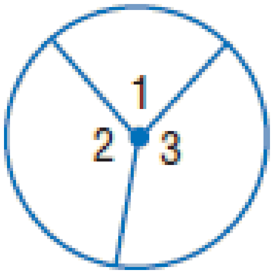
ANGLES, ARCS, AND CHORDS

Central Angle

- A **central angle** is an angle whose vertex is the center of a circle.

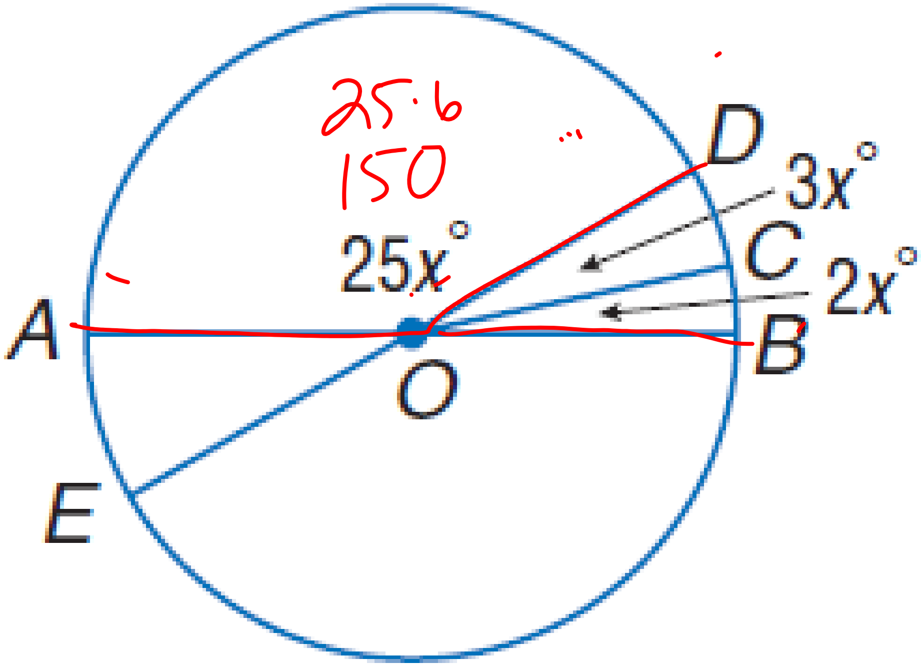
The sum of the measures of the central angles of a circle with no interior points in common is 360.

$m\angle 1 + m\angle 2 + m\angle 3 = 360$



Examples

- Find m$\angle AOD$. = 150



$$25x + 3x + 2x = 30x$$

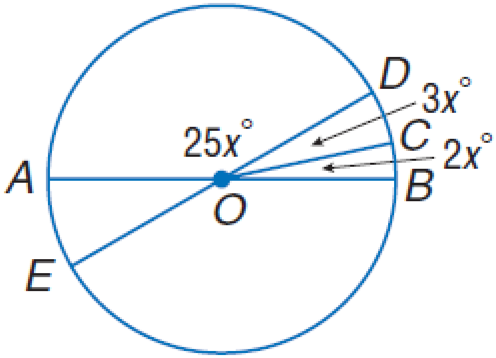
$$30x \times 2 = 60x$$

$$\frac{60x}{60} = \frac{360}{60}$$

$$x = 6$$

Examples

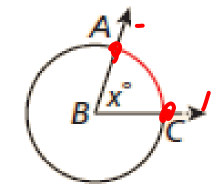
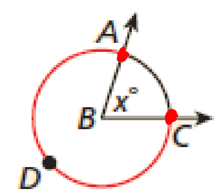
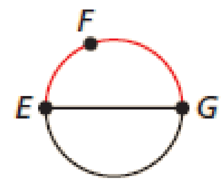
- Find m$\angle AOD$.



- m$\angle AOD$ + m$\angle DOB = 180$
- 25x + 3x + 2x = 180
- 30x = 180
- x = 6
- 25(6) = 150

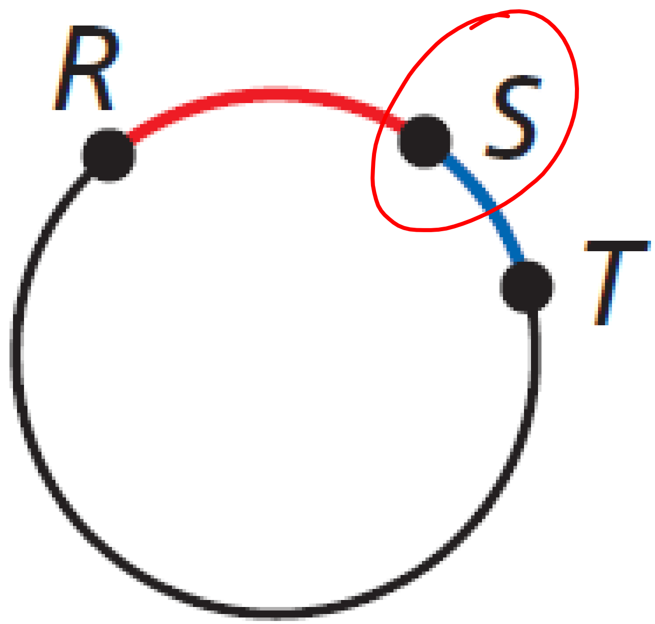
Arc

- An **arc** is an unbroken part of a circle consisting of two points called the endpoints and all the points on the circle between them.

ARC	MEASURE	DIAGRAM
A minor arc is an arc whose points are on or in the interior of a central angle.	The measure of a minor arc is equal to the measure of its central angle. $m\overset{\frown}{AC} = m\angle ABC = x^\circ$	
A major arc is an arc whose points are on or in the exterior of a central angle.	The measure of a major arc is equal to 360° minus the measure of its central angle. $m\overset{\frown}{ADC} = 360^\circ - m\angle ABC = 360^\circ - x^\circ$	
If the endpoints of an arc lie on a diameter, the arc is a semicircle.	The measure of a semicircle is equal to 180°. $m\overset{\frown}{EFG} = 180^\circ$	

Adjacent Arcs

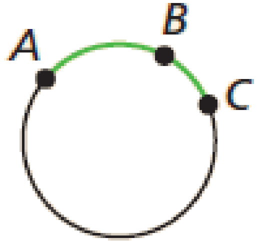
- **Adjacent arcs** are arcs of the same circle that intersect at exactly one point. $\widehat{RS}$ and $\widehat{ST}$ are adjacent arcs.



Arc Addition Postulate

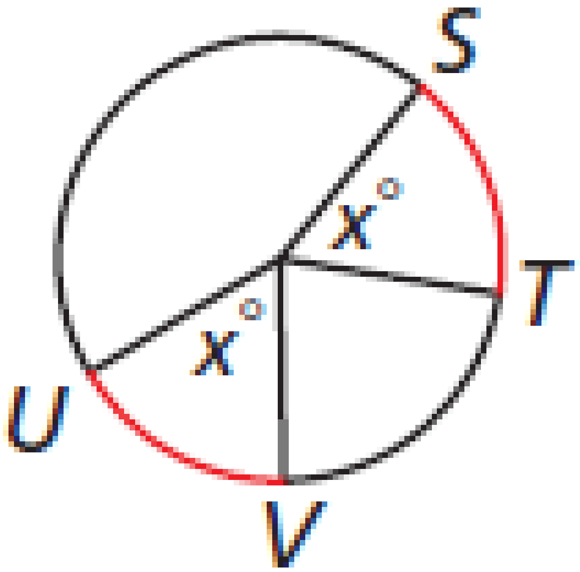
The measure of an arc formed by two adjacent arcs is the sum of the measures of the two arcs.

$$m\overset{\frown}{ABC} = m\overset{\frown}{AB} + m\overset{\frown}{BC}$$



Congruent Arcs

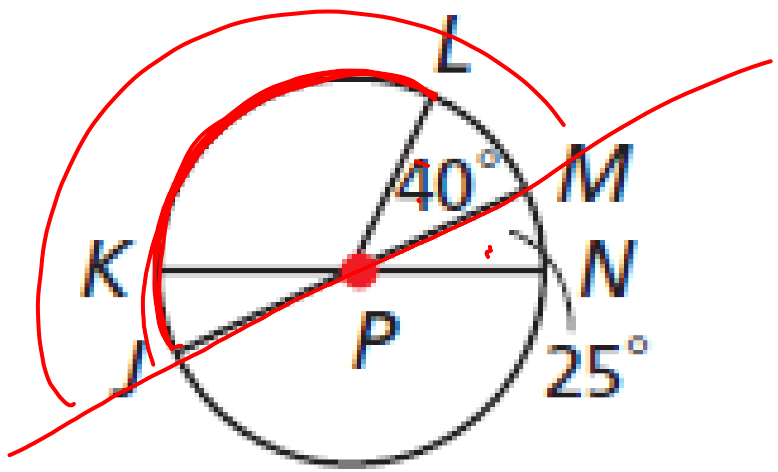
- **Congruent arcs** are two arcs that have the same measure. In the figure, $\widehat{ST} \cong \widehat{UV}$.



Examples

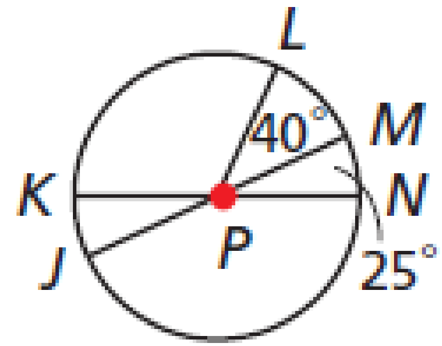
140

- Find the measure.
- m$\widehat{JKL}$



Examples

- Find the measure.
- m$\widehat{JKL}$

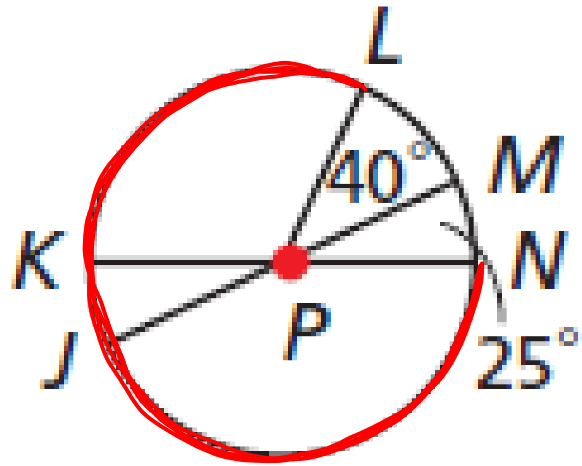


- $\widehat{JKM} = 180°$ semi-circle
- $\widehat{LM} = 40°$ given
- $\widehat{JKL} = 140°$ 180 - 40

295

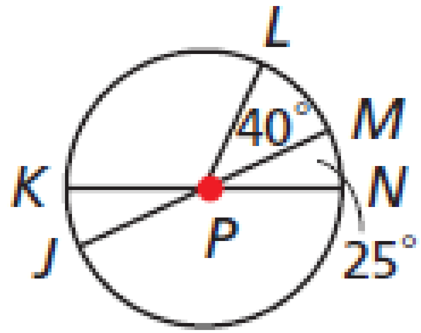
Examples

- Find the measure.
- $m\widehat{LJN}$



Examples

- Find the measure.
- m$\widehat{LJN}$



- $\widehat{LJN} + \widehat{NML} = 360°$ circle = 360°
- $\widehat{NML} = \widehat{NM} + \widehat{ML}$ arc addition post.
- $25° + 40° = 65°$ addition
- $\widehat{LJN} = 295°$ 360 – 65

Arc Length

- Another way to measure an arc is by its length. An arc is part of the circle, so the length of an arc is a part of the circumference.

$$\text{degree measure of arc} \rightarrow \frac{A}{360} = \frac{\ell}{2\pi r} \leftarrow \text{arc length} \atop \leftarrow \text{circumference}$$

degree measure of whole circle → 360

πd

This can also be expressed as $\frac{A}{360} \cdot C = \ell$.

Examples

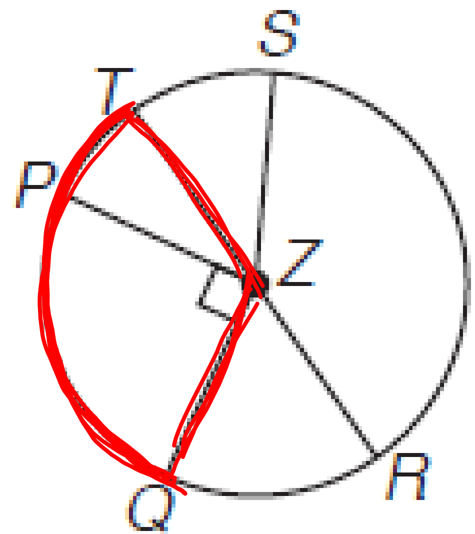
- The radius of ⊙Z is 13.5 units long. Find the length of each arc for the given angle measure.

- $\overset{\frown}{QPT}$ if $m\angle QZT = 120$

$$\ell = \frac{A}{360} \cdot C = \frac{120}{360} \cdot 27\pi$$

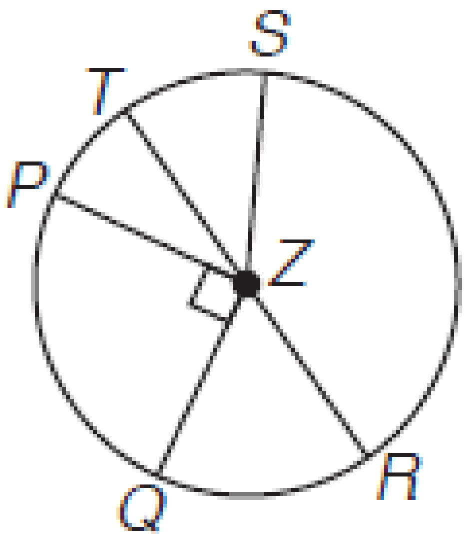
$$= \frac{1}{3} 27\pi$$

$$= 9\pi$$



Examples

- The radius of ⊙Z is 13.5 units long. Find the length of each arc for the given angle measure.

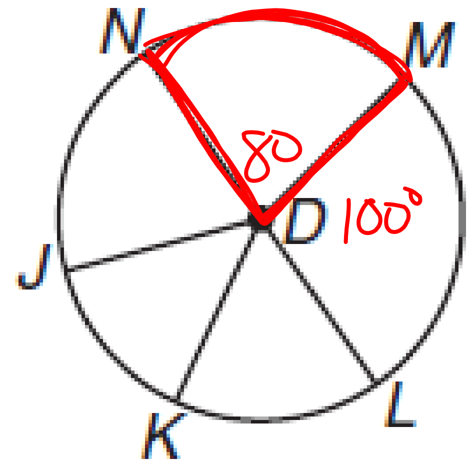


- $\widehat{QPT}$ if $m\angle QZT = 120$
- $\frac{120}{360} * 2\pi(13.5) = \ell$
- $\frac{1}{3} * 27\pi = \ell$
- $9\pi = \ell$
- $28.26 = \ell$

Examples

- The diameter of ⊙Z is 18 units long. Find the length of each arc for the given angle measure.

- $\widehat{MN}$ if $m\angle MDN = 80$



$$l = \frac{A}{360} \cdot \pi d = \frac{80}{360} \cdot 18\pi$$

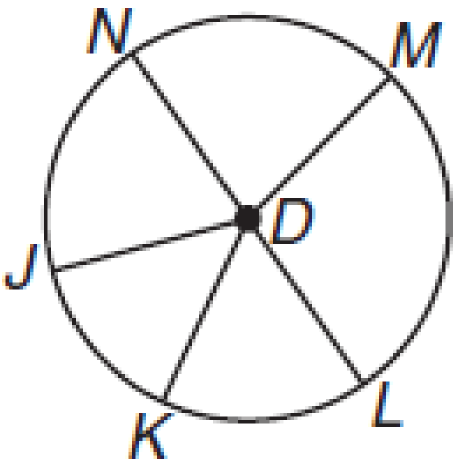
$$= \frac{2}{9} \cdot 18\pi$$

$$= 4\pi$$

4π
100
60

Examples

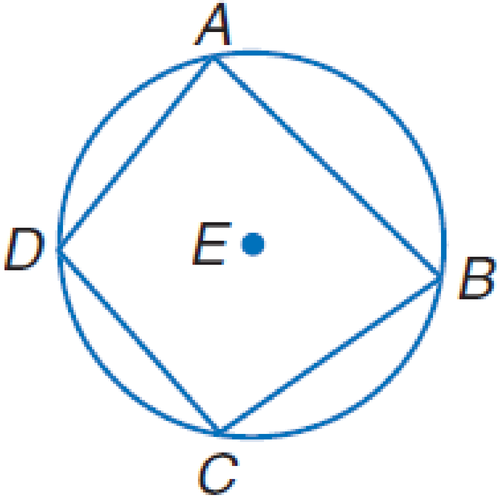
- The diameter of ⊙Z is 18 units long. Find the length of each arc for the given angle measure.



- $\widehat{MN}$ if $m\angle MDN = 80$
- $\frac{80}{360} * \pi(18) = \ell$
- $\frac{2}{9} * 18\pi = \ell$
- $4\pi = \ell$
- $12.26 = \ell$

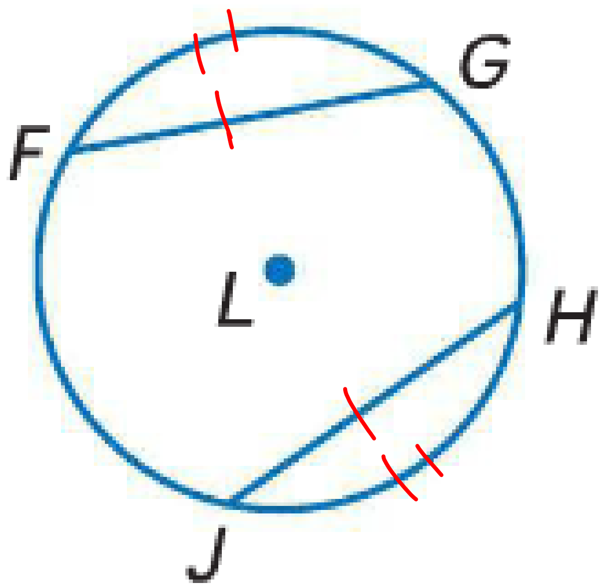
Chord

- Chord – a segment with endpoints on the circle.
- The endpoints of a chord are also endpoints of an arc.
- The chords of adjacent arcs can form a polygon. Quadrilateral *ABCD* is an **inscribed** polygon because all of its vertices lie on the circle.
- Circle *E* is **circumscribed** about the polygon because it contains all the vertices of the polygon.



Congruent Chords Theorem

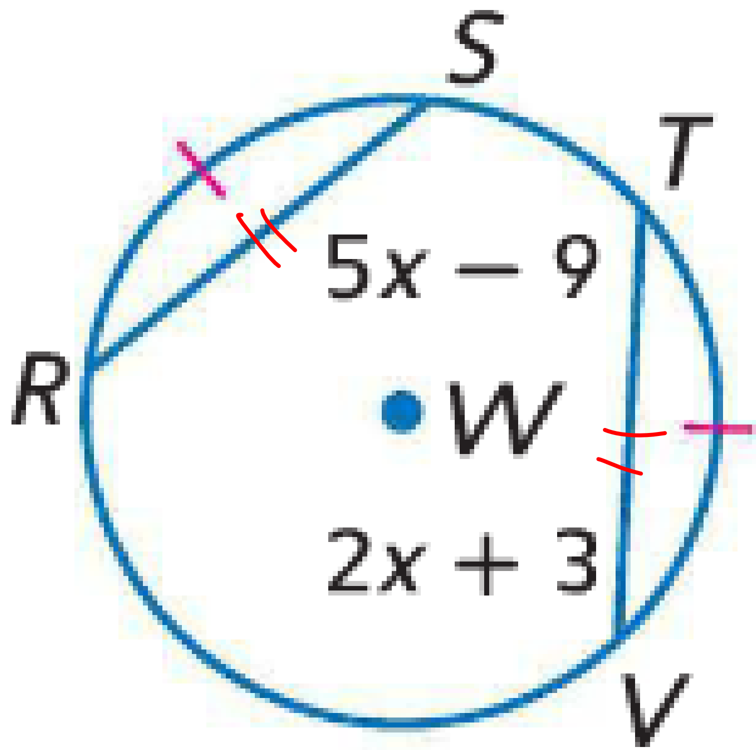
- In the same circle or in congruent circles, two minor arcs are congruent if and only if their corresponding chords are congruent.



Examples

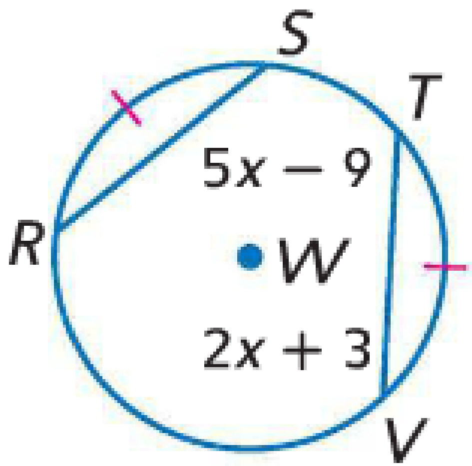
- In ⊙W, arc RS ≅ arc TV. Find RS.

$5x - 9 = 2x + 3$



Examples

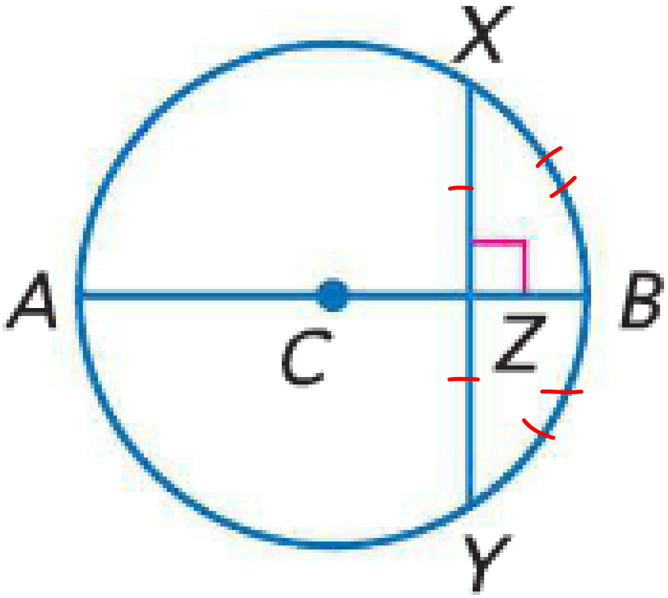
- In ⊙W, arc RS ≅ arc TV. Find RS.



- RS ≅ TV
- $5x - 9 = 2x + 3$
- $3x = 12$
- $x = 4$
- $RS = 5(4) - 9 = 20 - 9 = 11$

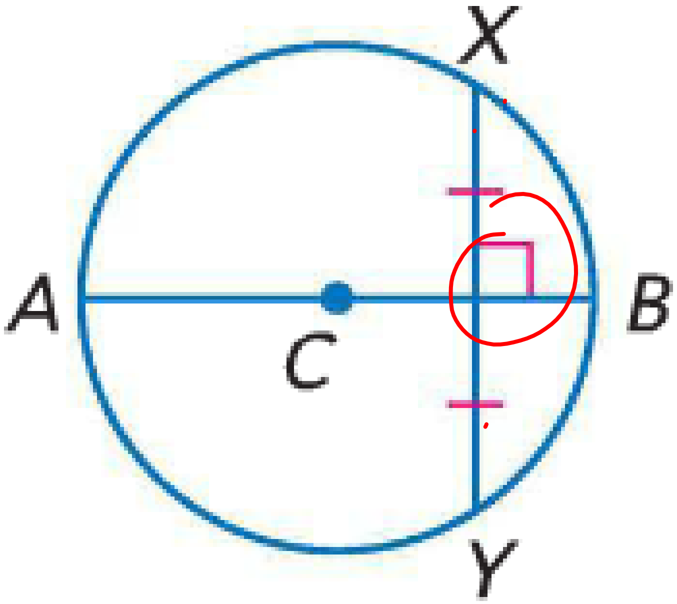
Perpendicular Bisector Theorem

- If a diameter (or radius) of a circle is perpendicular to a chord, then it bisects the chord and its arc.



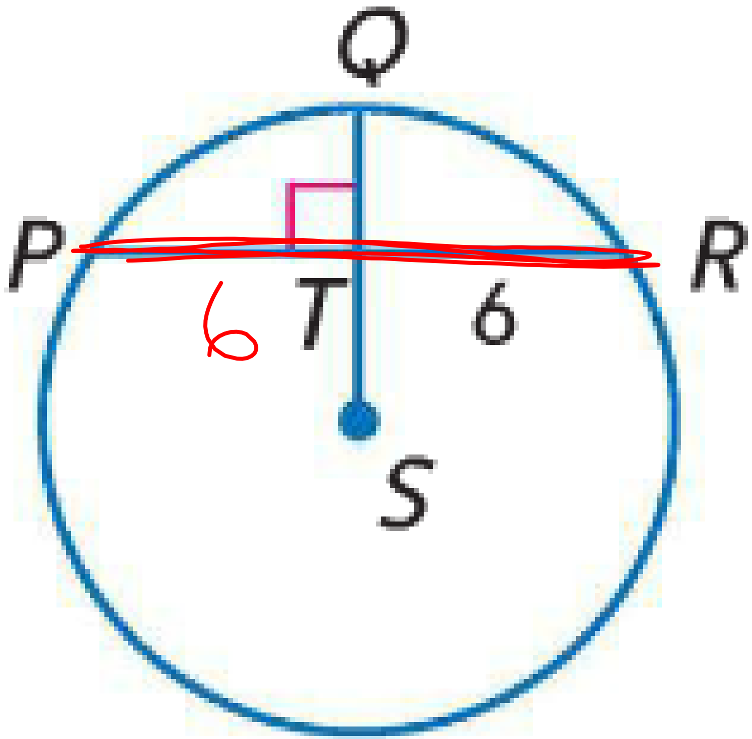
Converse of the Perpendicular Bisector Theorem

- The perpendicular bisector of a chord is a diameter (or radius) of the circle.



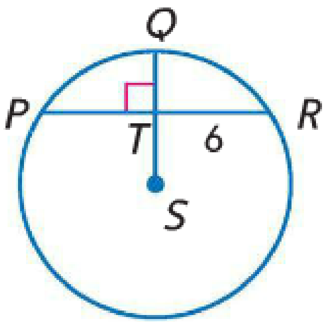
Examples

- In ⊙S, find PR. 12



Examples

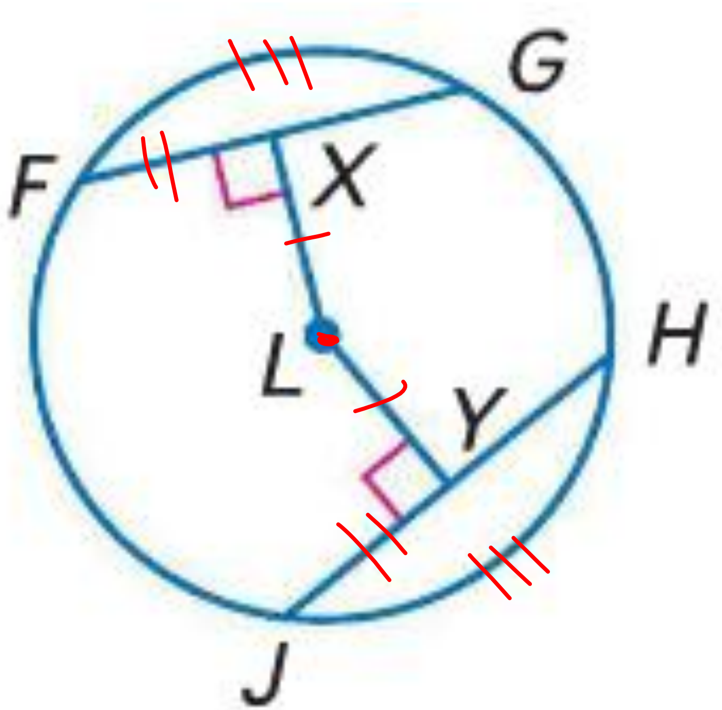
- In ⊙S, find PR.



- If TR = 6, then PR = 12

Theorem

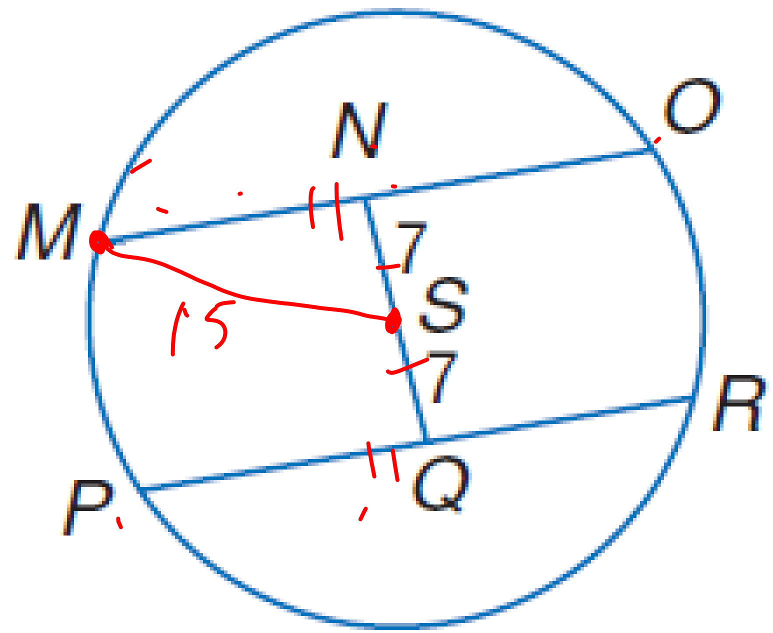
- In the same circle or in congruent circles, two chords are congruent if and only if they are equidistant from the center.



Examples

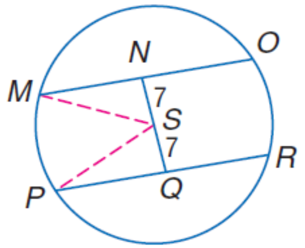
- Chords MO and PR are equidistant from the center. If the radius of $\odot S$ is 15, find MO and PQ.

$15^2 - 7^2 = MN^2$



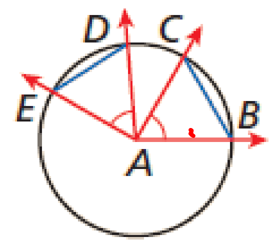
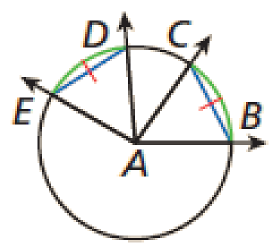
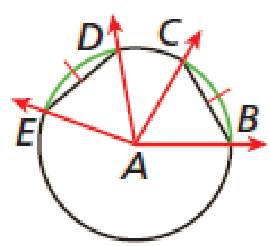
Examples

- Chords MO and PR are equidistant from the center. If the radius of $\odot S$ is 15, find MO and PQ.



- Since the radius of the circle is 15, then PS = 15.
- Use Pythagorean Theorem to solve the right triangle.
- $PQ = \sqrt{PS^2 - SQ^2} = \sqrt{15^2 - 7^2} = 13.27$
- $MO = 2PQ = 2 * 13.27 = 26.54$

Central Angles/Chords/Arcs

THEOREM	HYPOTHESIS		CONCLUSION
In a circle or congruent circles:			
(1) Congruent central angles have congruent chords.		$\angle EAD \cong \angle BAC$	$\overline{DE} \cong \overline{BC}$
(2) Congruent chords have congruent arcs.		$\overline{ED} \cong \overline{BC}$	$\overset{\frown}{DE} \cong \overset{\frown}{BC}$
(3) Congruent arcs have congruent central angles.		$\overset{\frown}{ED} \cong \overset{\frown}{BC}$	$\angle DAE \cong \angle BAC$