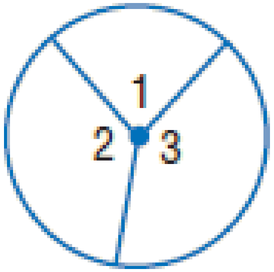
ANGLES, ARCS, AND CHORDS

Central Angle

- A **central angle** is an angle whose vertex is the center of a circle.

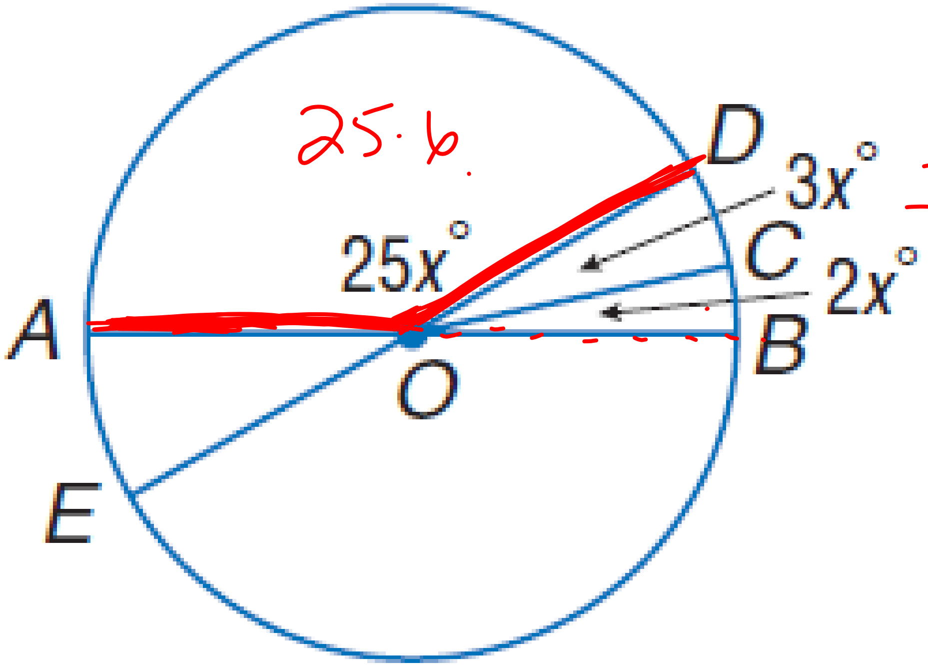
The sum of the measures of the central angles of a circle with no interior points in common is 360.

$$m\angle 1 + m\angle 2 + m\angle 3 = \boxed{360}$$



Examples

- Find m$\angle AOD$. $= 150$



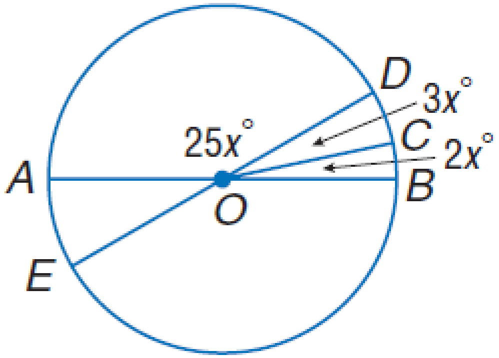
$$\begin{array}{r} 25x \\ 3x \\ +\ 2x \\ \hline \end{array}$$

$$\frac{3\cancel{0}x}{\cancel{3}0} = \frac{18\cancel{0}}{3\cancel{0}}$$

$x = 6$

Examples

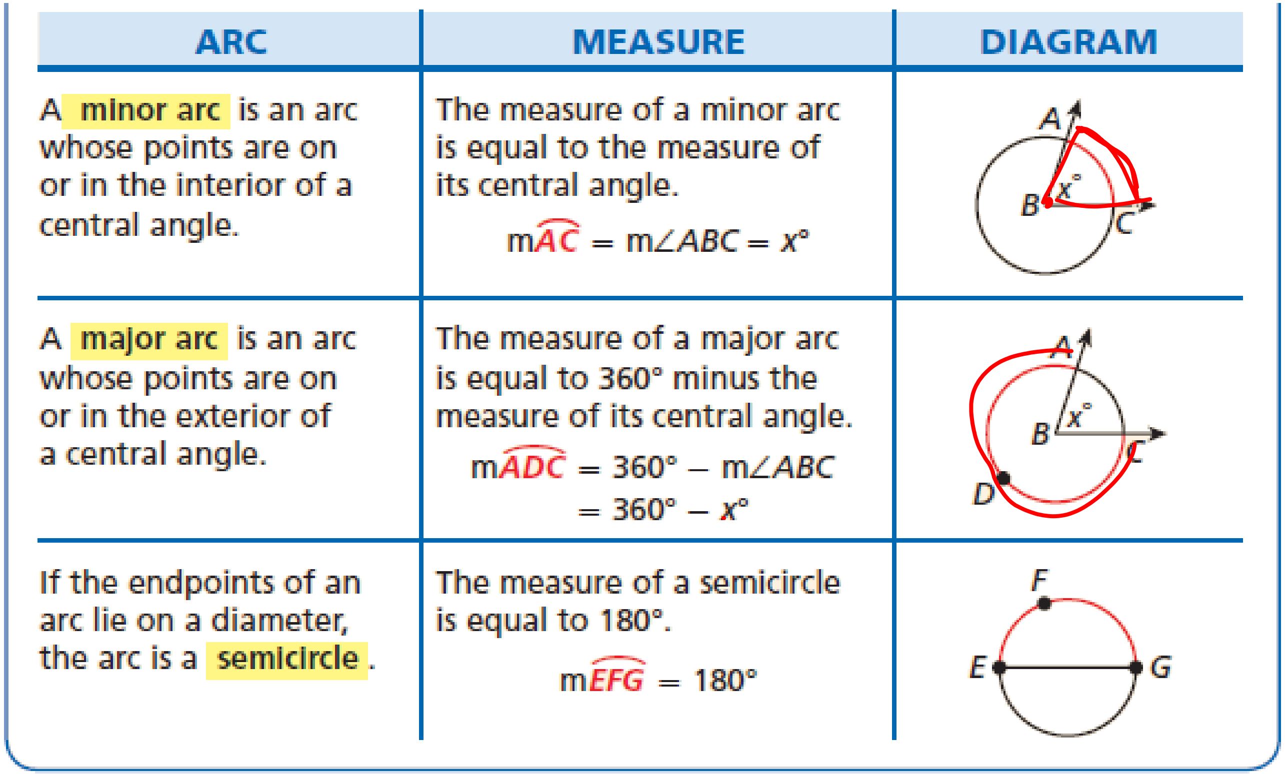
- Find m$\angle AOD$.



- m$\angle AOD$ + m$\angle DOB$ = 180
- 25x + 3x + 2x = 180
- 30x = 180
- x = 6
- 25(6) = 150

Arc

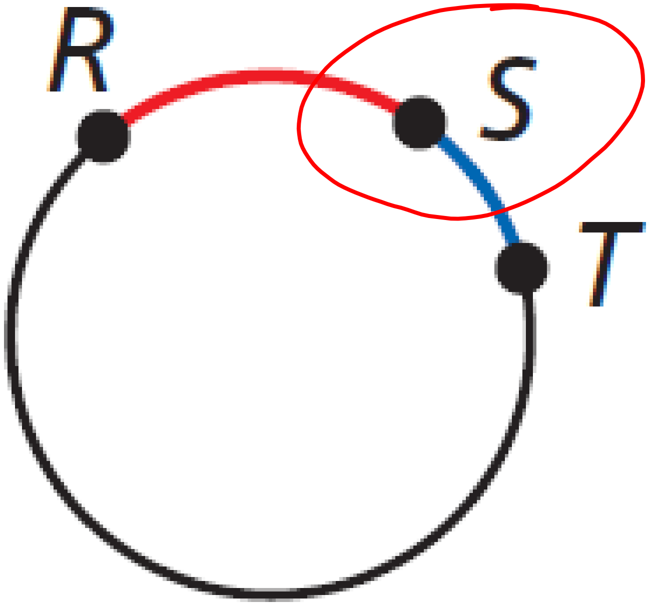
- An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.



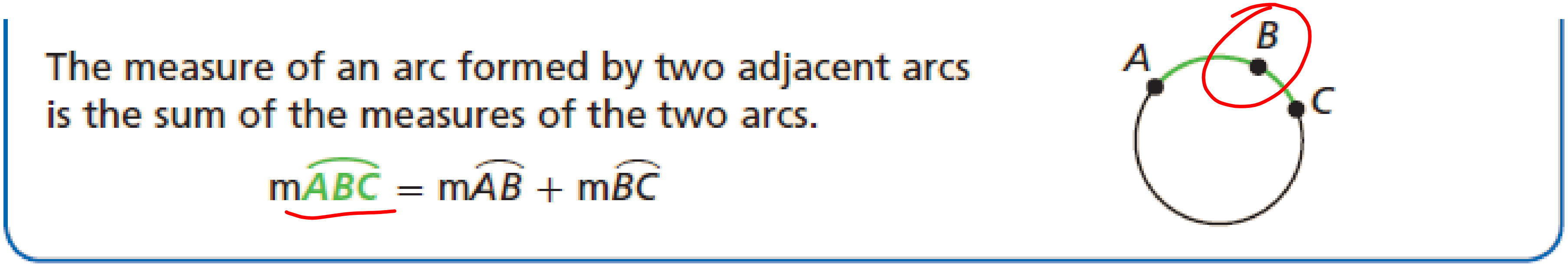
ARC	MEASURE	DIAGRAM
A **minor arc** is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\overset{\frown}{AC} = m\angle ABC = x°$	A B x° C
A **major arc** is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\overset{\frown}{ADC} = 360° - m\angle ABC$ $= 360° - x°$	A B x° C D
If the endpoints of an arc lie on a diameter, the arc is a **semicircle**.	The measure of a semicircle is equal to 180°. $m\overset{\frown}{EFG} = 180°$	F E G

Adjacent Arcs

- **Adjacent arcs** are arcs of the same circle that intersect at exactly one point. $\overset{\frown}{RS}$ and $\overset{\frown}{ST}$ are adjacent arcs.



Arc Addition Postulate

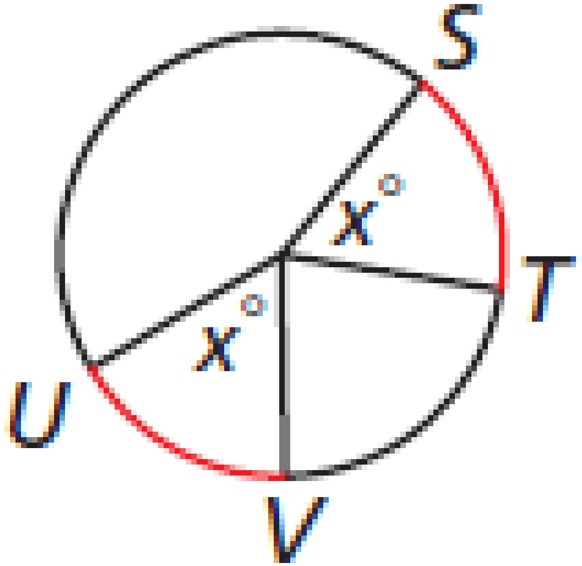


The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$\underline{m\overset{\frown}{ABC}} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$

Congruent Arcs

- **Congruent arcs** are two arcs that have the same measure. In the figure, $\widehat{ST} \cong \widehat{UV}$.



Examples

115

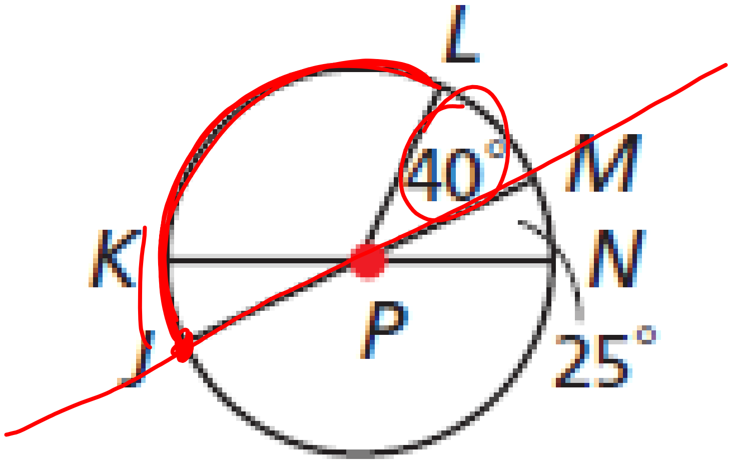
130

~~245~~

$\boxed{140}$

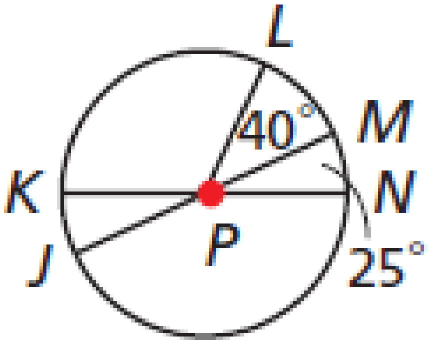
145

- □ Find the measure.
- □ m$\widehat{JKL}$



Examples

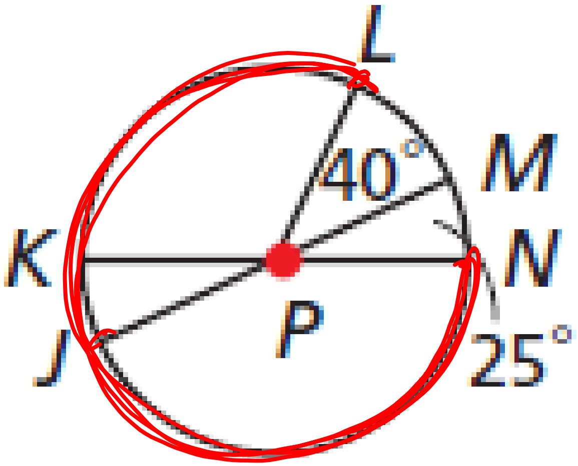
- Find the measure.
- m$\widehat{JKL}$



$\widehat{JKM} = 180°$	semi-circle
$\widehat{LM} = 40°$	given
$\widehat{JKL} = 140°$	180 - 40

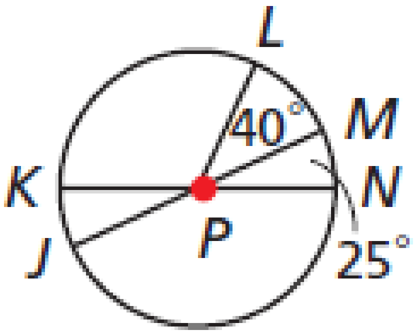
Examples

- ☐ Find the measure.
- ☐ m$\widehat{LJN}$



Examples

- Find the measure.
- $m\widehat{LJN}$



$\widehat{LJN} + \widehat{NML} = 360°$	circle = 360°
$\widehat{NML} = \widehat{NM} + \widehat{ML}$	arc addition post.
$25° + 40° = 65°$	addition
$\widehat{LJN} = 295°$	360 – 65

Arc Length

- Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

$$\begin{array}{r} \text{degree measure of arc} \longrightarrow \\ \text{degree measure of whole circle} \longrightarrow \end{array} \frac{A}{360} = \frac{\ell}{2\pi r} \begin{array}{l} \longleftarrow \text{arc length} \\ \longleftarrow \text{circumference} \end{array}$$

This can also be expressed as $\boxed{\frac{A}{360} \cdot C = \ell.}$

Examples

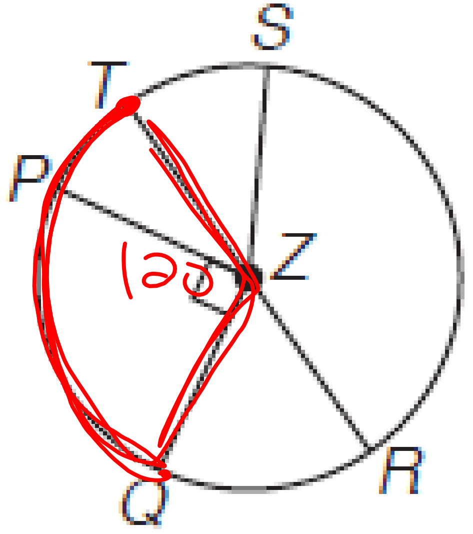
- The radius of ⊙Z is 13.5 units long. Find the length of each arc for the given angle measure.

- $\widehat{QPT}$ if $m\angle QZT = 120$

$$l = \frac{A}{360} 2\pi r = \frac{12\cancel{0}}{36\cancel{0}} \cdot 2\pi 13\ 5$$

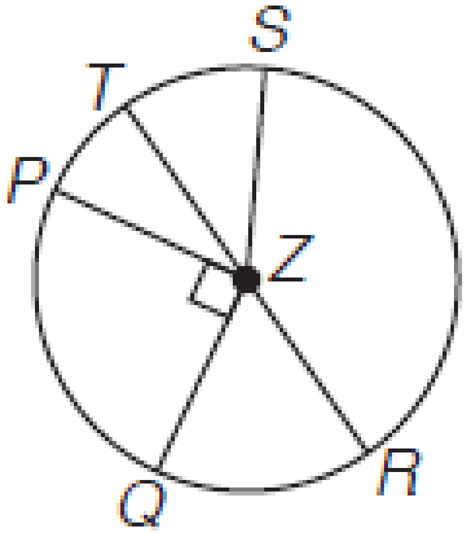
$$= \frac{1}{3} \cdot 27\pi$$

$$\boxed{= 9\pi}$$



Examples

- The radius of ⊙Z is 13.5 units long. Find the length of each arc for the given angle measure.



- $\widehat{QPT}$ if $m\angle QZT = 120$
- $\frac{120}{360} * 2\pi(13.5) = \ell$
- $\frac{1}{3} * 27\pi = \ell$
- $9\pi = \ell$
- $28.26 = \ell$

Examples

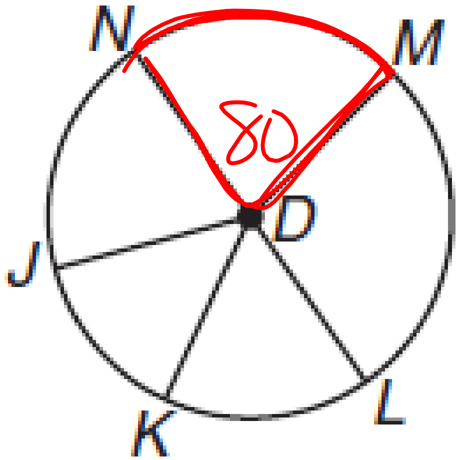
- ☐ The diameter of ⊙Z is 18 units long. Find the length of each arc for the given angle measure.

- ☐ $\widehat{MN}$ if $m\angle MDN = 80$

$$\ell = \frac{A}{360} \cdot \pi d = \frac{8\cancel{0}}{36\cancel{0}} \cdot 18\pi$$

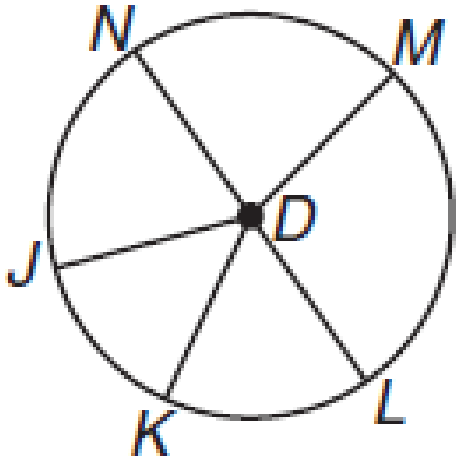
$$= \frac{2}{\cancel{9}} \cdot \overset{2}{\cancel{18}}\pi$$

$$= 4\pi$$



Examples

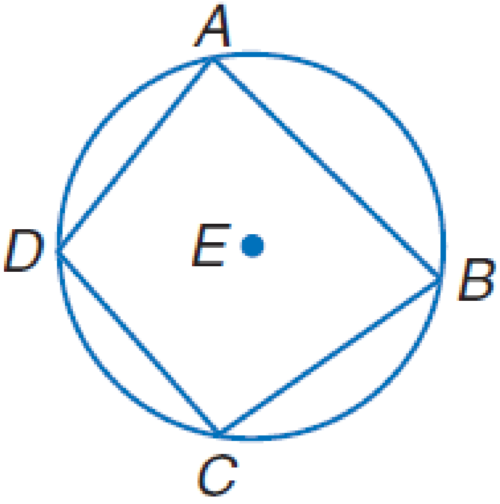
- The diameter of ⊙Z is 18 units long. Find the length of each arc for the given angle measure.



- $\widehat{MN}$ if $m\angle MDN = 80$
- $\frac{80}{360} * \pi(18) = \ell$
- $\frac{2}{9} * 18\pi = \ell$
- $4\pi = \ell$
- $12.26 = \ell$

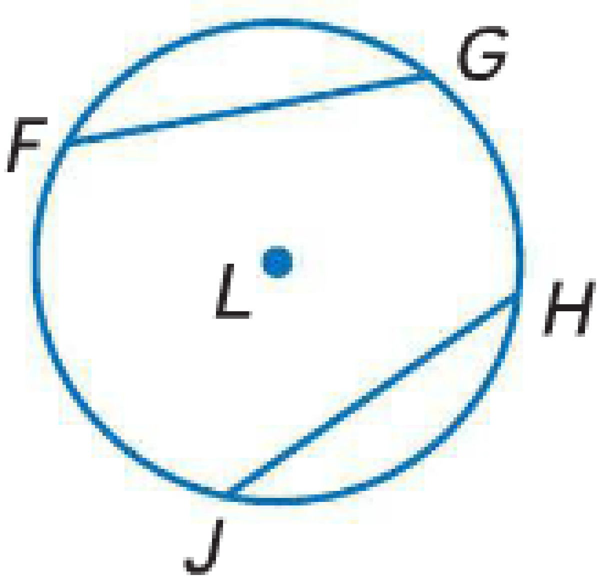
Chord

- Chord – a segment with endpoints on the circle.
- The endpoints of a chord are also endpoints of an arc.
- The chords of adjacent arcs can form a polygon. Quadrilateral *ABCD* is an **inscribed** polygon because all of its vertices lie on the circle.
- Circle *E* is **circumscribed** about the polygon because it contains all the vertices of the polygon.



Congruent Chords Theorem

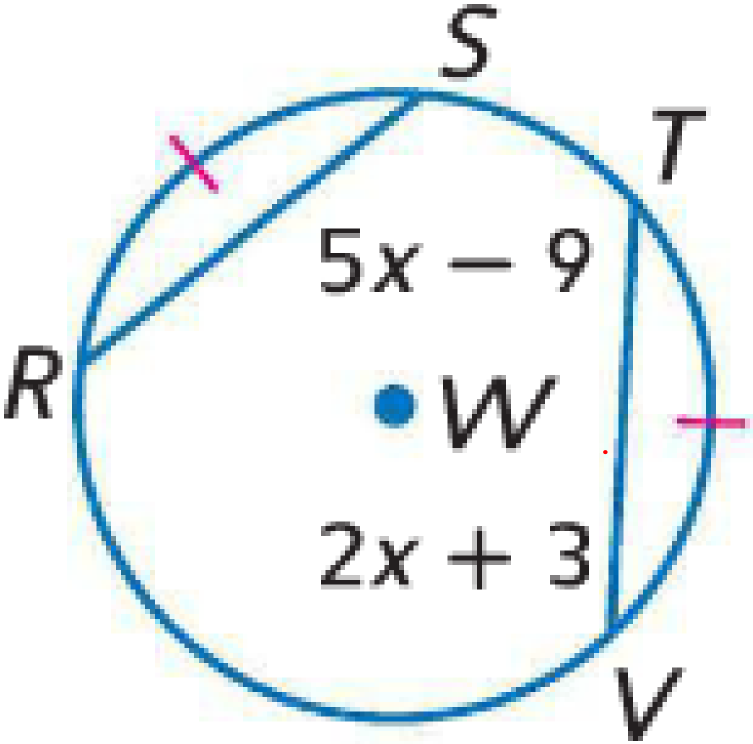
- In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



Examples

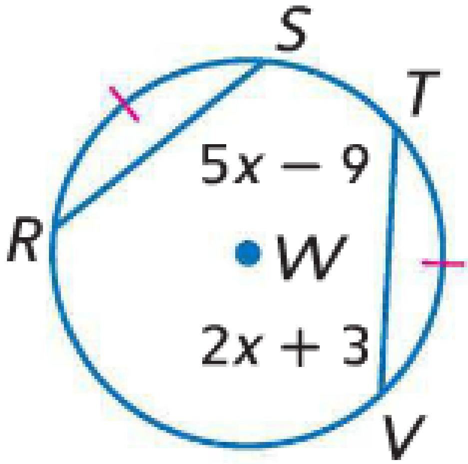
- In ⊙W, arc RS ≅ arc TV. Find RS.

$5x-9=2x+3$



Examples

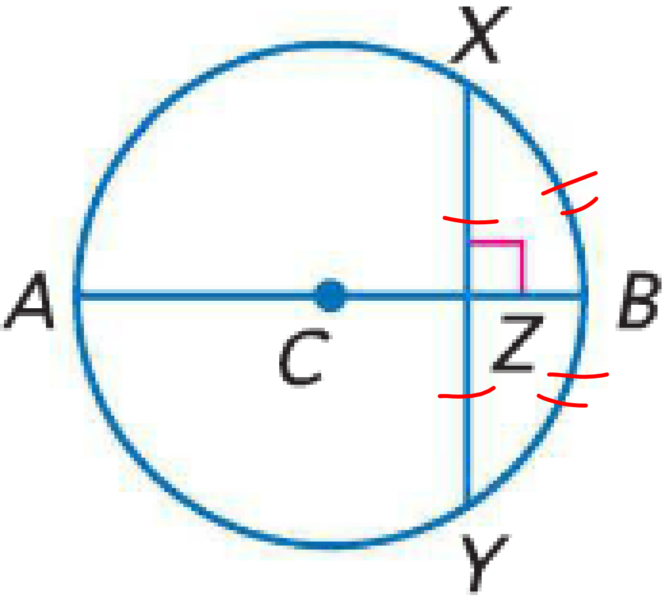
- In ⊙W, arc RS ≅ arc TV. Find RS.



- $RS \cong TV$
- $5x - 9 = 2x + 3$
- $3x = 12$
- $x = 4$
- $RS = 5(4) - 9 = 20 - 9 = 11$

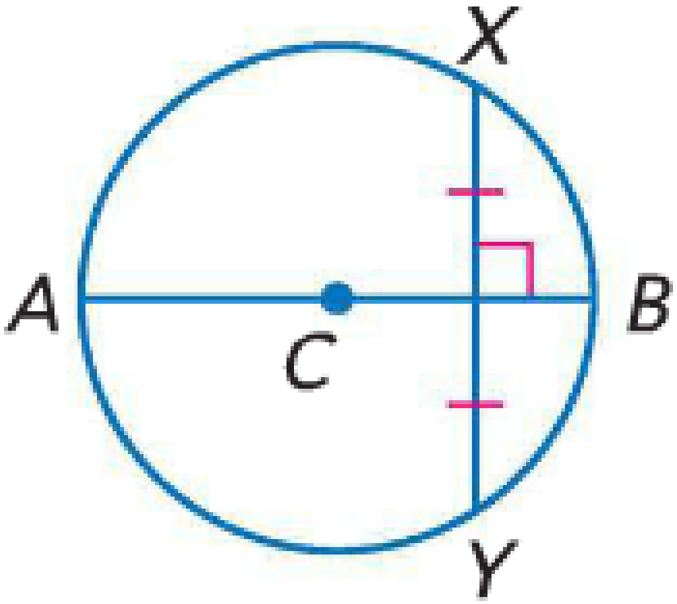
Perpendicular Bisector Theorem

- If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.



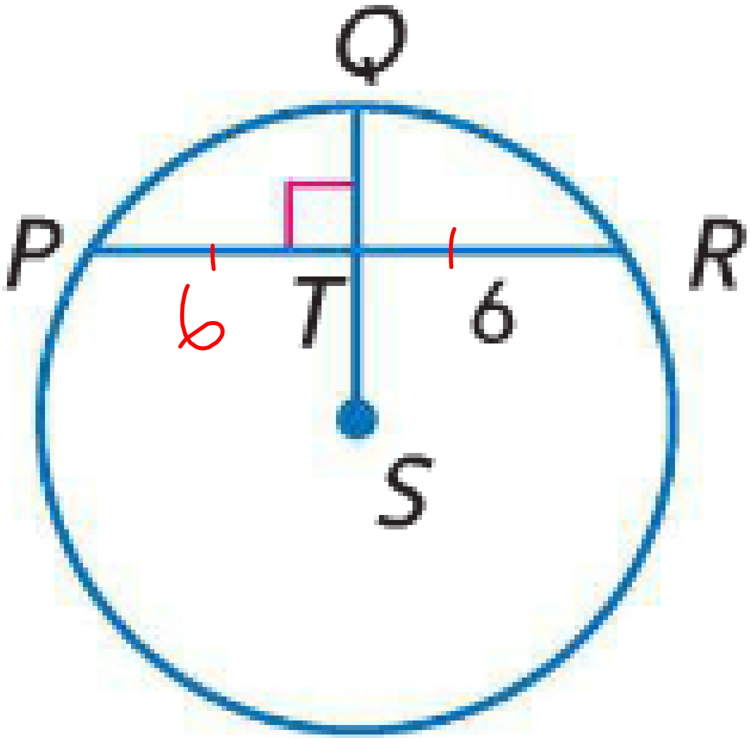
Converse of the Perpendicular Bisector Theorem

- The perpendicular bisector of a chord is a diameter (or radius) of the circle.



Examples

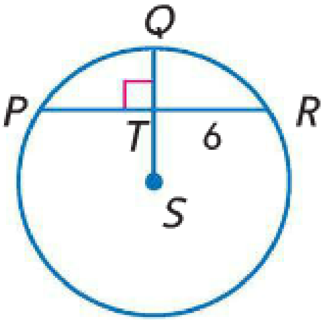
- In ⨀S, find PR.=12



Examples

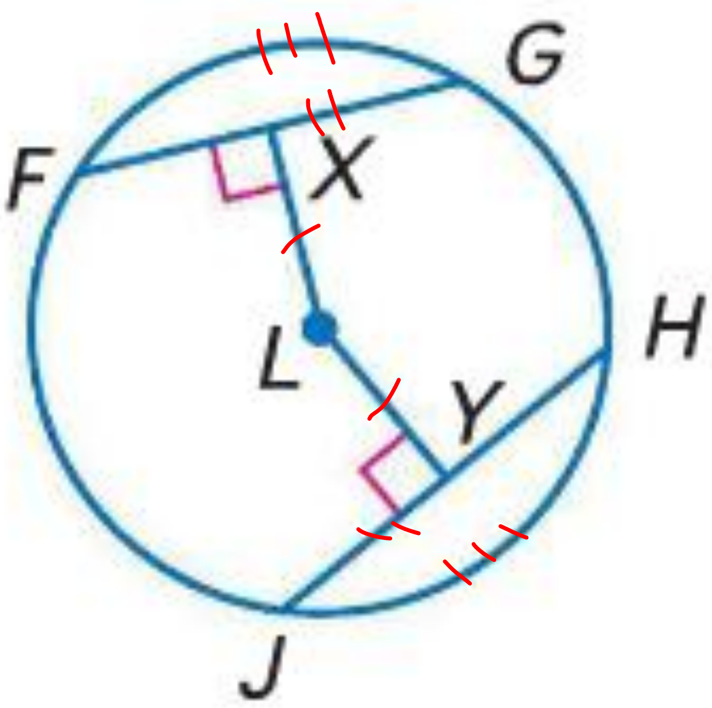
- In ⊙S, find PR.

- If TR = 6, then PR = 12



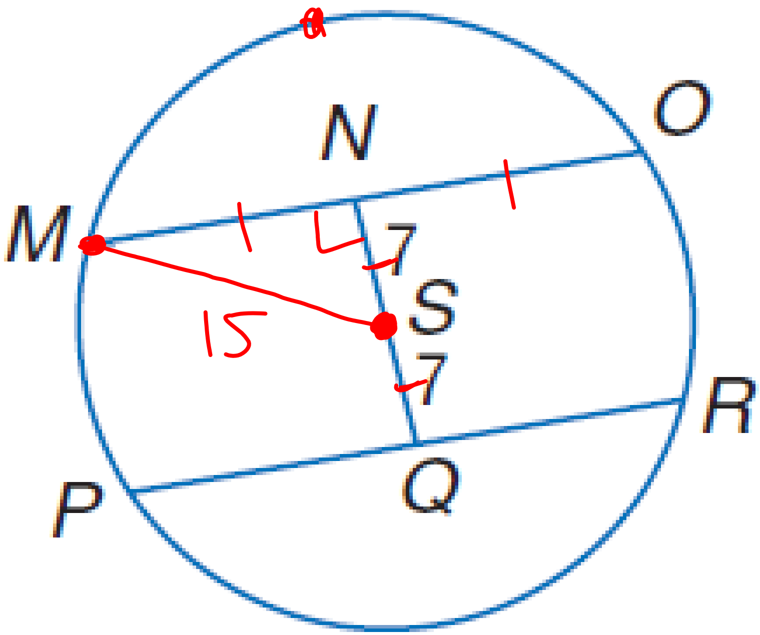
Theorem

- In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



Examples

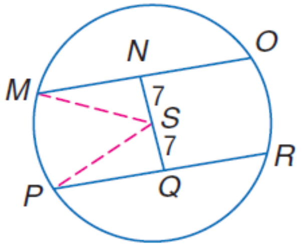
- Chords *MO* and *PR* are equidistant from the center. If the radius of ⊙*S* is 15, find *MO* and *PQ*.



$15^2 - 7^2 = MN^2$

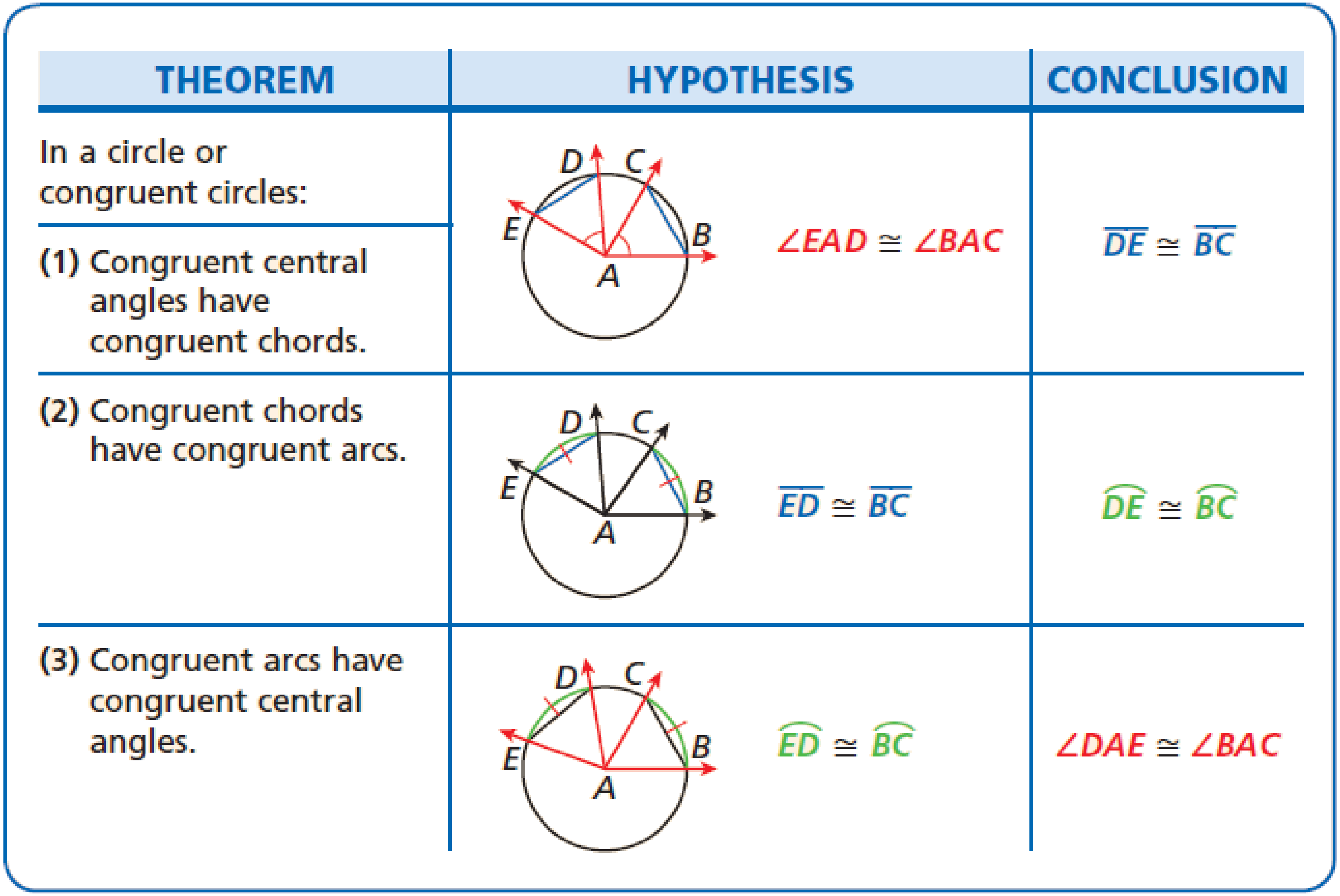
Examples

- Chords *MO* and *PR* are equidistant from the center. If the radius of ⊙*S* is 15, find *MO* and *PQ*.



- Since the radius of the circle is 15, then PS = 15.
- Use Pythagorean Theorem to solve the right triangle.
- PQ = $\sqrt{PS^2 - SQ^2} = \sqrt{15^2 - 7^2}$ = 13.27
- MO = 2PQ = 2 * 13.27 = 26.54

Central Angles/Chords/Arcs



THEOREM	HYPOTHESIS	CONCLUSION
In a circle or congruent circles:		
(1) Congruent central angles have congruent chords.	$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.	$\overline{ED} \cong \overline{BC}$	$\overset{\frown}{DE} \cong \overset{\frown}{BC}$
(3) Congruent arcs have congruent central angles.	$\overset{\frown}{ED} \cong \overset{\frown}{BC}$	$\angle DAE \cong \angle BAC$