Coordinate Proofs with Triangles

Coordinate Proof

- **Coordinate proof** uses figures in the coordinate plane and algebra to prove geometric concepts.
- The first step in a coordinate proof is placing the figure on the coordinate plane.

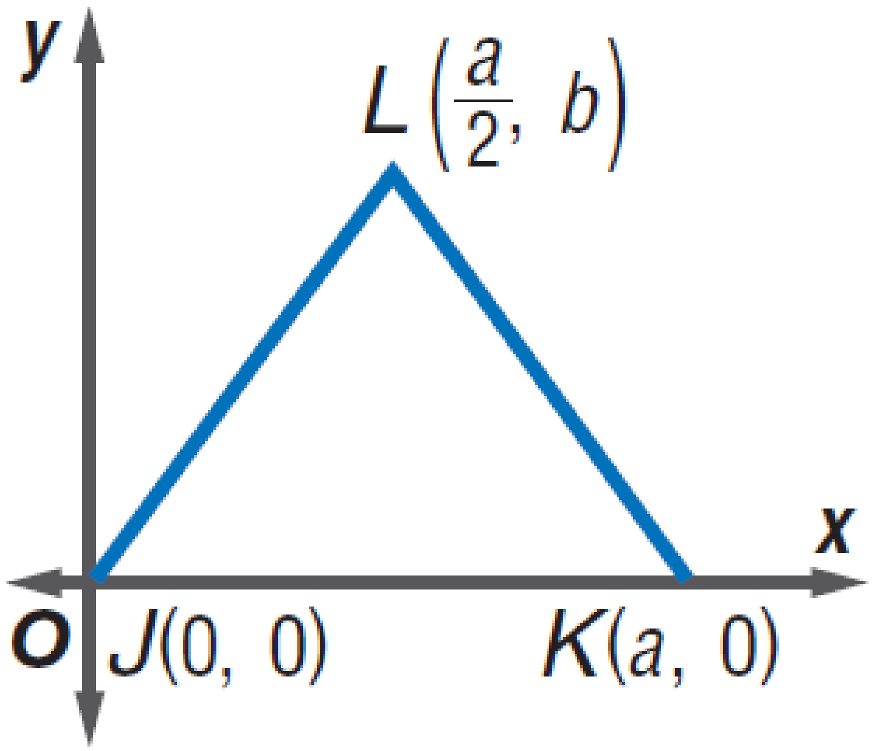
KEY CONCEPT ***Placing Figures on the Coordinate Plane***

1. Use the origin as a vertex or center of the figure.
2. Place at least one side of a polygon on an axis.
3. Keep the figure within the first quadrant if possible.
4. Use coordinates that make computations as simple as possible.

Examples

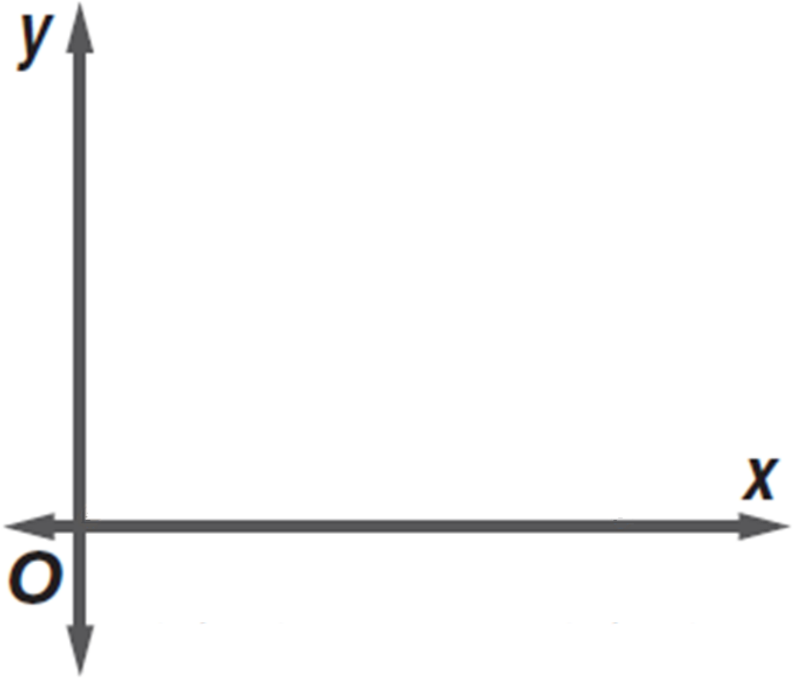
- **Position and label isosceles triangle *JKL* on a coordinate plane so that base *JK* is *a* units long.**

- Use the origin as vertex *J* of the triangle.
- Place the base of the triangle along the positive *x*-axis.
- Position the triangle in the first quadrant.
- Since *K* is on the *x*-axis, its *y*-coordinate is 0.
- Its *x*-coordinate is *a* because the base is *a* units long.
- *JKL* is isosceles, so the *x*-coordinate of *L* is halfway between 0 and *a* or $\frac{a}{2}$.
- We cannot write the *y*-coordinate in terms of *a*, so call it *b*.



Examples

- Position and label right triangle *HIJ* with legs *HI* and *IJ* on a coordinate plane so that *HI* is *a* units long and *IJ* is *b* units long.



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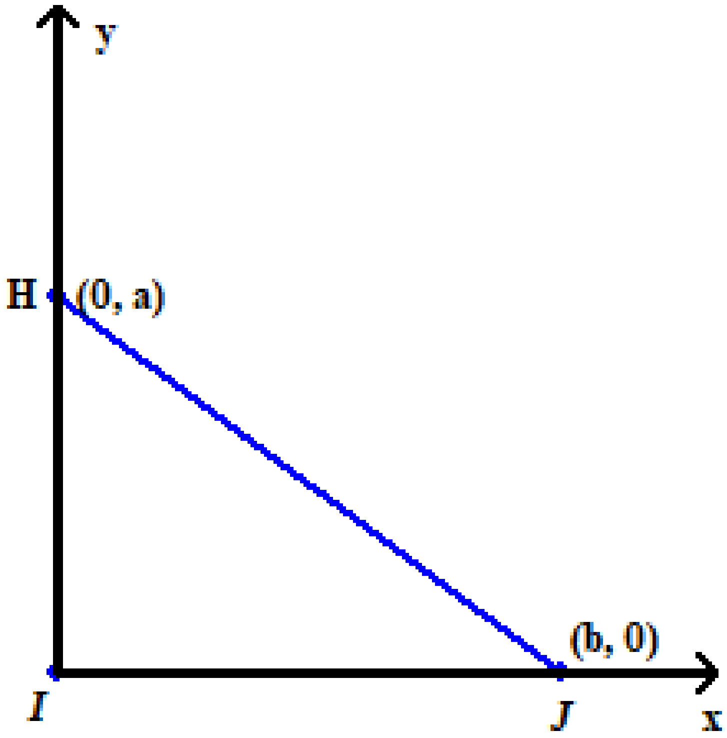
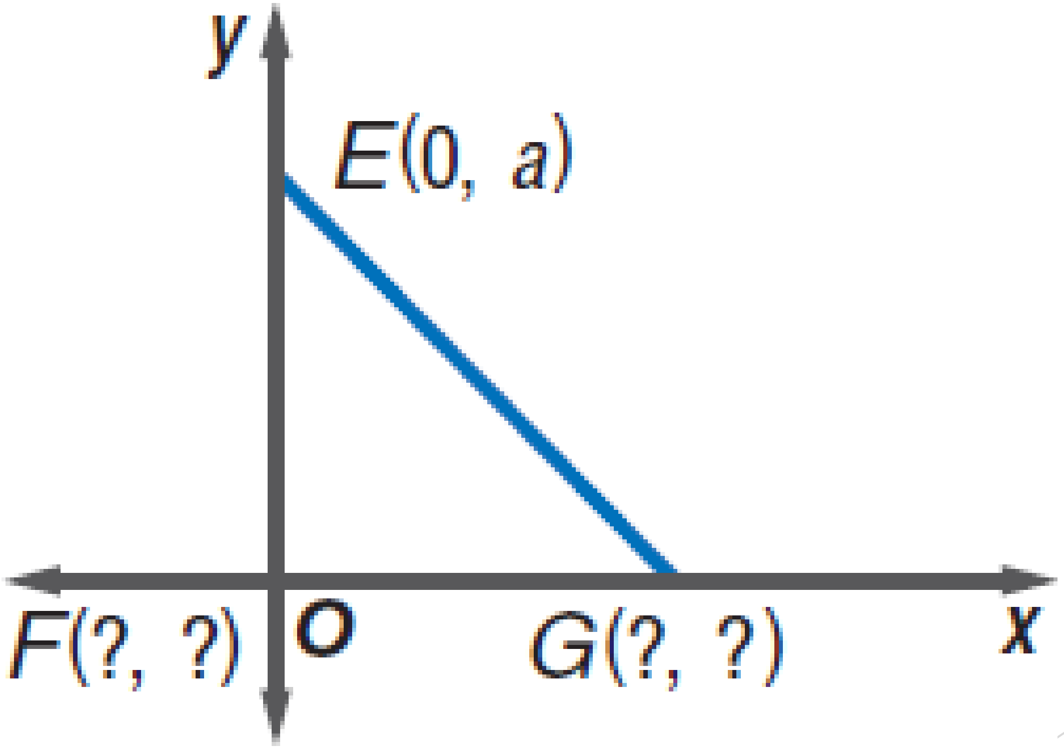


Examples

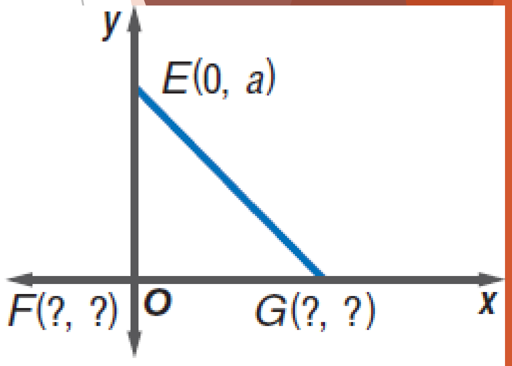
- Name the missing coordinates of isosceles right triangle *EFG*.



Examples

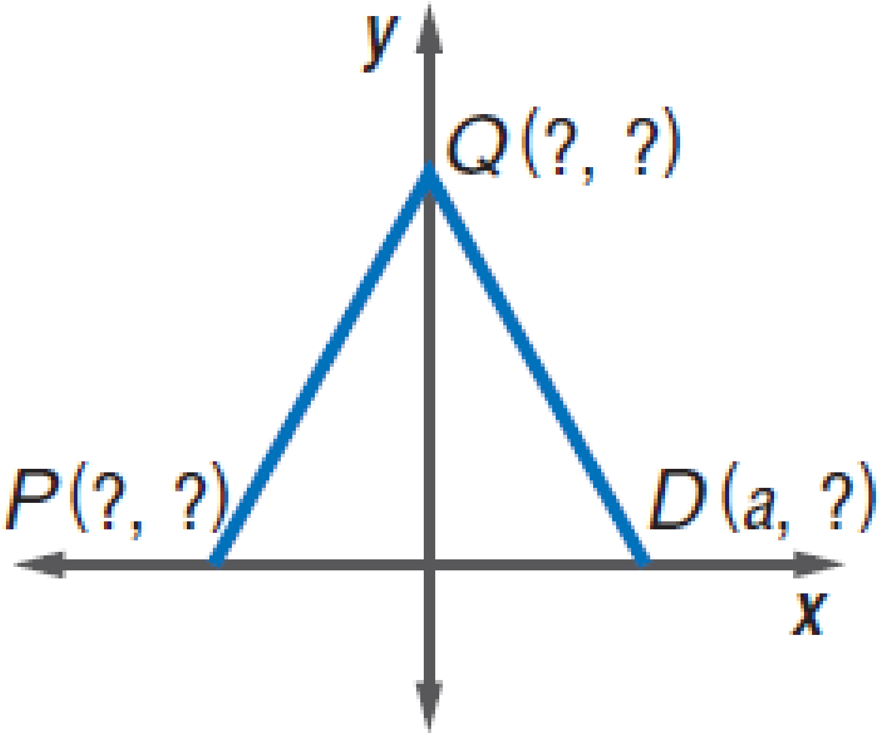
- **Name the missing coordinates of isosceles right triangle *EFG*.**

- Vertex *F* is positioned at the origin; its coordinates are (0, 0).
- Vertex *E* is on the *y*-axis, and vertex *G* is on the *x*-axis.
- So *EFG* is a right angle. Since *EFG* is isosceles, *EF* ≅ *GF*.
- *EF* is *a* units and *GF* must be the same. So, the coordinates of *G* are (*a*, 0).



Examples

- Name the missing coordinates of isosceles triangle *PDQ*.



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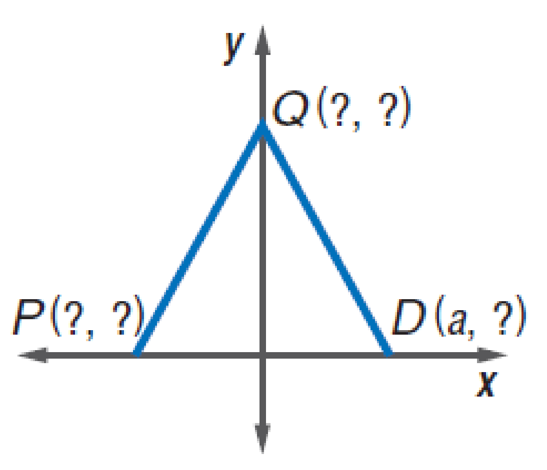


- Vertices *P* and *D* are positioned on the *x*-axis; their *y*-coordinates are 0.
- Since *PDQ* is isosceles, the distance from the origin to *P* is the same as the distance from the origin to *D*. So, the *x*-coordinate of *P* is –a. So the the coordinates of *P* are (-a, 0) and *D* are (a, 0).
- Vertex *Q* is on the *y*-axis, so its *x*-coordinate is 0. We cannot write the *y*-coordinate in terms of a, so call it b → *Q*(0, b)

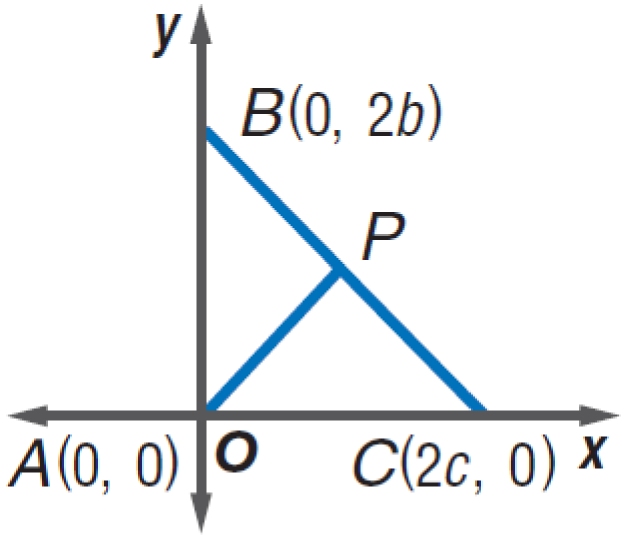
Writing Coordinate Proofs

- After a figure is placed on the coordinate plane and labeled, we can use coordinate proof to verify properties and to prove theorems.

Examples

- **Write a coordinate proof to prove that the measure of the segment that joins the vertex of the right angle in a right triangle to the midpoint of the hypotenuse is one-half the measure of the hypotenuse.**
- Place the right angle at the origin and label it *A*. Use coordinates that are multiples of 2 because the Midpoint Formula takes half the sum of the coordinates.

- **Given:** right $\triangle ABC$ with right $\angle BAC$
- P is the midpoint of $\overline{BC}$.
- **Prove:** $AP = \frac{1}{2}BC$



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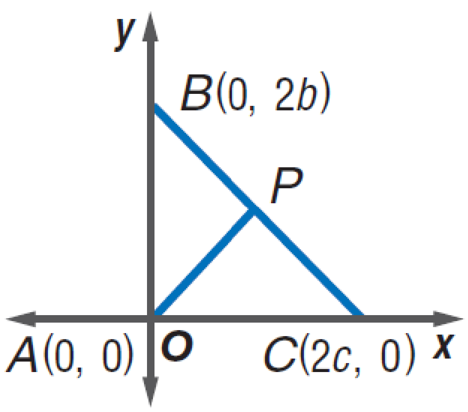

- **Proof:**
- By the Midpoint Formula, the coordinates of *P* are $(\frac{0+2c}{2}, \frac{2b+0}{2})$ or (*c*, *b*).
- Use the Distance Formula to find *AP* and *BC*.
- $AP = \sqrt{(c-0)^2 + (b-0)^2}$ $\quad BC = \sqrt{(2c-0)^2 + (0-2b)^2}$

  $= \sqrt{c^2 + b^2}$ $\quad = \sqrt{4c^2 + 4b^2}$ or $2\sqrt{c^2 + b^2}$

  $\frac{1}{2}BC = \sqrt{c^2 + b^2}$
- Therefore, $AP = \frac{1}{2}BC$

Examples

- Use coordinate geometry to classify a triangle with vertices located at the following coordinates: $A(0, 0)$, $B(0, 6)$, and $C(3, 3)$.

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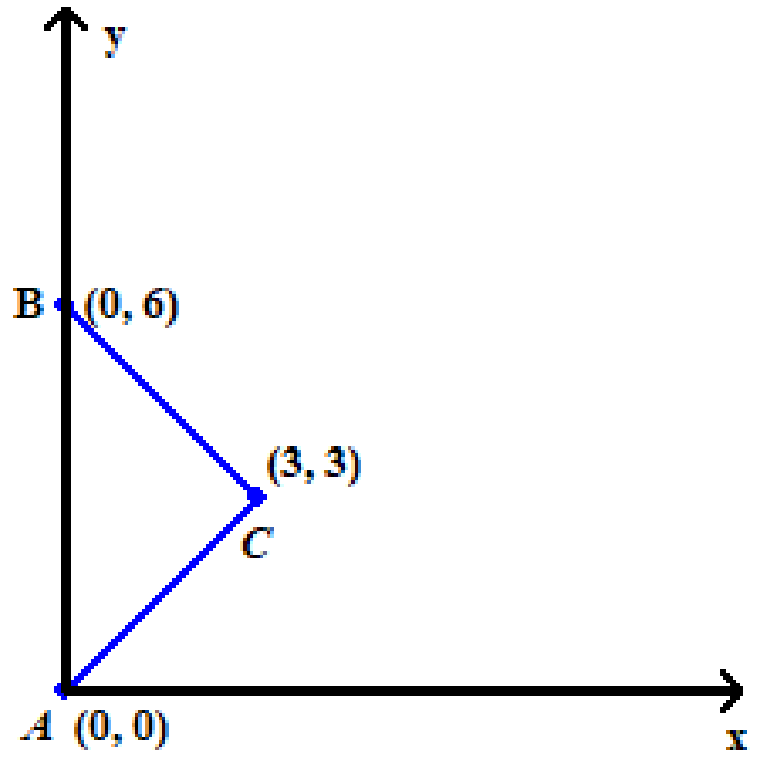
- Use coordinate geometry to classify a triangle with vertices located at the following coordinates: *A*(0, 0), *B*(0, 6), and *C*(3, 3).



- Proof:
- By the Distance Formula, AB = 6, BC moves 3 x 3,

  and AC moves 3 x 3
- Since BC and AC are congruent, the triangle is isosceles.