

# Ratios and Proportions

# Ratio

- A ratio is a comparison of two quantities using division. A ratio of quantities  $a$  and  $b$  can be expressed as  $a$  to  $b$ ,  $a:b$ , or  $a/b$ , where  $b \neq 0$ .
- Ratios are usually expressed in simplest form.

# Examples

- The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio.

$$\frac{546}{26} = \frac{26}{26} = \boxed{\frac{21}{1}}$$

# Examples

- The ratio of football players to high schools in Montgomery County is 546:26. What is the ratio of football players to high schools written as a unit ratio.
- $\frac{\textit{number of football players}}{\textit{number of high schools}} = \frac{546}{26}$  or  $\frac{21}{1}$

# Extended Ratios

- Extended ratios can be used to compare three or more quantities.
- $a:b:c$  means that the ratio of the first two quantities is  $a:b$ , the ratio of the last two quantities is  $b:c$ , and the ratio of the first and last quantities is  $a:c$ .

# Examples

- In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.

$$\begin{array}{ccc} 40 & 60 & 90 \\ 4x + 6x + 9x = 190 \end{array}$$

$$\boxed{90}$$

$$\begin{array}{r} 19x = 190 \\ \hline 19 \quad 19 \end{array}$$

$$x = 10$$

# Examples

- In a triangle, the ratio of the measures of three sides is 4:6:9, and its perimeter is 190 inches. Find the length of the longest side of the triangle.
- $4x + 6x + 9x = 190$
- $19x = 190$
- $x = 10$
- $9(10) = 90$  in

# Proportions

- A proportion is an equation that says two ratios are equal
- Equivalent fractions set equal to each other form a proportion
- $\frac{2}{3}$  *and*  $\frac{6}{9}$  are equivalent fractions,  $\frac{2}{3} = \frac{6}{9}$  is a proportion

# Proportions

$$\frac{a}{b} \times \frac{c}{d}$$

$$\frac{a}{b} \times \frac{d}{c}$$

- Every proportion has two cross products.
- The cross products in  $\frac{2}{3} = \frac{6}{9}$  are 2 times 9 and 3 times 6.
- The extremes of the proportion are 2 and 9, the means are 3 and 6.

$$a:b = c:d$$

$$\begin{array}{ccccc} \text{extreme} \rightarrow & \frac{a}{b} = \frac{c}{d} & \leftarrow & \text{mean} \\ \text{mean} \rightarrow & & \leftarrow & \text{extreme} \end{array}$$

- Extremes are on the outside, means are on the inside;  $a:b = c:d$
- The product of the means equals the product of the extremes.

# Equivalent Proportions

- Proportions will be equivalent as long as they have identical cross products.

The image shows four proportions, each with red lines indicating the cross-products. The first proportion is  $\frac{a}{b} = \frac{c}{d}$ , with red lines connecting 'a' to 'd' and 'c' to 'b'. The second is  $\frac{b}{a} = \frac{d}{c}$ , with red lines connecting 'b' to 'c' and 'd' to 'a'. The third is  $\frac{a}{c} = \frac{b}{d}$ , with red lines connecting 'a' to 'd' and 'b' to 'c'. The fourth is  $\frac{c}{a} = \frac{d}{b}$ , with red lines connecting 'c' to 'b' and 'd' to 'a'.

$$\frac{a}{b} = \frac{c}{d}, \quad \frac{b}{a} = \frac{d}{c}, \quad \frac{a}{c} = \frac{b}{d}, \quad \frac{c}{a} = \frac{d}{b}$$

# Examples

- Solve each proportion.

- $\frac{2}{3} = \frac{6}{9}$

$$2 \cdot 9 = 3 \cdot 6$$

$$18 = 18$$

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- $2(9) = 6(3)$

- $18 = 18$

# Examples

- Solve each proportion.

- $\frac{3}{5} = \frac{x}{75}$

$$3 \cdot 75 = 5 \cdot x$$

$$\frac{225}{5} = \frac{5x}{5}$$

$$45 = x$$

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$$\frac{3 \times 15}{5 \times 15} = \frac{x}{75} = 45$$

# Examples

- Solve each proportion.

- $\frac{3}{5} = \frac{x}{75}$

- $\frac{3}{5} = \frac{x}{75}$

- $3(75) = 5x$

- $225 = 5x$

- $45 = x$

# Examples

- Solve each proportion.

•  $\frac{3x-5}{4} = \frac{-13}{2}$   ~~$\times 2$~~

$$(3x-5)2 = 4(-13)$$

$$6x-10 = -52$$

$$\frac{6x}{6} = \frac{-42}{6}$$

$$x = -7$$

$$3x-5 = -24$$

$$\frac{3x}{3} = \frac{-21}{3}$$

$$x = -7$$

# Examples

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- $x = -7$