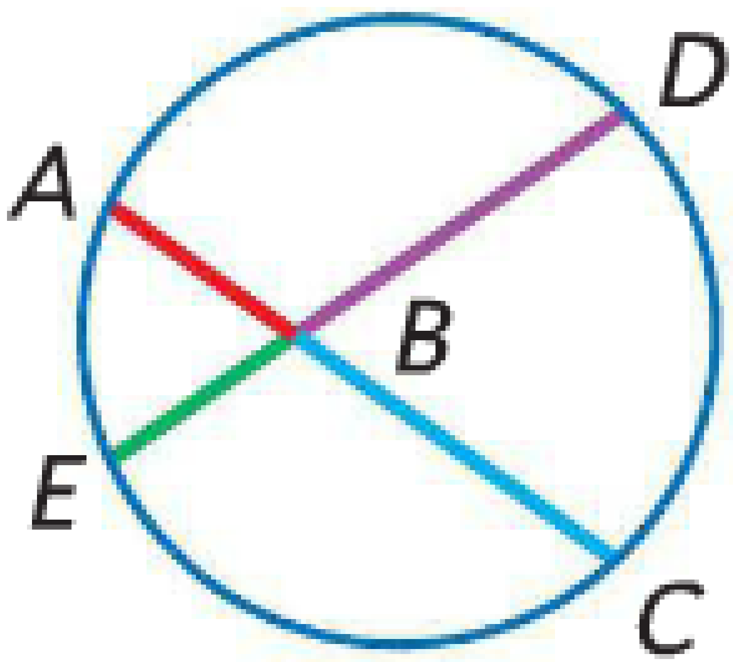
Special Segments in a Circle

Segments of Chords Theorem

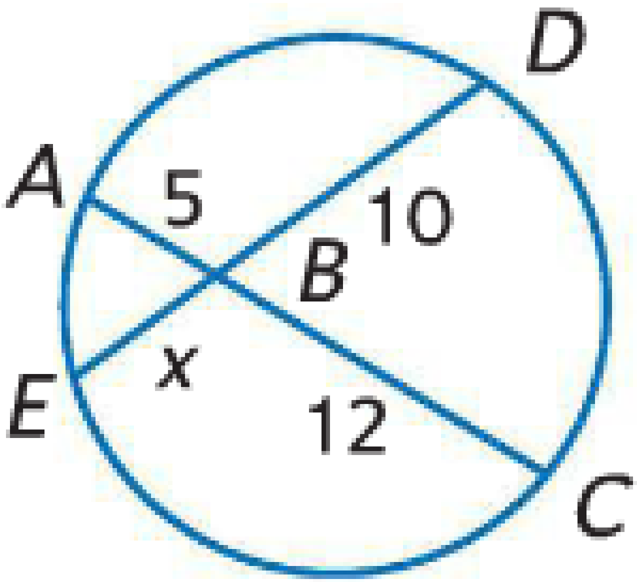
- If two chords intersect in a circle, then the products of the lengths of the chord segments are equal.

multiply



Examples

- Find x.



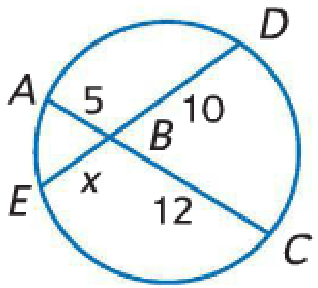
$$\frac{5 \cdot 12}{10} = \frac{x \cdot 10}{10}$$

60

$$6 = x$$

Examples

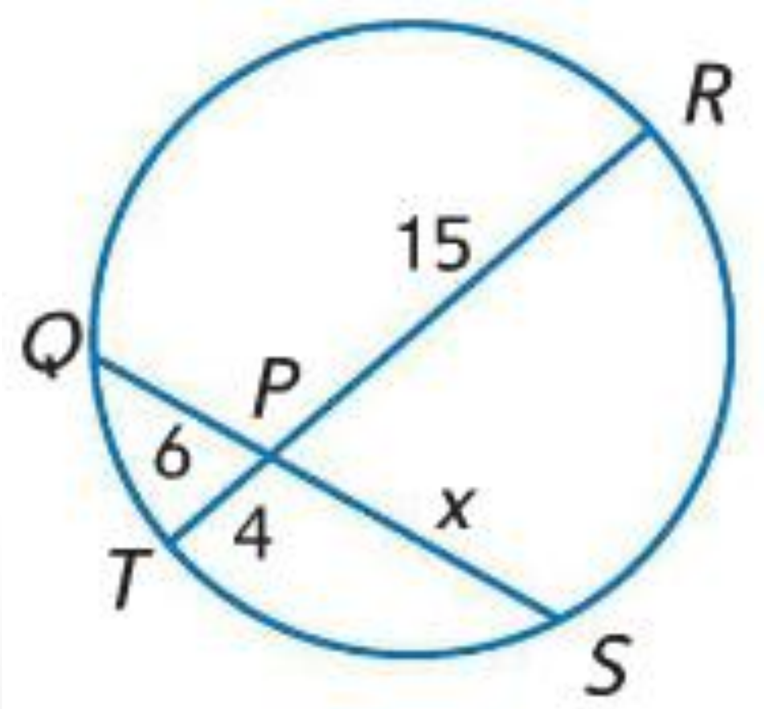
- Find x.



- 5 * 12 = x * 10 — The products of the lengths of the chord segments are equal.
- 60 = 10x — Multiply
- 6 = x — Divide both sides by 10

Examples

- Find x.



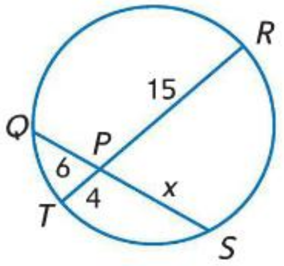
60

$$\frac{4 \cdot 15}{6} = \frac{6 \cdot x}{6}$$

$$10 = x$$

Examples

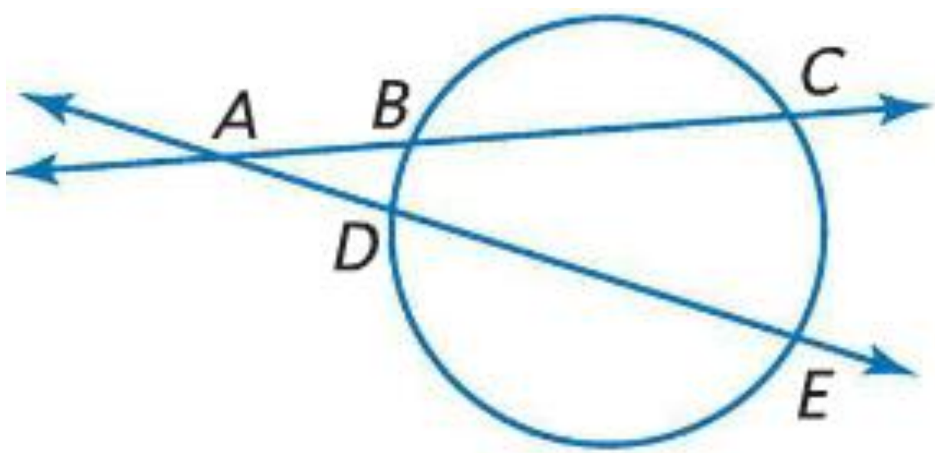
- Find x.



- 4 * 15 = 6 * x — The products of the lengths of the chord segments are equal.
- 60 = 6x — Multiply
- 10 = x — Divide both sides by 6

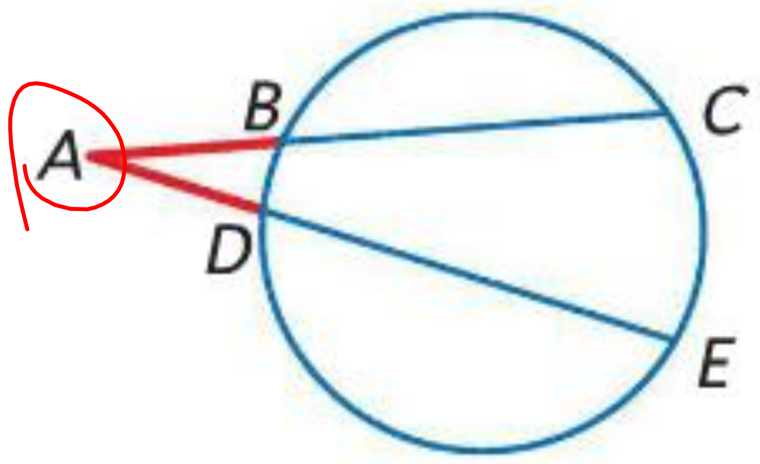
Secant Segments

- A secant segment is a segment of a secant line that has exactly one endpoint on the circle.
- A secant segment that lies in the exterior of the circle is called an external secant segment.



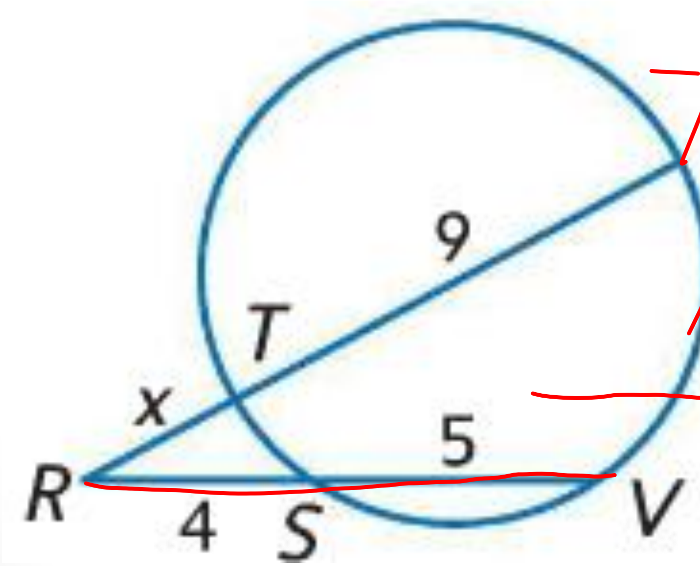
Secant Segments Theorem

- If two secants intersect in the exterior of a circle, then the product of the measures of one secant segment and its external secant is equal to the product of the measures of the other secant and its external secant segment.



Examples

- Find x.



$$(5+4)\cdot 4 = (9+x)x$$

$$36 = x^2 + 9x$$

$$-36 \qquad -36$$

$$0 = 1x^2 + 9x - 36$$

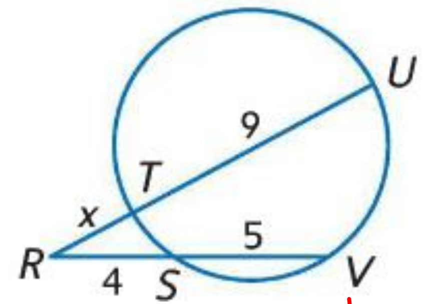
$$-36 = (x-3)(x+12)$$

1	36
2	18
-3	12

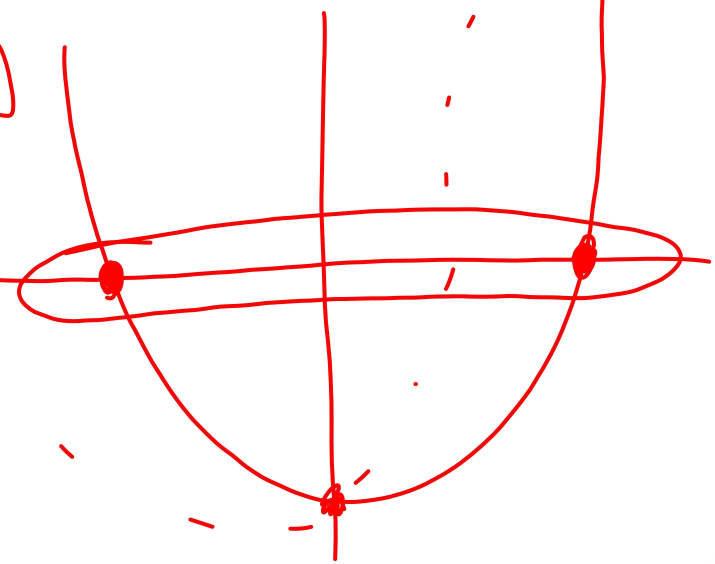
$$x = 3, \cancel{-12}$$

Examples

- Find x.

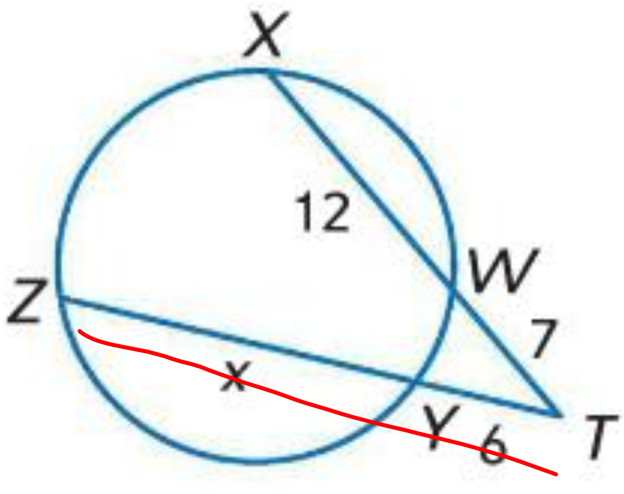


- $(9 + x) * x = 9 * 4$ — Whole line times outside piece on both sides
- $9x + x^2 = 36$ — Multiply
- $x^2 + 9x - 36 = 0$ — Set equation equal to zero
- $(x + 12)(x - 3) = 0$ — Factor the equation
- $x = -12, 3; x = 3$ — Choose sensible answer



Examples

- Find x.



$$(12+7)7 = (x+6)6$$

$$19 \cdot 7 = 6x + 36$$

$$133 = 6x + 36$$

$$-36 \qquad -36$$

$$\frac{97}{6} = \frac{6x}{6}$$

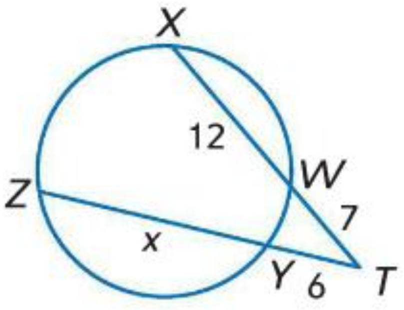
$$\frac{97}{6} = x$$

$$10\frac{1}{6}$$

$$\frac{97}{6}$$

Examples

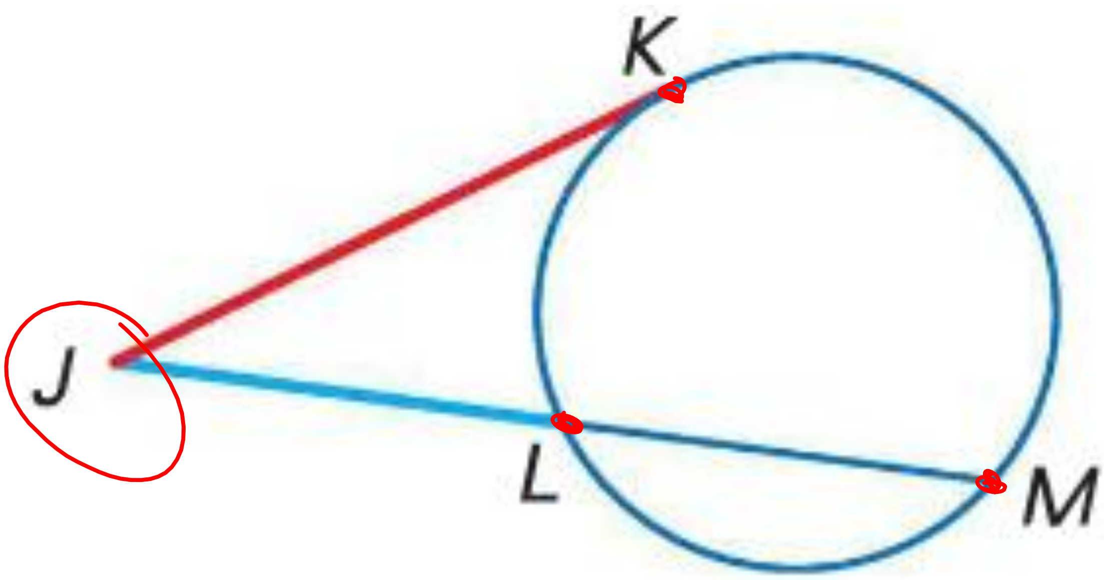
- Find x.



- (x + 6) * 6 = 19 * 7 — Whole line times outside piece on both sides
- 6x + 36 = 133 — Multiply
- 6x = 97 — Subtract 36 from both sides
- x = 16.17 — Divide both sides by 6

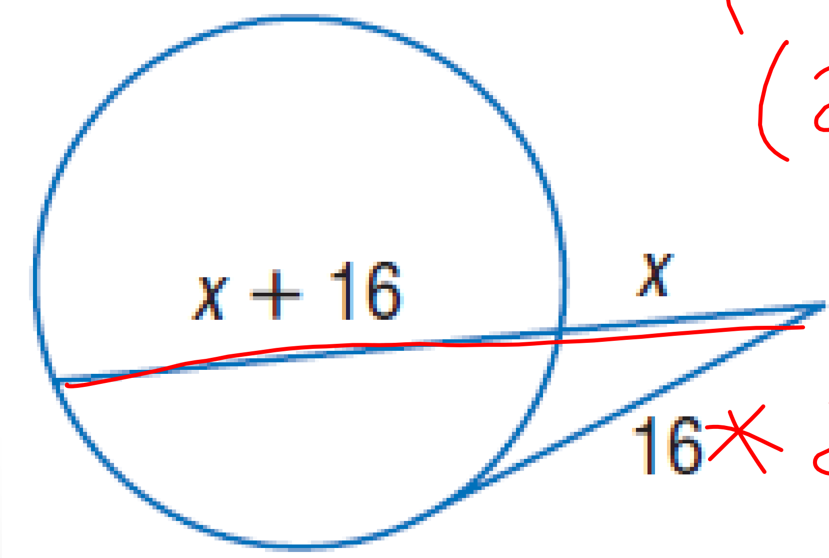
Tangent-Secant Segment Theorem

- If a tangent and a secant intersect in the exterior of a circle, then the square of the measure of the tangent is equal to the product of the measures of the secant and its external secant segment.



Examples

- Find x. Assume that segments that appear to be tangent are tangent.



16^2

$(x+16+x)x = 16 \cdot 16$

$(2x+16)x = 256$

$2x^2+16x = 256$

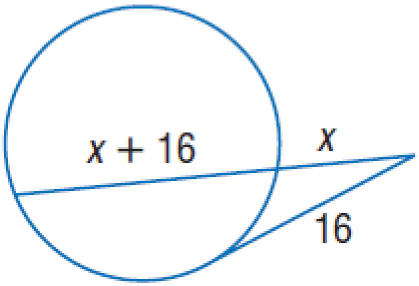
-256

$2x^2+16x-256 = 0$

~~-16~~, 8

Examples

- Find x. Assume that segments that appear to be tangent are tangent.



- $16^2 = x*(x + x + 16)$ Tangent-Secant Segment Theorem
- $256 = 2x^2 + 16x$ Simplify the equation
- $0 = 2x^2 + 16x - 256$ Subtract 256 from both sides
- $0 = x^2 + 8x - 128$ Reduce the equation
- $0 = (x + 16)(x – 8)$ Factor the equation
- $x = -16, 8$ Solve;
- $x = 8$ Choose sensible answer