CHAPTER 12

Vocabulary

The table contains important vocabulary terms from Chapter 12. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
arithmetic sequence			
explicit formula			
finite sequence			
geometric mean			
geometric sequence			
infinite geometric series			

Vocabulary

Know it! Note

The table contains important vocabulary terms from Chapter 12. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
arithmetic sequence	879	A sequence whose successive terms differ by the same nonzero number d, called the common difference.	4, 7, 10, 13, 16, … + 3 + 3 + 3 + 3 $d = 3$
explicit formula	863	A formula that defines the nth term a_n, or general term, of a sequence as a function of n.	Sequence: 4, 7, 10, 13, 16, … Recursive formula: $a_1 = 4$, $a_n = a_{n-1} + 3$ Explicit formula: $a_n = 1 + 3n$
finite sequence	862	A sequence with a finite number of terms.	0, 1, 2, 3, 4, 5, 6, 7, 8, 9
geometric mean	892	For positive numbers a and b, the positive number x such that $\frac{a}{x} = \frac{x}{b}$. In a geometric sequence, a term that comes between two given nonconsecutive terms of the sequence.	$\frac{a}{x} = \frac{x}{b}$ $x^2 = ab$ $x = \sqrt{ab}$
geometric sequence	890	A sequence in which the ratio of successive terms is a constant r, called the common ratio, where $r \neq 0$ and $r \neq 1$.	1, 2, 4, 8, 16, ... · 2 · 2 · 2 · 2 $r = 2$
infinite geometric series	900	A geometric series with infinitely many terms.	$\sum_{n=1}^{\infty} \frac{1}{2^n} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \dots$

Term	Page	Definition	Clarifying Example
infinite sequence			
limit			
partial sum			
recursive formula			
sequence			
series			
summation notation			
term of a sequence			

Term	Page	Definition	Clarifying Example
infinite sequence	862	A sequence with infinitely many terms.	1, 3, 5, 7, 9, 11, …
limit	900	A number (or infinity) that the terms of an infinite sequence or series approach as the term number increases.	The series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$ has a limit of 1.
partial sum	870	Indicated by $S_n = \sum_{i-1}^{n} a_i$, the sum of a specified number of terms n of a sequence whose total number of terms is greater than n.	For the sequence $a_n = n_2$, the fourth partial sum of the infinite series $\sum_{k-1}^{\infty} k^2$ is $\sum_{k-1}^{4} k^2 = 1^2 + 2^2 + 3^2 + 4^2 = 30.$
recursive formula	862	A formula for a sequence in which one or more previous terms are used to generate the next term.	Formula: $a_1 = 5$; $a_n = a_{n-1} + 2$ Sequence: 5, 7, 9, 11, …
sequence	862	A list of numbers that often form a pattern.	1, 2, 4, 8, 16, …
series	870	The indicated sum of the terms of a sequence.	Sequence: $a_n = 1, 2, 4, 8, 16$ Series: $1 + 2 + 4 + 8 + 16$ Sum of the series: 31
summation notation	870	A method of notating the sum of a series using the Greek letter Σ (capital sigma).	$\sum_{k-1}^{5} 3k = 3 + 6 + 9 + 12 + 15$, where the rule for the kth term of the sequence is $a_k = 3k$.
term of a sequence	862	An element or number in the sequence.	5 is the third term in the sequence 1, 3, 5, 7, …

CHAPTER 12

Chapter Review

12-1 Introduction to Sequences

Find the first 5 terms of each sequence.

1. $a_n = \frac{4}{5}n$

2. $a_n = 3^{n-1}$

3. $a_n = \frac{n}{n+1}$

4. $a_n = n^2 - 3n$

Write a possible explicit rule for the *n*th term of each sequence.

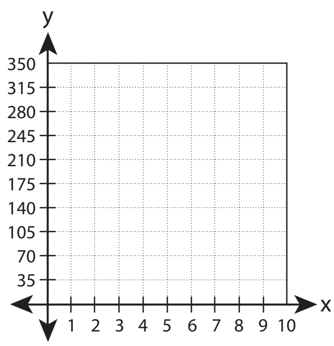
5. 1, 3, 5, 7, ...

6. −2, −4, −6, −8, ...

7. 400, 200, 100, 50, 25, ...

8. 366, 344, 322, 300, 278, ...

9. A car traveling at 65 mi/h passes a mile marker that reads mile 23. If the car maintains this speed for 5 hours, what mile marker should the car pass? Graph the sequence for *n* hours, and describe its pattern.



12-2 Series and Summation Notation

Expand each series and evaluate.

10. $\sum_{k=1}^{7} (2k + 1)$

11. $\sum_{k=1}^{4} \frac{k}{k+1}$

12. $\sum_{k=1}^{5} (k^2 - 1)$

CHAPTER 12

Chapter Review

12-1 Introduction to Sequences

Find the first 5 terms of each sequence.

1. $a_n = \frac{4}{5}n$

$\frac{4}{5}, \frac{8}{5}, \frac{12}{5}, \frac{16}{5}, 4$

2. $a_n = 3^{n-1}$

1, 3, 9, 27, 81

3. $a_n = \frac{n}{n+1}$

$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$

4. $a_n = n^2 - 3n$

$-2, -2, 0, 4, 10$

Write a possible explicit rule for the *n*th term of each sequence.

5. 1, 3, 5, 7, ...

$a_n = 2n - 1$

6. $-2, -4, -6, -8, \ldots$

$a_n = -2n$

7. 400, 200, 100, 50, 25, ...

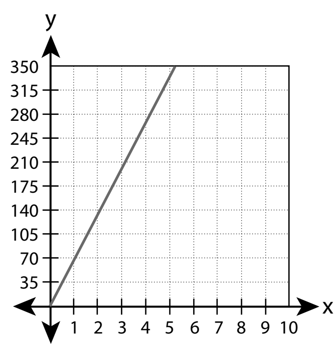
$a_n = 400\left(\frac{1}{2}\right)^{n-1}$

8. 366, 344, 322, 300, 278, ...

$a_n = 366 - 22(n - 1)$ or
$a_n = 388 - 22n$

9. A car traveling at 65 mi/h passes a mile marker that reads mile 23. If the car maintains this speed for 5 hours, what mile marker should the car pass? Graph the sequence for *n* hours, and describe its pattern.

348; The graph is steadily increasing.



12-2 Series and Summation Notation

Expand each series and evaluate.

10. $\sum_{k=1}^{7} (2k + 1)$

63

11. $\sum_{k=1}^{4} \frac{k}{k+1}$

$\frac{163}{60}$

12. $\sum_{k=1}^{5} (k^2 - 1)$

50

Evaluate each series.

13. $\sum_{k=1}^{6} \frac{1}{3}$

14. $\sum_{k=1}^{10} k^3$

15. $\sum_{k=1}^{20} k$

16. The first row of an auditorium has 28 seats, and each of the following rows has 4 more seats than the preceding row. How many seats are in the first 12 rows?

12-3 Arithmetic Sequences and Series

Find the 8th term of each arithmetic sequence.

17. 8.01, 8.02, 8.03, 8.04, …

18. $-2, -8, -14, -20, \ldots$

19. $a_3 = 14$ and $a_6 = 29$

20. $a_3 = -8$ and $a_7 = 32$

Find the missing terms in each arithmetic sequence.

21. -46, ____, ____, -178

22. 62, ____, ____, ____, 158

Find the indicated sum for each arithmetic series.

23. S_{10} for $80 + 60 + 40 + 20 + \ldots$

24. $\sum_{k=1}^{5} k^3$

25. $\sum_{k=1}^{8} (0.75k + 1.25)$

26. S_9 for $-8, -3, 2, 7, \ldots$

Evaluate each series.

13. $\sum_{k=1}^{6} \frac{1}{3}$

2

14. $\sum_{k=1}^{10} k^3$

3025

15. $\sum_{k=1}^{20} k$

210

16. The first row of an auditorium has 28 seats, and each of the following rows has 4 more seats than the preceding row. How many seats are in the first 12 rows?

600 seats

12-3 Arithmetic Sequences and Series

Find the 8th term of each arithmetic sequence.

17. 8.01, 8.02, 8.03, 8.04, …

8.08

18. $-2, -8, -14, -20, \ldots$

-44

19. $a_3 = 14$ and $a_6 = 29$

39

20. $a_3 = -8$ and $a_7 = 32$

42

Find the missing terms in each arithmetic sequence.

21. -46, ___, ___, -178

$-90, -134$

22. 62, ___, ___, ___, 158

86, 110, 134

Find the indicated sum for each arithmetic series.

23. S_{10} for $80 + 60 + 40 + 20 + \ldots$

-100

24. $\sum_{k=1}^{5} k^3$

225

25. $\sum_{k=1}^{8} (0.75k + 1.25)$

37

26. S_9 for $-8, -3, 2, 7, \ldots$

108

27. Suppose that you make a bank deposit for \$2.50 the first week in January, \$3.00 the second week, \$3.50 the third week, and so on. How much will you contribute to the account on the last week of the year (52nd week)? What is the total amount that you have deposited in the bank after one year?

12-4 Geometric Sequences and Series

Find the 8th term of each geometric sequence.

28. $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

29. $2, -6, 18, -54, \ldots$

30. $-16, -8, -4, -2, \ldots$

31. $6, 36, 216, 1296, \ldots$

Find the 10th term of each geometric sequence with the given terms.

32. $a_2 = -20$ and $a_4 = -5$

33. $a_2 = 5$ and $a_4 = \frac{1}{5}$

34. $a_2 = -10$ and $a_9 = 781{,}250$

35. $a_3 = -48$ and $a_6 = 486$

Find the geometric mean of each pair of numbers.

36. $\frac{1}{6}$ and $\frac{1}{24}$

37. -25 and -121

38. 108 and $\frac{1}{3}$

Find the indicated sum for each geometric series.

39. S_6 for $1 + 4 + 16 + 64 + \ldots$

40. S_9 for $\frac{1}{18} - \frac{1}{6} + \frac{1}{2} - \ldots$

41. $\sum_{k=0}^{10} 3^k$

42. $\sum_{k=0}^{9} 12\left(-\frac{1}{2}\right)^k$

27. Suppose that you make a bank deposit for \$2.50 the first week in January, \$3.00 the second week, \$3.50 the third week, and so on. How much will you contribute to the account on the last week of the year (52nd week)? What is the total amount that you have deposited in the bank after one year?

\$28; \$793

12-4 Geometric Sequences and Series

Find the 8th term of each geometric sequence.

28. $1, \frac{1}{4}, \frac{1}{16}, \frac{1}{64}, \ldots$

$\mathbf{\frac{1}{16{,}384}}$

29. $2, -6, 18, -54, \ldots$

$\mathbf{-4374}$

30. $-16, -8, -4, -2, \ldots$

$\mathbf{-\frac{1}{8}}$

31. $6, 36, 216, 1296, \ldots$

1,679,616

Find the 10th term of each geometric sequence with the given terms.

32. $a_2 = -20$ and $a_4 = -5$

$\mathbf{\frac{-5}{64}}$

33. $a_2 = 5$ and $a_4 = \frac{1}{5}$

$\mathbf{\frac{1}{78{,}125}}$

34. $a_2 = -10$ and $a_9 = 781{,}250$

$\mathbf{-3{,}906{,}250}$

35. $a_3 = -48$ and $a_6 = 486$

10,633

Find the geometric mean of each pair of numbers.

36. $\frac{1}{6}$ and $\frac{1}{24}$

$\mathbf{\pm\frac{1}{12}}$

37. -25 and -121

$\mathbf{\pm 55}$

38. 108 and $\frac{1}{3}$

$\mathbf{\pm 6}$

Find the indicated sum for each geometric series.

39. S_6 for $1 + 4 + 16 + 64 + \ldots$

1365

40. S_9 for $\frac{1}{18} - \frac{1}{6} + \frac{1}{2} - \ldots$

$\mathbf{\frac{4921}{18}}$

41. $\sum_{k=0}^{10} 3^k$

88,573

42. $\sum_{k=0}^{9} 12\left(-\frac{1}{2}\right)^k$

7.9922

43. A small business has spent $100,000 for gas. The cost of the gas is expected to increase at an annual rate of 3%.

a. What are the gas costs in year 15?

b. How much in total will be paid for gas over the first 15 years?

12-5 Mathematical Induction and Infinite Geometric Series

Find the sum of each infinite series, if it exists.

44. $4 + 2 + 1 + \ldots$

45. $8 + 40 + 200 + \ldots$

46. $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}$

47. $\sum_{k=0}^{\infty} 500(1.11)^{-k}$

Use mathematical induction to prove $4 + 8 + 12 + \ldots + 4n = 2n(n + 1)$.

48. Step 1

49. Step 2

50. Step 3

51. A ping pong ball is dropped from a height of 16 feet. The ball rebounds to 25% of its previous height after each bounce.

a. Write an infinite geometric series to represent the distance that the ball travels after it initially hits the ground. (*Hint:* The ball travels up and down on each bounce.)

b. What is the total distance that the ball travels after it initially hits the ground?

43. A small business has spent \$100,000 for gas. The cost of the gas is expected to increase at an annual rate of 3%.

a. What are the gas costs in year 15?

\$151,258.97

b. How much in total will be paid for gas over the first 15 years?

\$1,859,891.39

12-5 Mathematical Induction and Infinite Geometric Series

Find the sum of each infinite series, if it exists.

44. $4 + 2 + 1 + \ldots$

8

45. $8 + 40 + 200 + \ldots$

no sum exists

46. $\sum_{k=1}^{\infty} \left(\frac{1}{2}\right)^{k-1}$

2

47. $\sum_{k=0}^{\infty} 500(1.11)^{-k}$

5045.4545

Use mathematical induction to prove $4 + 8 + 12 + \ldots + 4n = 2n(n + 1)$.

48. Step 1

Basis step: Since $2 \cdot 1 \cdot (1 + 1) = 2 \cdot 2 = 4$, S_1 is true.

49. Step 2

$S_k = 2k(k + 1)$

50. Step 3

$4 + 8 + 12 + \ldots + 4k + 4(k + 1)$
$= 2k(k + 1) + 4(k + 1)$
$= (k + 1)(2k + 1)$
$= 2(k + 1)(k + 2)$

51. A ping pong ball is dropped from a height of 16 feet. The ball rebounds to 25% of its previous height after each bounce.

a. Write an infinite geometric series to represent the distance that the ball travels after it initially hits the ground. (*Hint:* The ball travels up and down on each bounce.)

$\sum_{k=1}^{\infty} 16\left(\frac{1}{4}\right)^{k-1}$

b. What is the total distance that the ball travels after it initially hits the ground?

$21\frac{1}{3}$

Big Ideas

Answer these questions to summarize the important concepts from Chapter 12 in your own words.

1. Explain the difference between an arithmetic and geometric sequence.

2. How is sigma notation used?

3. What is the difference between a series and a sequence?

4. What is an infinite geometric series?

For more review of Chapter 12:

- Complete the Chapter 12 Study Guide and Review on pages 912–915 of your textbook.
- Complete the Ready to Go On quizzes on pages 889 and 909 of your textbook.

CHAPTER 12

Big Ideas

Answer these questions to summarize the important concepts from Chapter 12 in your own words.

1. Explain the difference between an arithmetic and geometric sequence.

Answers may vary. An arithmetic sequence has a common difference found by subtracting two consecutive terms. A geometric sequence has a common ratio that is a ratio of two consecutive terms.

2. How is sigma notation used?

Answers may vary. Sigma notation is a shortcut way to denote a sum when the general term of a sequence is a formula.

3. What is the difference between a series and a sequence?

Answers may vary. A sequence is a list of numbers separated by commas. In a series the terms are separated by addition or subtraction signs.

4. What is an infinite geometric series?

The sum of the terms of an infinite geometric sequence is an infinite geometric series. Some geometric sequences do not have a sum. An infinite geometric series with a common ratio with $|r| < 1$ converges to a sum. Otherwise, it does not have a sum.

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