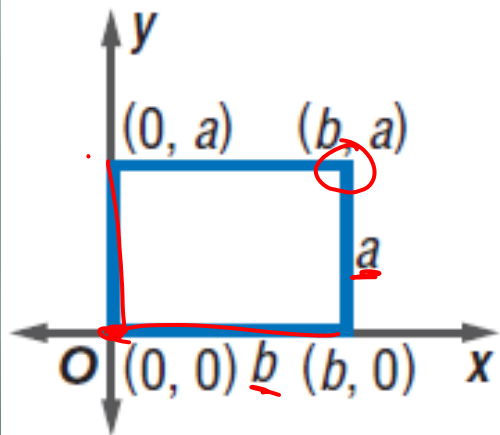


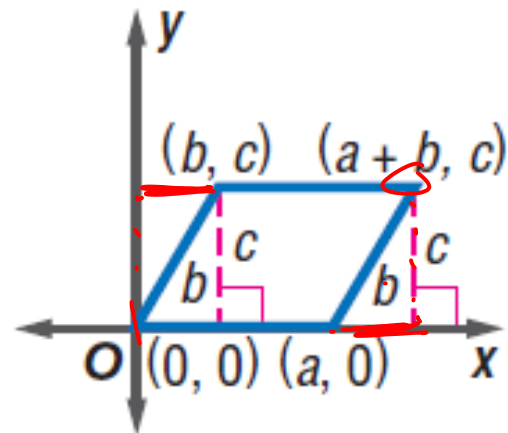
# COORDINATE PROOFS WITH QUADRILATERALS

# Positioning

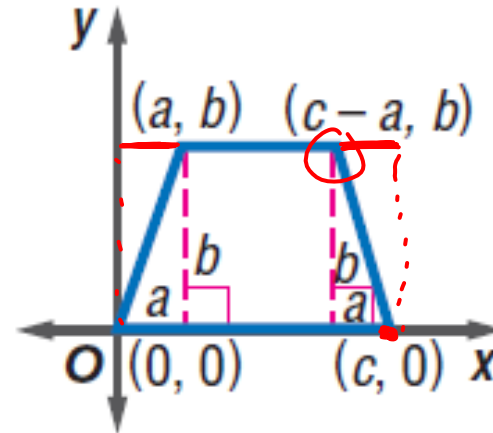
- The first step to using a coordinate proof is to place the figure on the coordinate plane. The placement of the figure can simplify the steps of the proof.



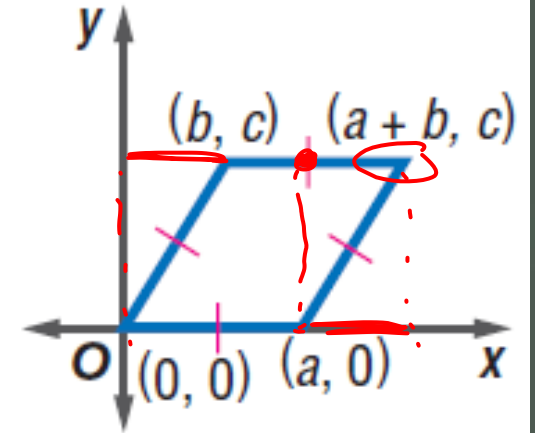
rectangle



parallelogram



isosceles trapezoid



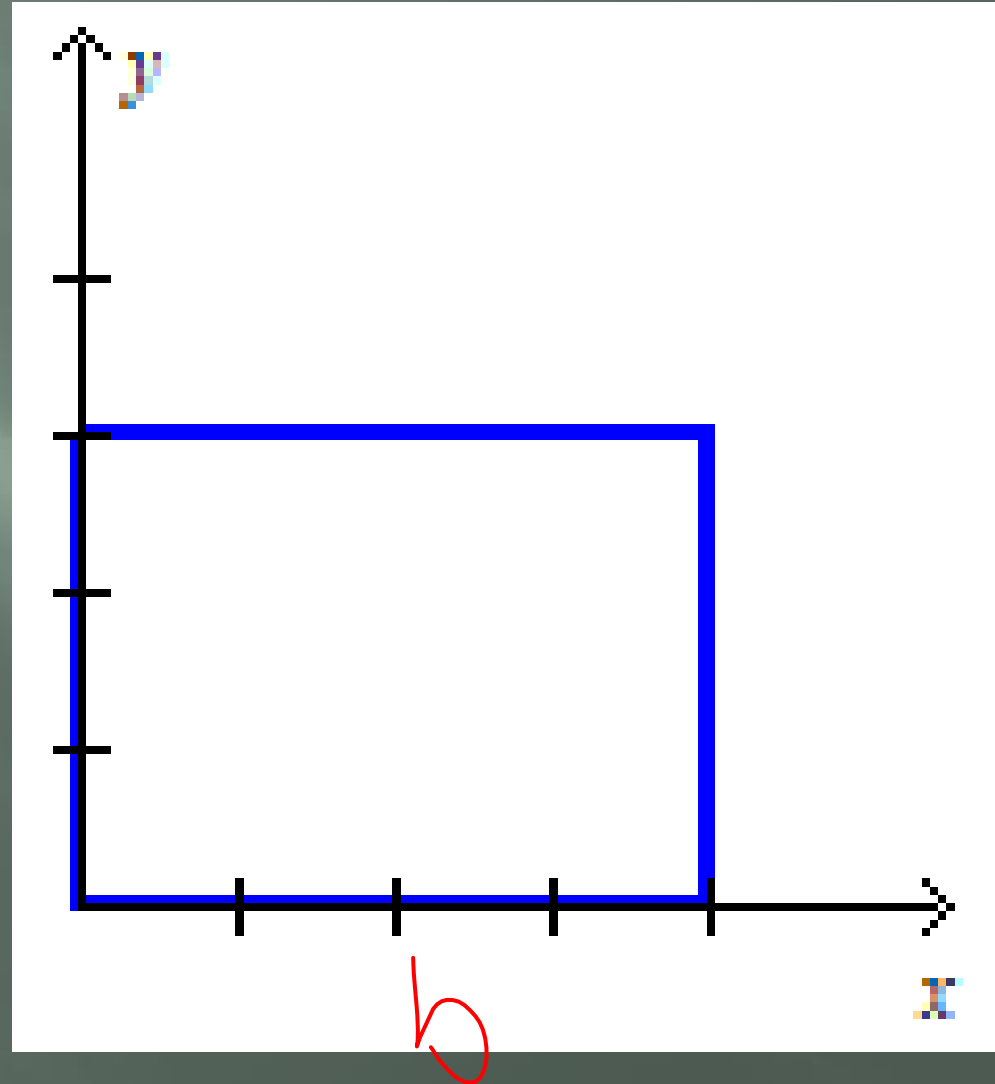
rhombus

## Positioning

Let  $A$ ,  $B$ ,  $C$ , and  $D$  be vertices of a quadrilateral with a length of  $a$  and a height of  $b$ .

Place the quadrilateral with vertex  $A$  at the origin,  $AB$  along the positive  $x$ -axis, and  $AD$  along the  $y$ -axis. Label the vertices  $A$ ,  $B$ ,  $C$ , and  $D$ .

$a$

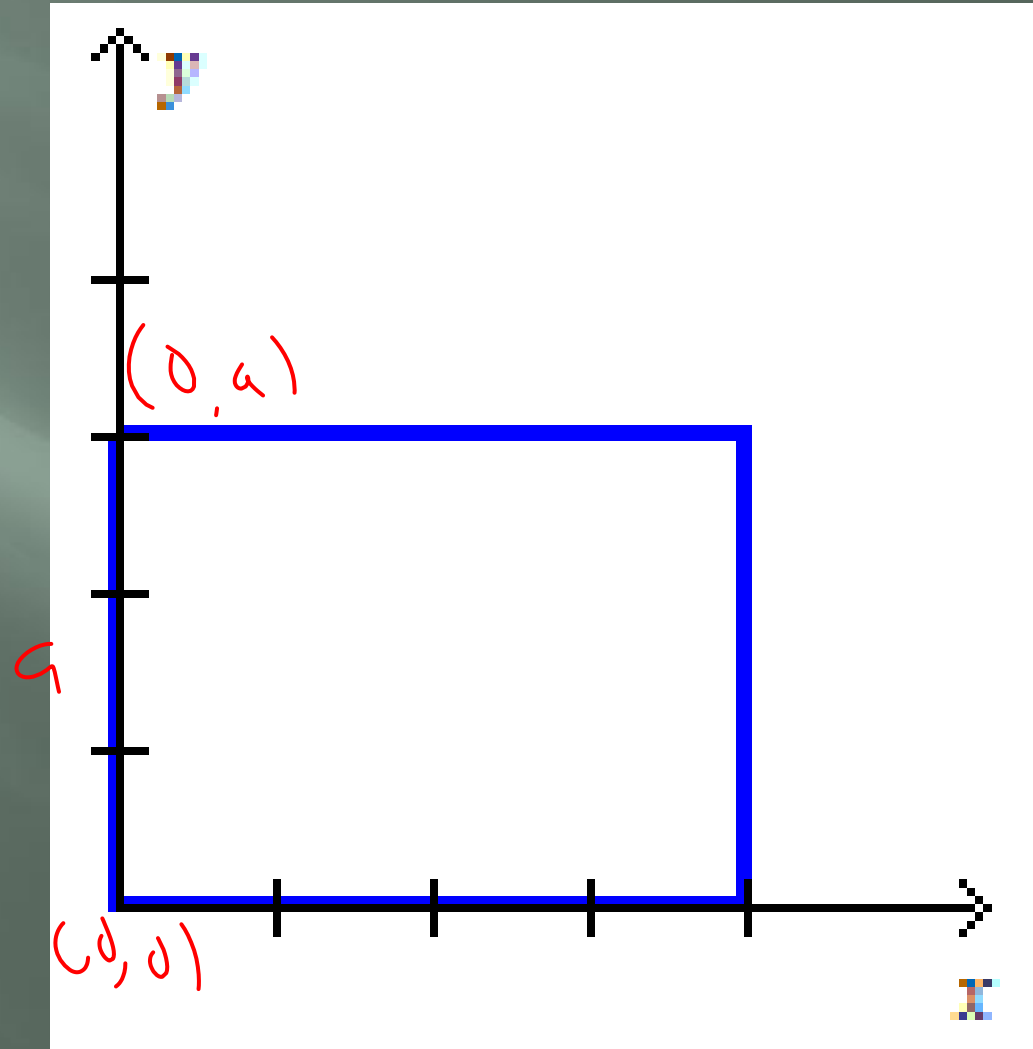


## Positioning

The  $y$ -coordinate of  $B$  is 0 because the vertex is on the  $x$ -axis. Since the side length is  $a$ , the  $x$ -coordinate is  $a$ .

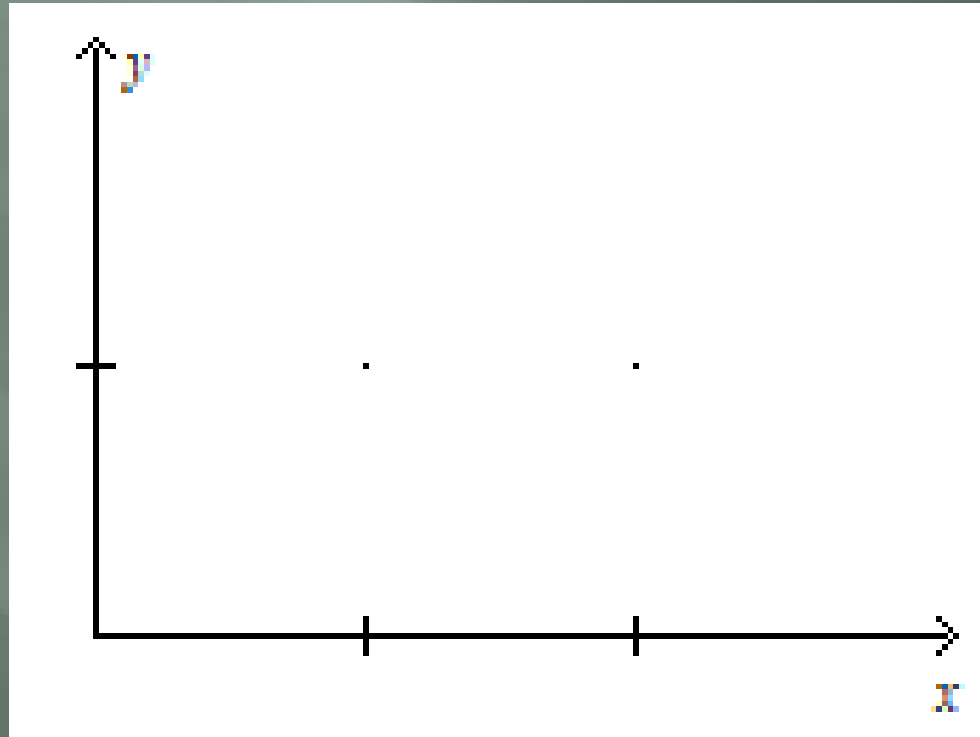
$D$  is on the  $y$ -axis so the  $x$ -coordinate is 0. The  $y$ -coordinate is  $0 + b$  or  $b$ .

The  $x$ -coordinate of  $C$  is also  $a$ . The  $y$ -coordinate is  $0 + b$  or  $b$  because the side  $BC$  is  $b$  units long.



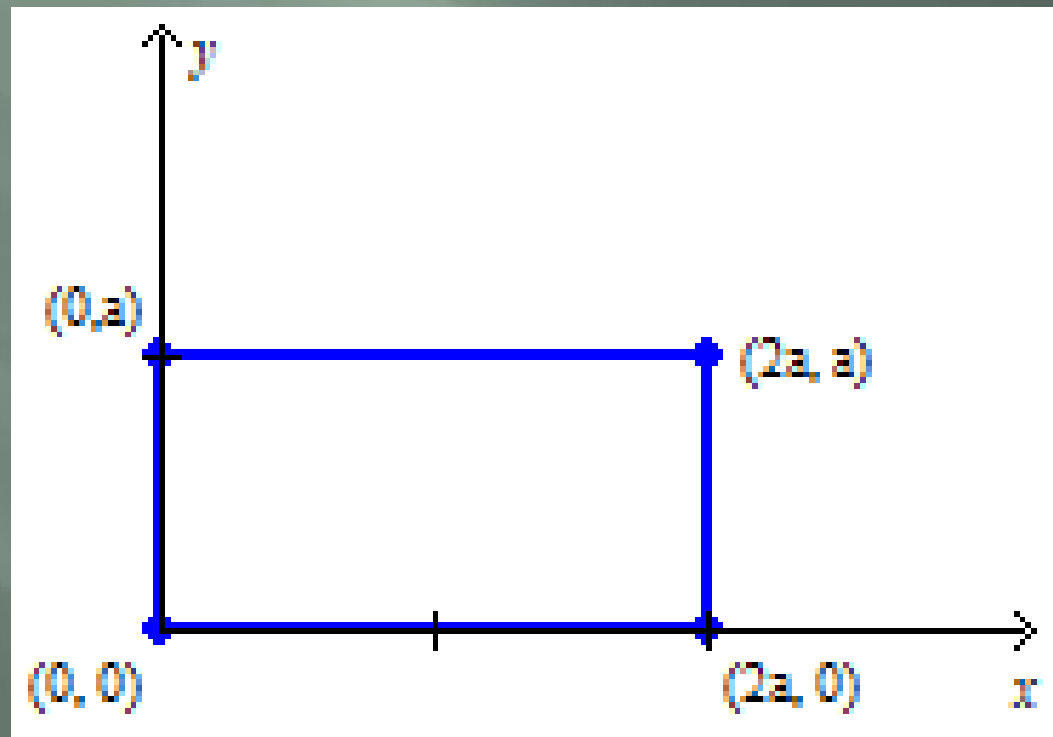
# Examples

- Position and label a rectangle with a length of  $2a$  units and a width of  $a$  units.



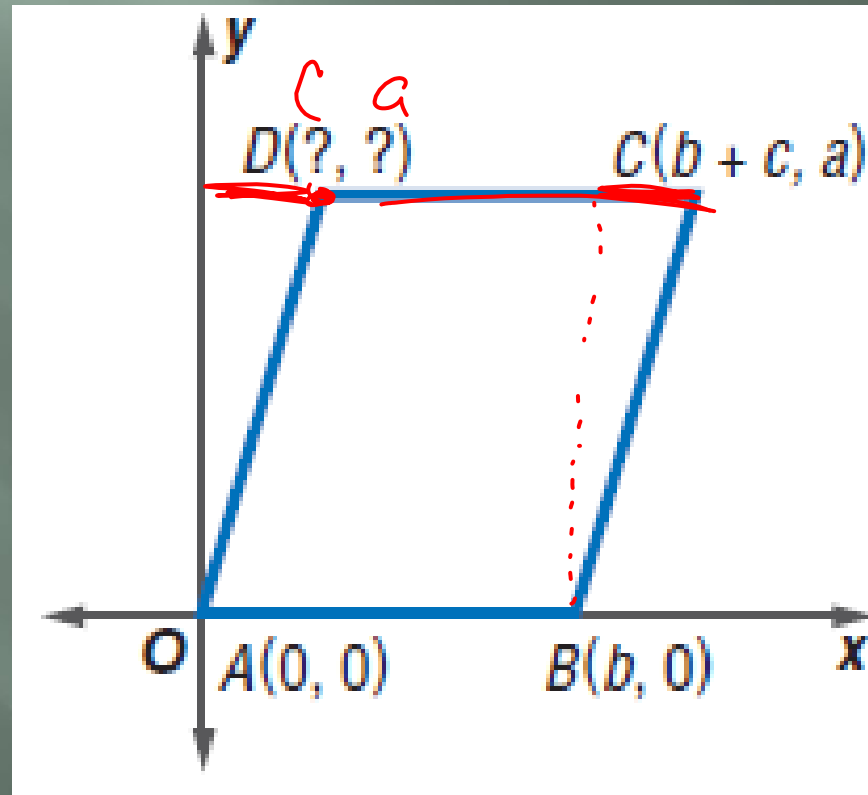
# Examples

- Position and label a rectangle with a length of  $2a$  units and a width of  $a$  units.



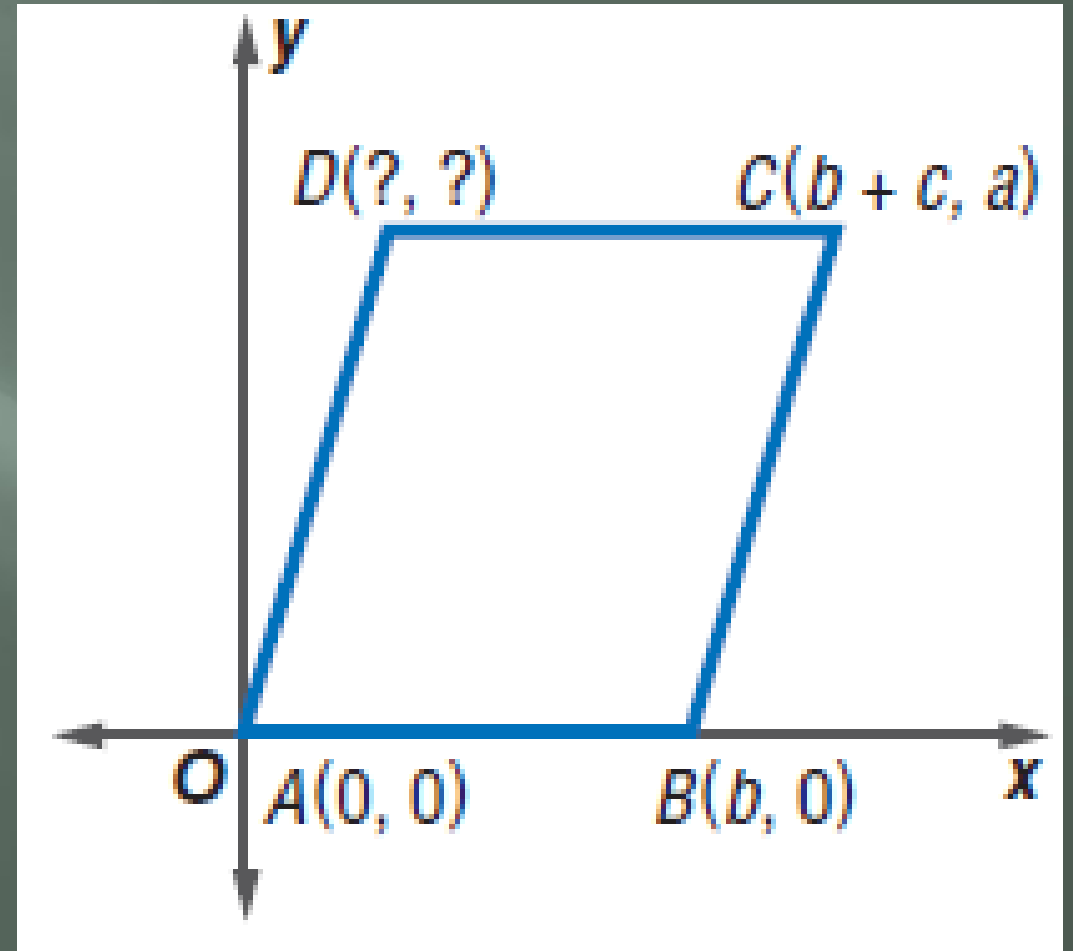
# Examples

- ▣ Name the missing coordinates for the parallelogram.



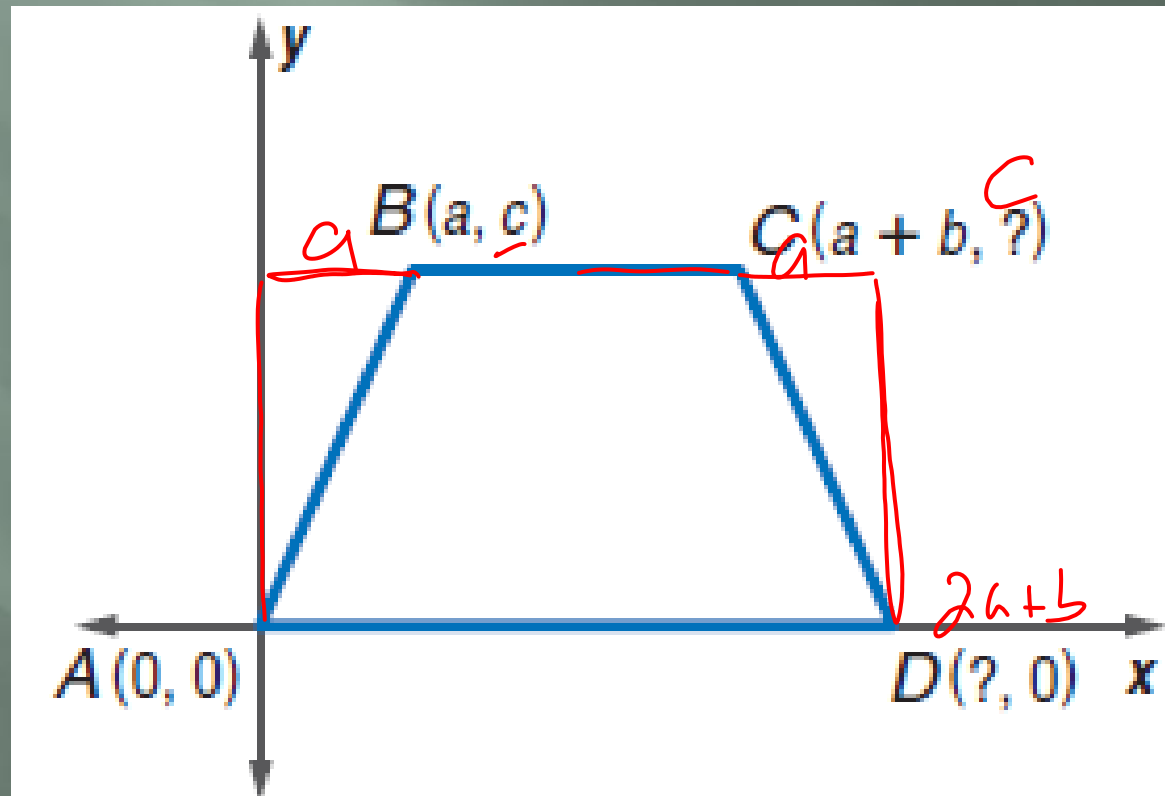
# Examples

- ▣ Name the missing coordinates for the parallelogram.
- ▣ Opposite sides of a parallelogram are congruent and parallel. So, the  $y$ -coordinate of  $D$  is  $a$ .
- ▣ The length of  $AB$  is  $b$ , and the length of  $DC$  is  $b$ .
- ▣ So, the  $x$ -coordinate of  $D$  is  $(b + c) - b$  or  $c$ .
- ▣ The coordinates of  $D$  are  $(c, a)$ .



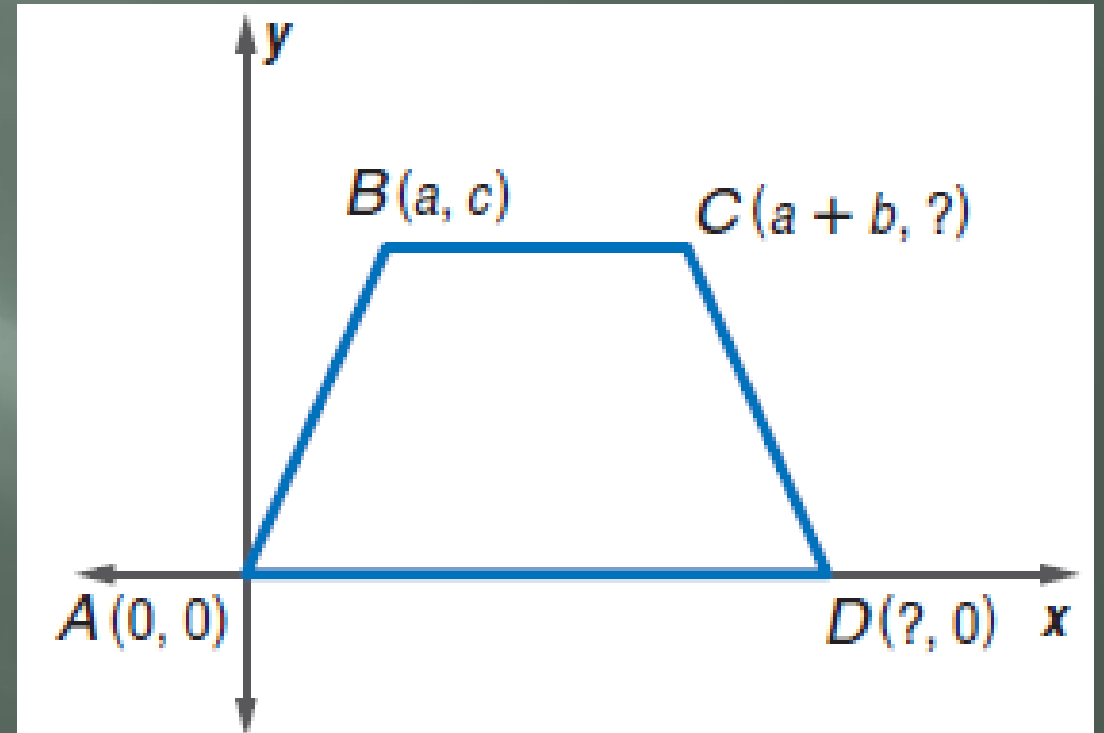
# Examples

- ▣ Name the missing coordinates for the isosceles trapezoid.



# Examples

- Name the missing coordinates for the isosceles trapezoid.
- Bases of a trapezoid are parallel, so  $C$  must have a  $y$ -value of  $c$ .
- The distance in the  $x$ -direction from  $A$  to  $B$  is  $a$ . Since the trapezoid is isosceles, the distance in the  $x$ -direction from  $C$  to  $D$  should also be  $a$ . The length of the top base is  $(a + b) - a$  or  $b$ , so the length from  $A$  to  $D$  is  $a + b + a$ , or  $2a + b$ .
- The coordinates of  $C$  are  $(a + b, c)$ .
- The coordinates of  $D$  are  $(2a + b, 0)$ .



# Coordinate Proof

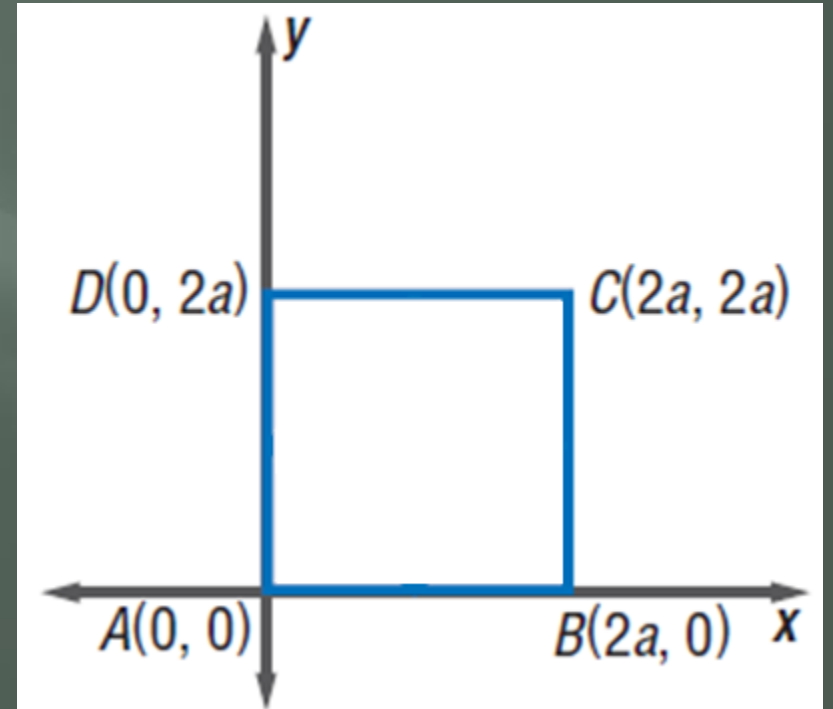
Once a figure has been placed on the coordinate plane, we can prove theorems using the Slope, Midpoint, and Distance Formulas.

# Examples

- ▣ Place a square on a coordinate plane. Label the midpoints of the sides,  $M$ ,  $N$ ,  $P$ , and  $Q$ . Write a coordinate proof to prove that  $MNPQ$  is a square.

# Examples

- ▣ Place a square on a coordinate plane. Label the midpoints of the sides,  $M$ ,  $N$ ,  $P$ , and  $Q$ . Write a coordinate proof to prove that  $MNPQ$  is a square.
- ▣ The first step is to position a square on the coordinate plane. Label the vertices to make computations as simple as possible.



# Examples

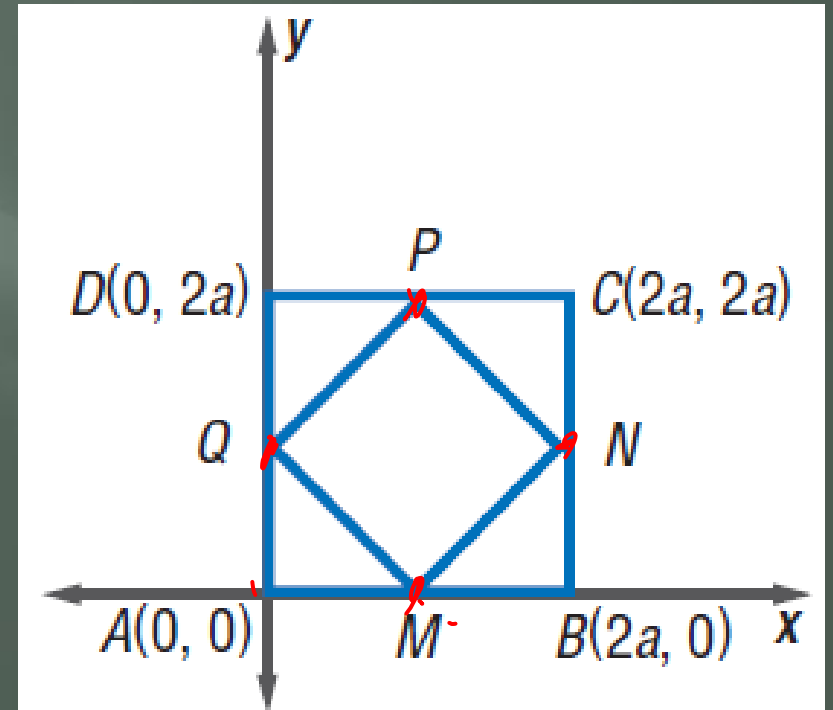
- Place a square on a coordinate plane. Label the midpoints of the sides,  $M$ ,  $N$ ,  $P$ , and  $Q$ . Write a coordinate proof to prove that  $MNPQ$  is a square.
- By the Midpoint Formula, the coordinates of  $M$ ,  $N$ ,  $P$ , and  $Q$  are as follows.

$$M\left(\frac{2a + 0}{2}, \frac{0 + 0}{2}\right) = (a, 0)$$

$$N\left(\frac{2a + 2a}{2}, \frac{2a + 0}{2}\right) = (2a, a)$$

$$P\left(\frac{0 + 2a}{2}, \frac{2a + 2a}{2}\right) = (a, 2a)$$

$$Q\left(\frac{0 + 0}{2}, \frac{0 + 2a}{2}\right) = (0, a)$$



# Examples

- Place a square on a coordinate plane. Label the midpoints of the sides,  $M$ ,  $N$ ,  $P$ , and  $Q$ . Write a coordinate proof to prove that  $MNPQ$  is a square.

- Find the slopes of  $\overline{QP}$ ,  $\overline{MN}$ ,  $\overline{QM}$ , and  $\overline{PN}$ .

$$\begin{array}{ll} \text{slope of } \overline{QP} = \frac{2a - a}{a - 0} \text{ or } 1 & \text{slope of } \overline{MN} = \frac{a - 0}{2a - a} \text{ or } 1 \\ \text{slope of } \overline{QM} = \frac{0 - a}{a - 0} \text{ or } -1 & \text{slope of } \overline{PN} = \frac{a - 2a}{2a - a} \text{ or } -1 \end{array}$$

- Each pair of opposite sides have the same slope, so they are parallel.
- Consecutive sides form right angles because their slopes are negative reciprocals.

# Examples

- Place a square on a coordinate plane. Label the midpoints of the sides,  $M$ ,  $N$ ,  $P$ , and  $Q$ . Write a coordinate proof to prove that  $MNPQ$  is a square.

- Use the distance formula to find the lengths of  $\overline{QP}$  and  $\overline{QM}$

$$\begin{aligned} QP &= \sqrt{(0 - a)^2 + (a - 2a)^2} & QM &= \sqrt{(0 - a)^2 + (a - 0)^2} \\ &= \sqrt{a^2 + a^2} & &= \sqrt{a^2 + a^2} \\ &= \sqrt{2a^2} \text{ or } a\sqrt{2} & &= \sqrt{2a^2} \text{ or } a\sqrt{2} \end{aligned}$$

- $MNPQ$  is a square because each pair of opposite sides is parallel, and consecutive sides form right angles and are congruent.