

ANGLES, ARCS, AND CHORDS

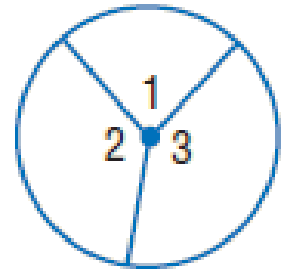


Central Angle

- A **central angle** is an angle whose vertex is the center of a circle.

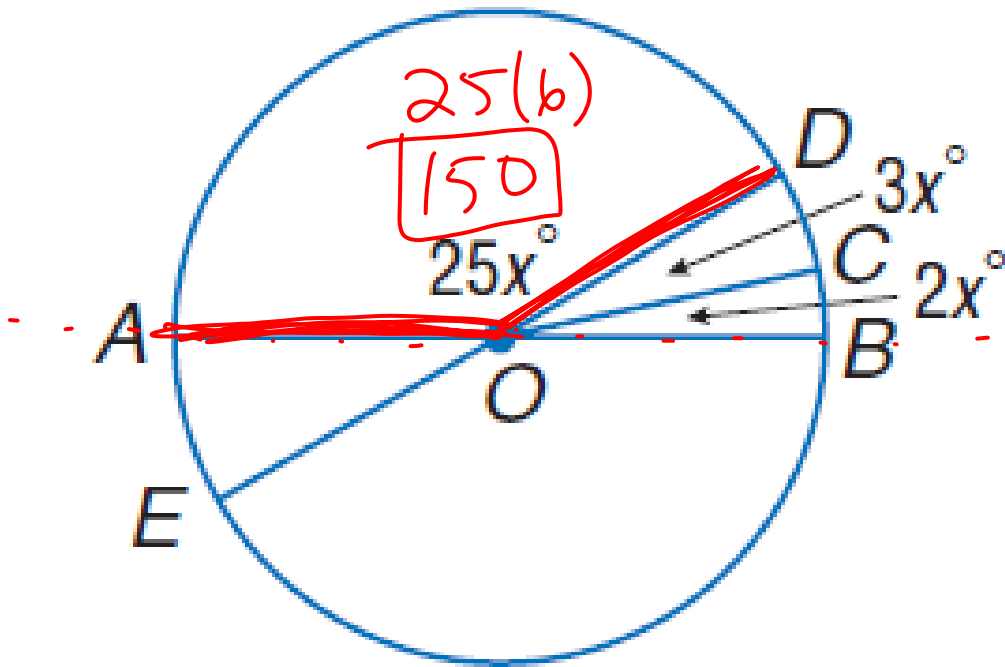
The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$m\angle 1 + m\angle 2 + m\angle 3 = 360$$



Examples

- Find $m\angle AOD$.

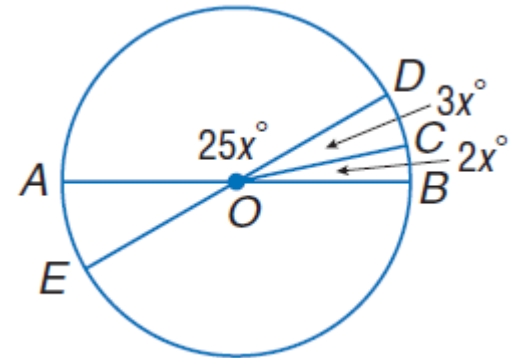


$$\begin{array}{r} 25x \\ 3x \\ + 2x \\ \hline 30x = \frac{180}{30} \end{array}$$

$$x = 6$$

Examples

- Find $m\angle AOD$.
- $m\angle AOD + m\angle DOB = 180$
- $25x + 3x + 2x = 180$
- $30x = 180$
- $x = 6$
- $25(6) = 150$



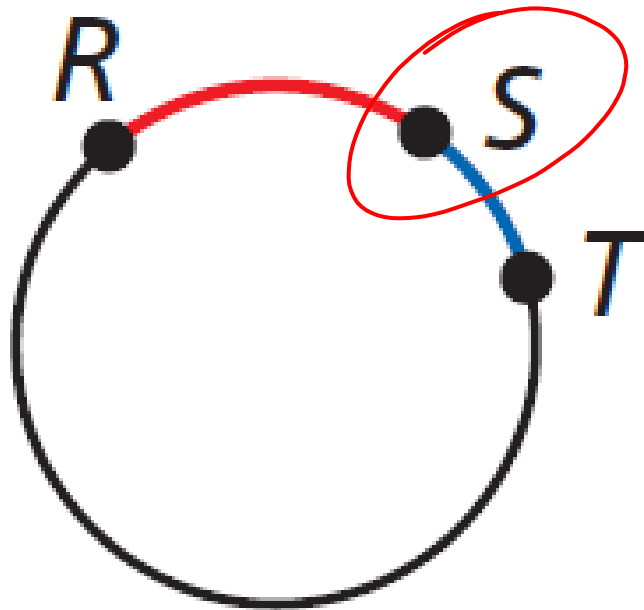
Arc

- An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

ARC	MEASURE	DIAGRAM
A minor arc is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\widehat{AC} = m\angle ABC = x^\circ$	
A major arc is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\widehat{ADC} = 360^\circ - m\angle ABC = 360^\circ - x^\circ$	
If the endpoints of an arc lie on a diameter, the arc is a semicircle .	The measure of a semicircle is equal to 180° . $m\widehat{EFG} = 180^\circ$	

Adjacent Arcs

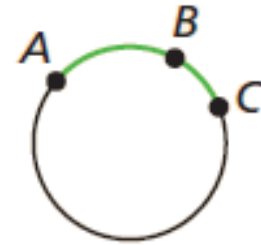
- **Adjacent arcs** are arcs of the same circle that intersect at exactly one point. \widehat{RS} and \widehat{ST} are adjacent arcs.



Arc Addition Postulate

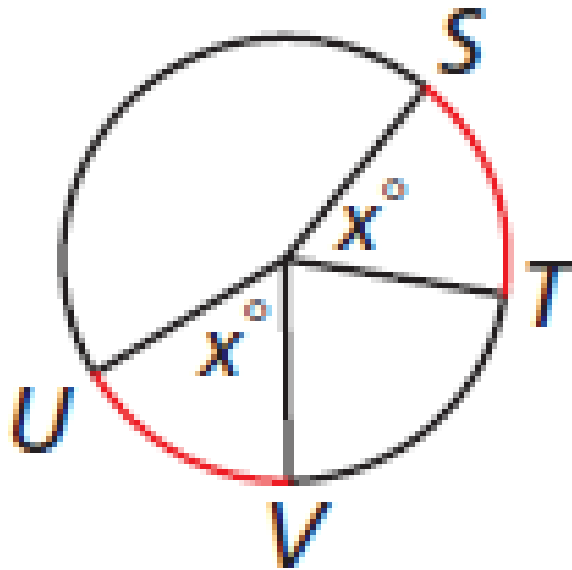
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\widehat{ABC} = m\widehat{AB} + m\widehat{BC}$$



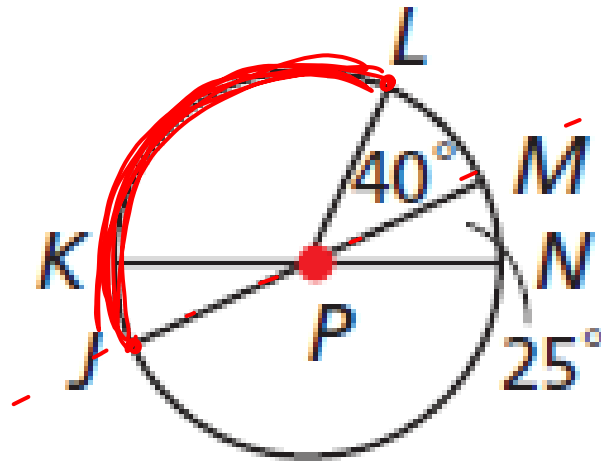
Congruent Arcs

- **Congruent arcs** are two arcs that have the same measure. In the figure, $\widehat{ST} \cong \widehat{UV}$.



Examples

- Find the measure.
- $m\widehat{JKL}$



$$\begin{array}{r} 40 \\ + 25 \\ \hline 65 \end{array} \quad \begin{array}{r} 180 \\ - 40 \\ \hline 140 \end{array}$$

Examples

□ Find the measure.

□ $m\widehat{JKL}$

□ $\widehat{JKM} = 180^\circ$

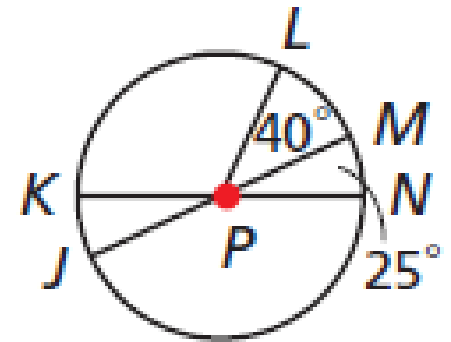
semi-circle

□ $\widehat{LM} = 40^\circ$

given

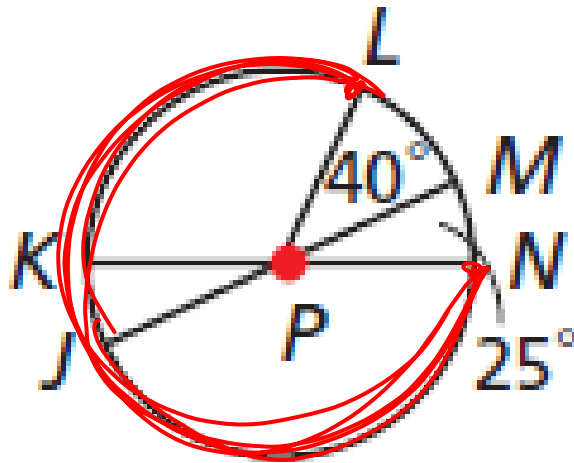
□ $\widehat{JKL} = 140^\circ$

$180 - 40$



Examples

- Find the measure.
- $m\widehat{LJN}$



$$\begin{array}{r} 40 \\ + 25 \\ \hline 65 \end{array}$$

$$\begin{array}{r} 360 \\ - 65 \\ \hline 295 \end{array}$$

Examples

□ Find the measure.

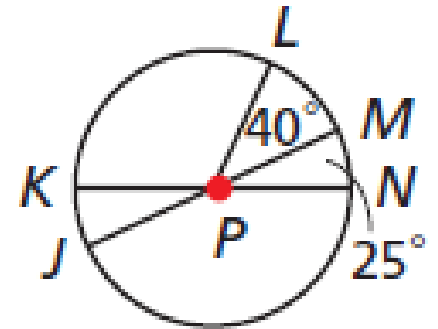
□ $m\widehat{LJN}$

□ $\widehat{LJN} + \widehat{NML} = 360^\circ$

□ $\widehat{NML} = \widehat{NM} + \widehat{ML}$

□ $25^\circ + 40^\circ = 65^\circ$

□ $\widehat{LJN} = 295^\circ$



circle = 360°

arc addition post.

addition

$360 - 65$

Arc Length

- Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

$$\begin{array}{l} \text{degree measure of arc} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \text{arc length} \\ \text{degree measure of whole circle} \rightarrow \end{array}$$

This can also be expressed as $\frac{A}{360} \cdot C = \ell.$

Examples

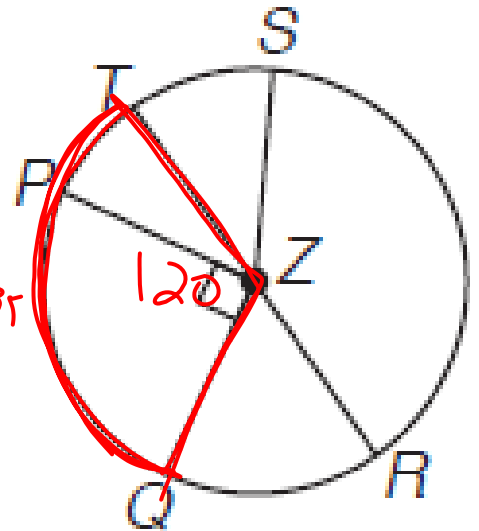
- The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.

- \widehat{QPT} if $m\angle QZT = 120$

$$l = \frac{A}{360} \cdot 2\pi r = \frac{120}{360} \cdot 2 \cdot \pi \cdot 13.5$$

$$= \frac{1}{3} \cdot 27\pi$$

$$= 9\pi$$



Examples

- The radius of $\odot Z$ is 13.5 units long. Find the length of each arc for the given angle measure.

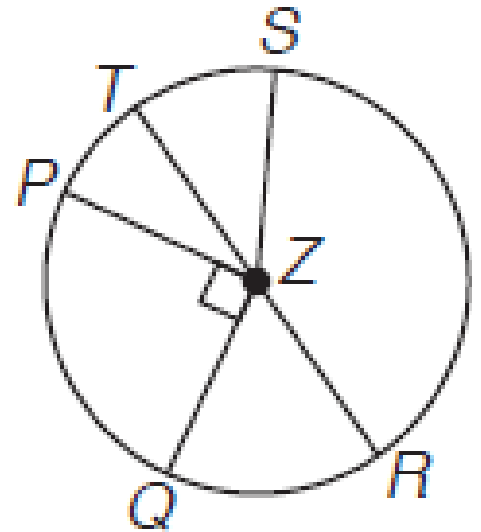
- \widehat{QPT} if $m\angle QZT = 120$

- $\frac{120}{360} * 2\pi(13.5) = \ell$

- $\frac{1}{3} * 27\pi = \ell$

- $9\pi = \ell$

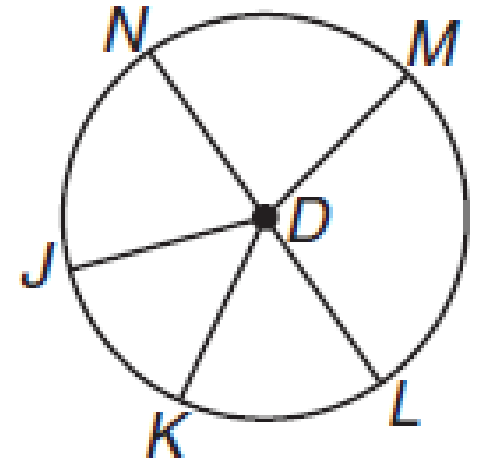
- $28.26 = \ell$



Examples

- The diameter of $\odot Z$ is 18 units long. Find the length of each arc for the given angle measure.

- \widehat{MN} if $m\angle MDN = 80$



Examples

- The diameter of $\odot Z$ is 18 units long. Find the length of each arc for the given angle measure.

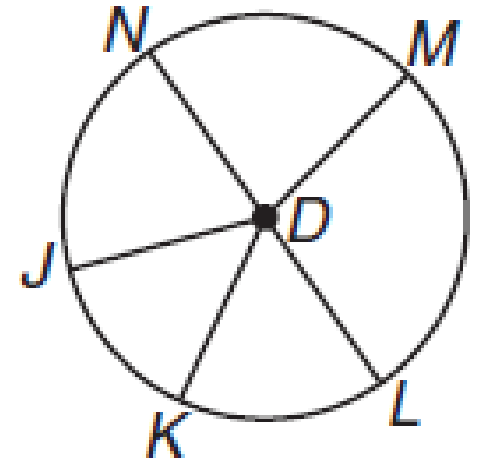
- \widehat{MN} if $m\angle MDN = 80$

- $\frac{80}{360} * \pi(18) = \ell$

- $\frac{2}{9} * 18\pi = \ell$

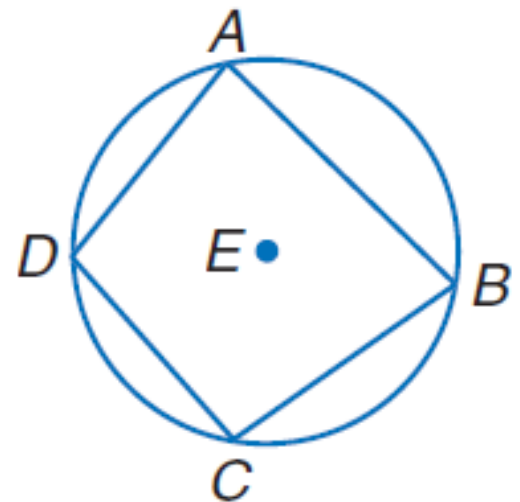
- $4\pi = \ell$

- $12.26 = \ell$



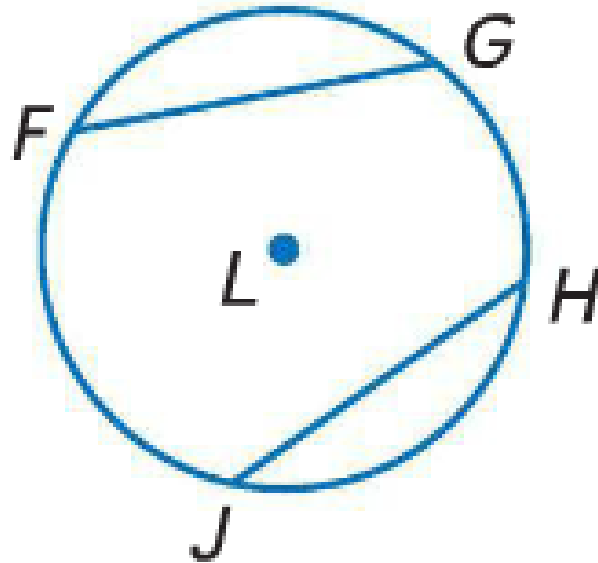
Chord

- Chord – a segment with endpoints on the circle.
- The endpoints of a chord are also endpoints of an arc.
- The chords of adjacent arcs can form a polygon.
Quadrilateral $ABCD$ is an **inscribed** polygon because all of its vertices lie on the circle.
- Circle E is **circumscribed** about the polygon because it contains all the vertices of the polygon.



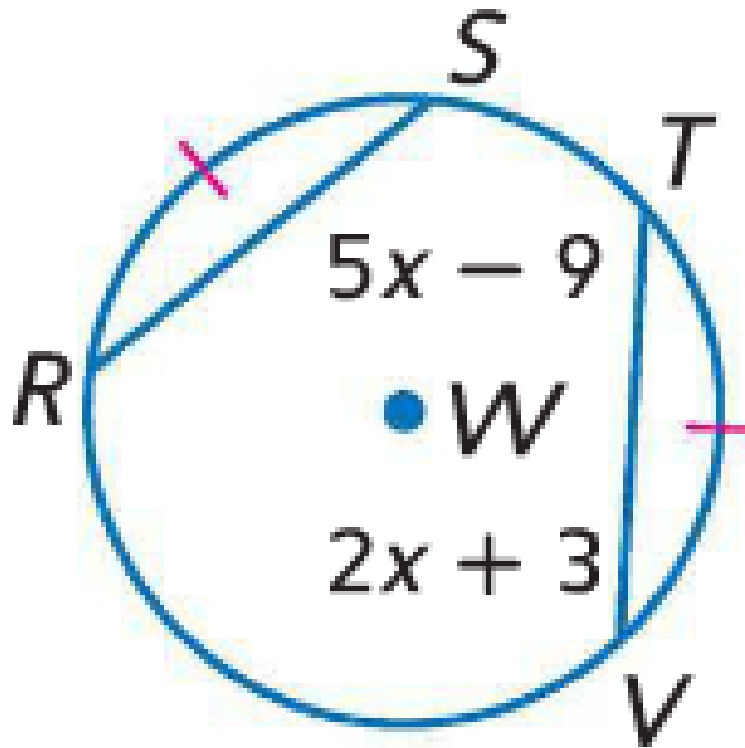
Congruent Chords Theorem

- In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



Examples

- In $\odot W$, arc $RS \cong$ arc TV . Find RS .



Examples

□ In $\odot W$, arc $RS \cong$ arc TV . Find RS .

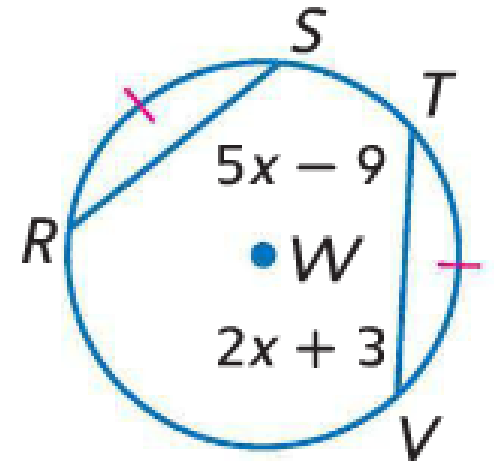
□ $RS \cong TV$

□ $5x - 9 = 2x + 3$

□ $3x = 12$

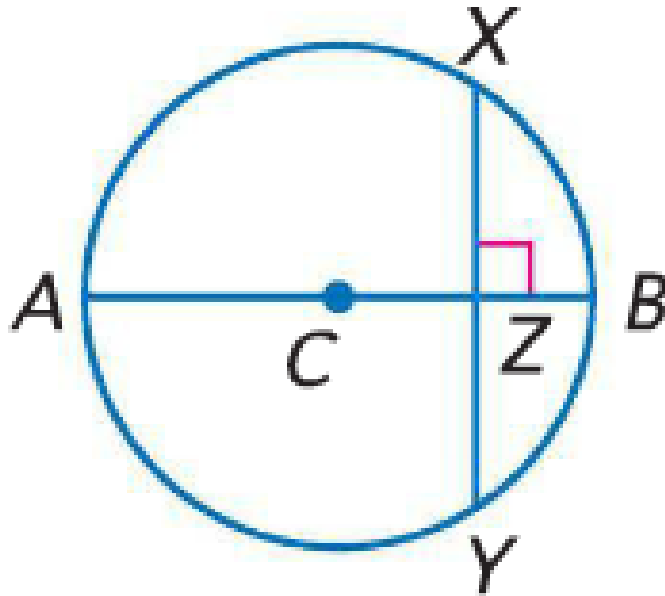
□ $x = 4$

□ $RS = 5(4) - 9 = 20 - 9 = 11$



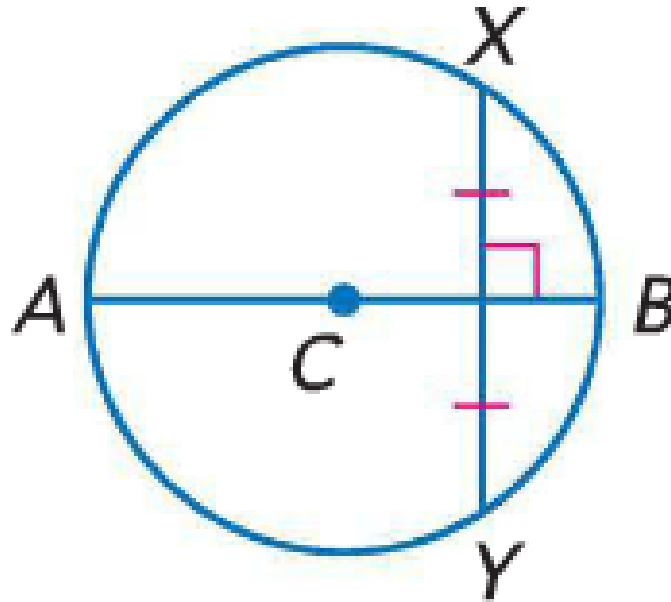
Perpendicular Bisector Theorem

- If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.



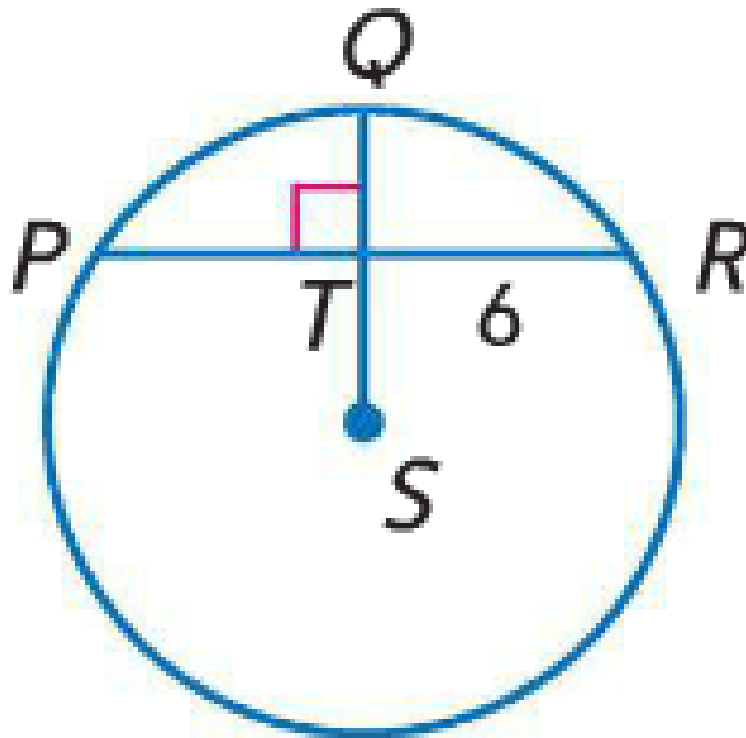
Converse of the Perpendicular Bisector Theorem

- The perpendicular bisector of a chord is a diameter (or radius) of the circle.



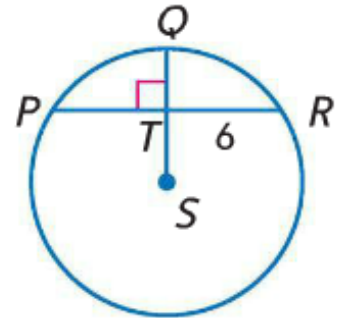
Examples

- In $\odot S$, find PR .



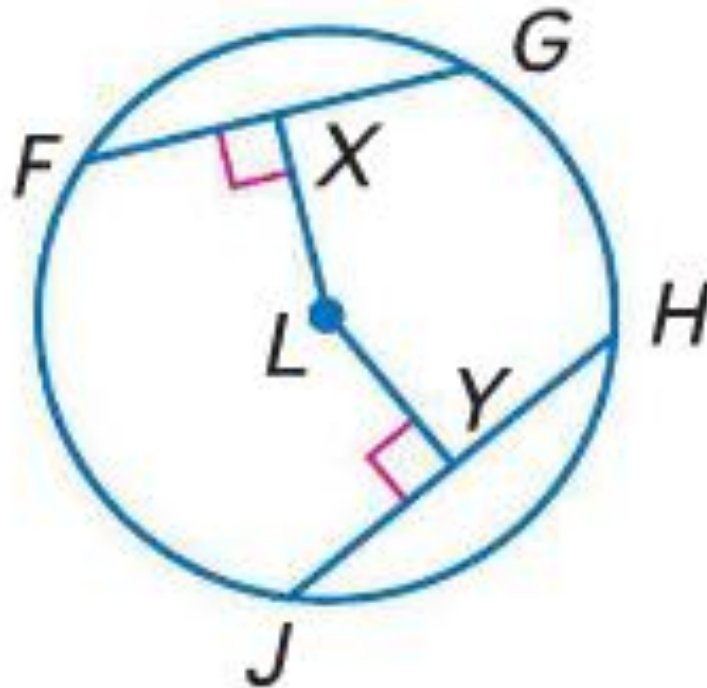
Examples

- In $\odot S$, find PR .
- If $TR = 6$, then $PR = 12$



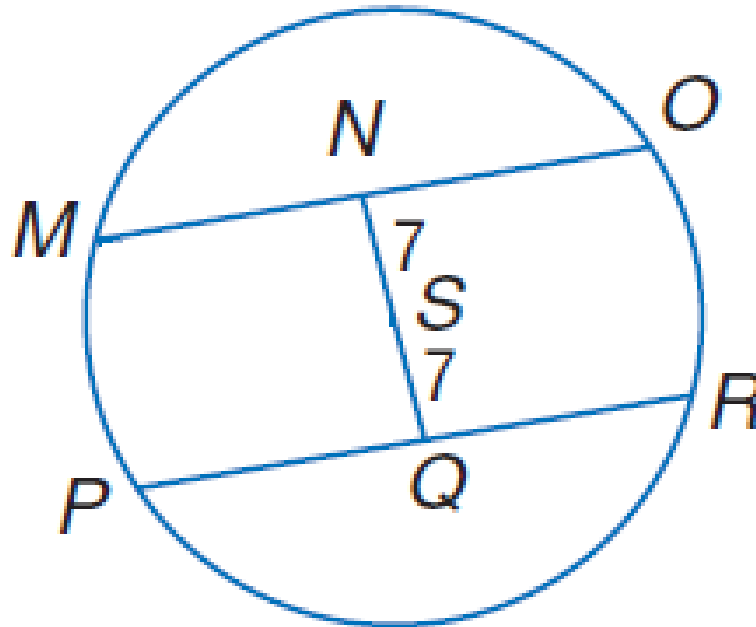
Theorem

- In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



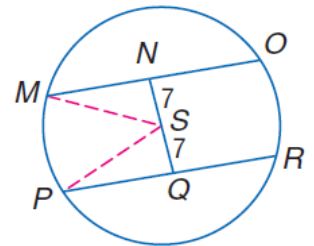
Examples

- Chords MO and PR are equidistant from the center. If the radius of $\odot S$ is 15, find MO and PQ .



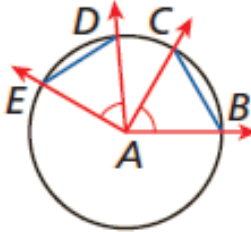
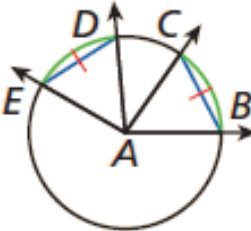
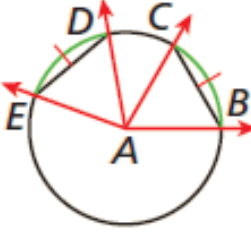
Examples

- Chords MO and PR are equidistant from the center. If the radius of $\odot S$ is 15, find MO and PQ .



- Since the radius of the circle is 15, then $PS = 15$.
- Use Pythagorean Theorem to solve the right triangle.
- $PQ = \sqrt{PS^2 - SQ^2} = \sqrt{15^2 - 7^2} = 13.27$
- $MO = 2PQ = 2 * 13.27 = 26.54$

Central Angles/Chords/Arcs

THEOREM	HYPOTHESIS	CONCLUSION
<p>In a circle or congruent circles:</p> <p>(1) Congruent central angles have congruent chords.</p>	 <p>$\angle EAD \cong \angle BAC$</p>	<p>$\overline{DE} \cong \overline{BC}$</p>
<p>(2) Congruent chords have congruent arcs.</p>	 <p>$\overline{ED} \cong \overline{BC}$</p>	<p>$\widehat{DE} \cong \widehat{BC}$</p>
<p>(3) Congruent arcs have congruent central angles.</p>	 <p>$\widehat{ED} \cong \widehat{BC}$</p>	<p>$\angle DAE \cong \angle BAC$</p>