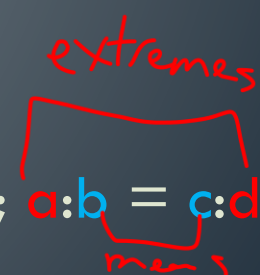
Diagram showing a proportion with red lines connecting 'a' to 'd' and 'b' to 'c', illustrating the cross-products.

- Every proportion has two cross products.
- The cross products in  $\frac{2}{3} = \frac{6}{9}$  are 2 times 9 and 3 times 6.
- The extremes of the proportion are 2 and 9, the means are 3 and 6.

$$18 = 18$$

$$\begin{array}{l} \text{extreme} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{mean} \\ \text{mean} \rightarrow \frac{a}{b} = \frac{c}{d} \leftarrow \text{extreme} \end{array}$$

- Extremes are on the outside, means are on the inside;  $a:b = c:d$
- The product of the means equals the product of the extremes.



# Equivalent Proportions

- Proportions will be equivalent as long as they have identical cross products.

The image shows four proportions, each with red lines indicating the cross products. The first proportion is  $\frac{a}{b} = \frac{c}{d}$ , with red lines connecting 'a' to 'd' and 'b' to 'c'. The second is  $\frac{b}{a} = \frac{d}{c}$ , with red lines connecting 'b' to 'c' and 'a' to 'd'. The third is  $\frac{a}{c} = \frac{b}{d}$ , with red lines connecting 'a' to 'd' and 'c' to 'b'. The fourth is  $\frac{c}{a} = \frac{d}{b}$ , with red lines connecting 'c' to 'b' and 'a' to 'd'.

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{b}{a} = \frac{d}{c}, \quad \frac{a}{c} = \frac{b}{d}, \quad \frac{c}{a} = \frac{d}{b}$$

# Examples

- Solve each proportion.

- $\frac{2}{3} = \frac{6}{9}$

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- $2(9) = 6(3)$

- $18 = 18$

# Examples

REMDAS

- Solve each proportion.

$$\bullet \frac{3}{5} = \frac{x}{75}$$

*Handwritten annotations: A red '45' is written above the fraction. Red 'x15' is written above the numerator '3' and below the denominator '5'. A red 'x' is written between the two fractions. A red arrow points from the '45' down to the 'x' in the final equation.*

$$\frac{225}{5} = \frac{5x}{5}$$
$$45 = x$$

# Examples

- Solve each proportion.

- $\frac{3}{5} = \frac{x}{75}$

- $\frac{3}{5} = \frac{x}{75}$

- $3(75) = 5x$

- $225 = 5x$

- $45 = x$

# Examples

- Solve each proportion.

$$\bullet \frac{3x-5}{4} = \frac{-13}{2} \quad \times 2 \rightarrow$$

$$(3x-5)2 = 4(-13)$$

$$6x - 10 = -52$$
$$\quad \quad \quad +10 \quad \quad +10$$

$$6x = -42$$
$$\quad \quad \quad \frac{\quad}{6} \quad \quad \frac{\quad}{6}$$

$$x = -7$$

$$3x-5 = -20$$
$$\quad \quad \quad +5 \quad \quad +5$$

$$3x = -21$$
$$\quad \quad \quad \frac{\quad}{3} \quad \quad \frac{\quad}{3}$$

$$x = -7$$

# Examples

- Solve each proportion.

- $\frac{3x - 5}{4} = \frac{-13}{2}$

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- $(3x - 5)2 = 4(-13)$

- $6x - 10 = -52$

- $6x = -42$

- $x = -7$