Name ______________________________ Date __________ Class ___________

SECTION 12A

Ready To Go On? Skills Intervention

12-1 Introduction to Sequences

Find these vocabulary words in Lesson 12-1 and the Multilingual Glossary.

Vocabulary		
sequence	infinite sequence	finite sequence

Finding Terms of a Sequence by Using a Recursive Formula

Find the first five terms of each sequence.

A. $a_n = a_{n-1} + 1$, where $n \geq 2$ and $a_1 = 1$

$a_1 = 1$ — The first term is given.

$a_2 =$ ____ $+ 1 =$ ____ — Substitute $a_1 = 1$ into the rule $a_n = a_{n-1} + 1$ to find a_2.

$a_3 =$ ____ $+ 1 =$ ____ — Repeat to find a_3.

$a_{\_\_} =$ ____ $+ 1 =$ ____ — Repeat to find the next term.

$a_5 = 4 +$ ____ $=$ ____ — Repeat to find the fifth term.

B. $a_n = 3a_{n-1} - 1$, where $n \geq 2$ and $a_1 = 1$

$a_1 = 1$ — The first term is given.

$a_2 = 3(1) - 1 =$ ____ — Substitute $a_1 = 1$ into the rule $a_n = 3a_{n-1} - 1$ to find a_2.

$a_3 = 3($____$) - 1 =$ ____ — Repeat to find a_3.

$a_{__} = 3($____$) - 1 =$ ____ — Repeat to find the next term.

$a_5 = 3(14) -$ ____ $=$ ____ — Repeat to find the fifth term.

Finding Terms of a Sequence by Using the Explicit Formula

Find the first three terms of each sequence.

A. $a_n = 2n - 1$, where $n \geq 1$

$a_1 = 2 \cdot 1 - 1 =$ ____ — Let $n = 1$. Substitute this value into the formula $a_n = 2n - 1$ to find a_1.

$a_2 = 2($____$) - 1 =$ ____ — Let $n = 2$. Substitute this value into the formula $a_n = 2n - 1$ to find a_2.

$a_3 = 2($____$) - 1 =$ ____ — Let $n = 3$. Substitute this value into the formula $a_n = 2n - 1$ to find a_3.

B. $a_n = 3^n - n$, where $n \geq 1$

$a_1 = 3^1 - 1 =$ ____ — Let $n = 1$. Substitute this value into the formula to find a_1.

Repeat to find the remaining terms: $a_2 = 3^{\_\_} - 2 =$ ____; $a_3 = 3^{\_\_} -$ ____ $=$ ____

Name ________________________________ Date ___________ Class ___________

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-1 Introduction to Sequences

A recursive formula is a rule to describe a sequence where one or more previous terms are used to generate the next term.

Grace has an action figure from a popular science fiction movie still in its original packaging. It is currently worth \$75. Action figures from this movie typically increase in value by about 7% a year.

a. Write a recursive rule predicting the value of the figure each year.

b. Use the recursive formula to predict the value of the figure in 10 years.

Understand the Problem

1. What is the initial value of the figure? ________________________

2. Will the value of the figure increase or decrease? ________________________

3. By how much will the value increase each year? ________________________

____

Make a Plan

4. What do you need to determine? ________________________

____

5. Let a_n represent the value of the figure in year n. What symbol represents the value in the previous year? ____________

6. By how much did the value of the figure increase in year $n - 1$? ____________

7. Write a recursive rule to model the value of the figure.

____

Solve

8. Use the recursive rule to predict the value of the figure in 10 years. __________

Look Back

9. You can check your solution by using an explicit formula. The explicit rule for this pattern is the same as the formula for compound interest.

$a_n = a_0(1 + r)^t = 75(1 + \_\_\_\_\_)^{\_\_} = \_\_\_\_\_\_\_\_$

Does your answer check? ______

Name ________________________________ Date __________ Class ___________

SECTION 12A

Ready To Go On? Skills Intervention

12-2 Series and Summation Notation

Find these vocabulary words in Lesson 12-2 and the Multilingual Glossary.

Vocabulary		
series	summation notation	infinite series

Using Summation Notation

Write this series using summation notation. $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625}$

Find a rule for the kth term of the sequence.

Look at the numerators:

Each numerator is ____.

Look at the denominators:

Each denominator is the base ____ to a power of 0, 1, 2, 3, and ____.

Write the rule for the sequence: $a_k = \frac{\square}{5^{\square}}$

Write the notation for the first five terms: $\sum_{k=0}^{\square} \frac{\square}{5^k}$

Evaluating a Series

The value of π can be approximated by a partial sum of an infinite series.

Expand the series $\pi \approx 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ **and evaluate.**

Write out the first five terms.

$$\pi \approx 4\left(\frac{(\square)^0}{2 \cdot 0 + 1} + \frac{(-1)^{\square}}{2 \cdot \square + 1} + \frac{(-1)^{\square}}{\square \cdot 2 + 1} + \frac{(\square)^3}{2 \cdot \square + 1} + \frac{(\square)^{\square}}{2 \cdot 4 + \square}\right)$$

Simplify each term.

$$\pi \approx 4\left(\frac{1}{\square} + \frac{\square}{3} + \frac{1}{5} + \frac{-1}{\square} + \frac{\square}{9}\right)$$

Simplify.

$\pi \approx$ __________

Name ____ Date ___________ Class ____________

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-2 Series and Summation Notation

A series can be represented by summation notation, indicated by Σ, and then evaluated to find the sum of the series.

A hot air balloon rises 30 m in the first minute. In each minute thereafter, the balloon rises 70% as far as it did the previous minute. How far does the balloon rise in 5 minutes?

Understand the Problem

1. What is the initial height? _________
2. What is the height gain during the second minute? ________________________
3. What is the total height after the second minute? __________________________
4. What is the height gain during the third minute?

   ____

5. What is the total height after the third minute? ____________________________

Make a Plan

6. Write a rule that represents the height gain during the kth minute.

   ____

7. Write a summation that represents the total height after n minutes

   ____

Solve

8. Use the summation to calculate the total height of the balloon after 5 minutes.

   ____

   ____

   ____

Look Back

9. You can check your solution by using the formula for the sum of an infinite geometric series. For this series, $|r| = 0.70$, which is less than, so it is a convergent series. The sum is: $S_n = \frac{a_1}{1 - r} = \frac{\square}{1 - 0.70} = \frac{\square}{\square} = \square$

   Since the result in Exercise 8 is less than the sum of the series, is your answer reasonable? ______

Name ______________________________________ Date ___________ Class ____________

SECTION 12A

Ready To Go On? Skills Intervention

12-3 Arithmetic Sequences and Series

Find this vocabulary word in Lesson 12-3 and the Multilingual Glossary.

Vocabulary
arithmetic series

Identifying Arithmetic Sequences

Determine whether the sequence, 5, 9, 13, 17, ... , is arithmetic. If so, determine the common difference and the next term.

Find the difference of the first two terms: 9 − ____ = 4

Find the difference of the next two terms: 13 − ____ = ____

Find the difference of the next two terms: ____ − ____ = ____

Is the difference constant? _____; If so, what is the common difference? ____

Find the next term: 17 + ____ = ____

Finding the *n*th Term Given an Arithmetic Sequence

Find the ninth term of the sequence: $\frac{7}{2}$, 4, $\frac{9}{2}$, 5, …

The formula for the *n*th term in a sequence is $a_n = a_1 + (n - 1)d$, where a_1 is the first term.

What is a_1? ____ What is n? ____

Find the common difference: ____ $- \frac{7}{2} =$ ____

Substitute $d =$ ____; $a_1 = \frac{7}{\square}$; and $n =$ ____ into the formula and solve.

$$a_9 = \frac{7}{\square} + (\square - 1)\frac{1}{\square} = \frac{7}{2} + (8)\frac{1}{\square} = \frac{7}{\square} + \frac{\square}{2} = \frac{\square}{2}$$

Finding the Sum of an Arithmetic Sequence

Find the sum of the first 10 terms of the sequence: $\frac{7}{2}$, 4, $\frac{9}{2}$, 5, ...

First find the 10th term of the sequence.

What is a_1? ____ What is n? ____ What is the common difference: ____ $- \frac{7}{2} =$ ____

Substitute known values into the formula to find the 10th term: $a_{10} = \frac{7}{2} + ($____ $- 1)\frac{1}{2} =$ ____

The formula for the sum of the first n terms in an arithmetic sequence is $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

Find S_{10}: $S_n = n\left(\frac{a_1 + a_n}{2}\right) = \square\left(\frac{\frac{7}{2} + \square}{2}\right) =$ _______

Name ______________________________________ Date ___________ Class ____________

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-3 Arithmetic Sequences and Series

Thanh has a college fund of $1200. He withdraws $10 from this fund the first week of college and increases the withdrawal amount by $10 each week as his expenses increase. If Thanh then withdraws $150 from his account and his statement shows an account balance of $0, how many weeks have passed?

Understand the Problem

1. Is this an arithmetic series? Explain. ____________________________________

2. What is the formula for the sum of an arithmetic series? ____________________________

3. What is the unknown in this problem? ____________________________________

Make a Plan

4. What is the first term in the series?____________________________________

5. What is the arithmetic difference? ____________________________________

6. What is the last term in the series? ____________________________________

7. Solve the formula for the sum of an arithmetic series for n.

____

Solve

8. Substitute the known values into the formula and solve for n.

$$n = \frac{2S_n}{a_1 + a_n} = \frac{2\square}{\square + \square} = \frac{\square}{\square + \square} = \square$$

9. How many weeks have passed for Thanh to now have an account balance of $0? ______________________

Look Back

10. Use another method to check your solution: Write out the first 15 terms of the series, then find the sum of all 15 terms.

____

____

11. Is your solution the same using both methods: the series formula in Exercise 8 and adding up the terms of the series in Exercise 11? ______

Name ______________________ Date ___________ Class ___________

SECTION **12A**

Ready To Go On? Quiz

12-1 Introduction to Sequences

Find the first five terms of each sequence.

1. $a_n = (-2)^n$

____

2. $a_n = -n - 5$

____

3. $a_n = \frac{n}{n + 1}$

____

4. $a_n = a_{n-1} + 1, \quad a_1 = 1$

____

5. A piece of paper is 0.03 mm thick. It is torn in two equal pieces and stacked. The stack is torn in two and stacked again. The process is repeated 5 times. How thick is the pile?

____

Write an explicit rule for the *n*th term of each sequence.

6. 1, 2, 3, 4, 5, . . .

____

7. 1, 6, 13, 22, 33,

____

____

____

8. −4, −1, 4, 11, 20, . . .

____

9. $\frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots$

____

12–2 Series and Summation Notation

Write the following sums using summation notation.

10. 1 + 6 + 13 + 22 + 33

____

11. $\frac{2}{9} + \frac{3}{11} + \frac{4}{13} + \frac{5}{15}$

____

Name ________________________________ Date ___________ Class ____________

SECTION **12A**

Ready To Go On? Quiz continued

Expand and evaluate each series

12. $\sum_{n=1}^{5}\left(\frac{n}{2n+3}\right)$

13. $\sum_{n=3}^{6}\left(-\frac{1}{2}\right)^{n}$

________________________________ ________________________________

14. Bowling pins are set up so that the first row has one pin and each subsequent row has one more pin than the previous. There are four rows. Use summation notion to write an expression for the number of bowling pins. Evaluate the expression.

____

12–3 Arithmetic Sequences and Series

Find the eighth term of each arithmetic sequence.

15. 13, 21.5, 30, 38.5, 47, . . .

16. −5, 7, 19, 31, . . .

________________________ ________________________

Find the missing terms in each arithmetic sequence.

17. −11, −6, ____, ____, ____, 14, 19, . . .

18. 9, 18, 27, ____, ____, 54, 63, . . .

________________________ ________________________

Find the indicated sum for each series.

19. 5, 9, 13, 17, . . .

S_{50} = ________________________

20. $\sum_{n=1}^{5}(5n+2)$

____

21. Honus loves to collect baseball cards. His younger brother Ty decides to give him baseball cards for his birthday. As a surprise, Ty plans to give Honus only two cards today, but then give four cards tomorrow, six cards the day after, and continue the sequence for one week. What is the total number of cards Ty gives to Honus?

____

Name ______________________________ Date __________ Class __________

SECTION 12A

Ready To Go On? Enrichment

Physics and Sequences

It is easy to make a simple device that will demonstrate Galileo's rule for falling objects. If small weights are tied to a cord so that the distances between the weights has the following pattern, 1, 3, 5, 7, 9, then when the string is dropped, the time interval between when the weights hit the ground will be the same for all weights. To see why, complete this table.

Column 1	Column 2	Column 3
Weight Number	**Distance to Next Weight**	**Total Distance**
1	1 unit	1 unit
2	3 units	4 units
3	5 units	________
4	7 units	________
5	9 units	________

Use the table to answer each question.

1. Are the numbers in Column 2 an arithmetic series? If so, calculate *d*.

____

2. Are the numbers in Column 3 an arithmetic series? If so, calculate *d*.

____

3. Write an explicit rule for the numbers in Column 2.

____

4. Write an explicit rule for the numbers in Column 3.

____

5. The formula for distance traveled as a function of time, according to Galileo (using modern notation), is $d = \frac{1}{2}at^2$, where *a* is a constant quantity called acceleration and *t* is time. In the table, which column of numbers represents time?

____

Name ______________________________ Date __________ Class __________

SECTION 12B

Ready To Go On? Skills Intervention

12-4 Geometric Sequences and Series

Find these vocabulary words in Lesson 12-4 and the Multilingual Glossary.

Vocabulary		
geometric sequence	geometric series	geometric means

Identifying Geometric Sequences

Determine if each sequence is arithmetic or geometric. If it is arithmetic, find the common difference. If it is geometric, find the common ratio.

A. −7, −3, 1, 5, 9, 13, 17

$5 - 1 = \_\_\_\_, 9 - \_\_\_\_ = \_\_\_\_$ Find the difference between two pairs of successive terms.

$\frac{5}{\square} = \square, \ \frac{9}{\square} = \square$ Find the ratio of two pairs of successive terms.

Is the sequence arithmetic or geometric? __________

Is there a common difference or common ratio? ________________

What is it? ____

B. $\frac{1}{3}, \frac{1}{12}, \frac{1}{48}, \frac{1}{192}$

$\frac{1}{12} - \frac{1}{\square} = -\frac{1}{\square}, \ \frac{1}{\square} - \frac{1}{12} = -\frac{1}{\square}$ Find the difference between two pairs of successive terms.

$\frac{1/\square}{1/3} = \frac{1}{\square}, \ \frac{1/\square}{1/12} = \frac{1}{\square}$ Find the ratio of two pairs of successive terms.

Is the sequence arithmetic or geometric? __________

Is there a common difference or common ratio?________________

What is it? ____

Finding the *n*th term Given a Geometric Sequence

Find the 8th term of a geometric sequence if $a_1 = 16$ and $r = 0.5$.

$a_n = a_1 r^{n-1}$ Write the formula for geometric sequence.

$a_n = \_\_\_\_(0.5)^{\square}$ Substitute known values and evaluate.

$= \_\_\_\_\_\_$

$= \frac{1}{\square}$

The 8th term in the sequence is $\frac{1}{8}$.

Name ______________________________ Date __________ Class ___________

SECTION 12B

Ready To Go On? Problem Solving Intervention

12-4 Geometric Sequences and Series

Each term in a geometric sequence is the product of the previous term and the common ratio.

Carlos planted three tomato plants. He exposed the plants to artificial light for 12 hours per day and measured their height each week. His measurements showed that the plants increased in height by an average of 8% per week. The average initial height of the three plants was 22 cm. Predict the average height of the plants at the end of 10 weeks.

Understand the Problem

1. Does the measured height of the plants form a geometric series? Explain.

____

2. What formula gives the *n*th term of a geometric sequence? ____________________

3. What is the first term of the sequence? ________

4. What is the second term of the sequence? ______________________________

5. What is the common ratio? ____

Make a Plan

6. In the expression for the *n*th term of a geometric sequence, what are the known values? ____

7. In the expression for the *n*th term of a geometric sequence, what is the unknown value? ____

Solve

8. Substitute the appropriate values into the geometric sequence formula.

____

9. Evaluate the formula.

____

10. What is the average height of the plants after 10 weeks? ______________________

Look Back

11. Use estimation to determine if your answer is reasonable. 8% can be rounded up to 10%. What is 10% of 22 cm? _________Multiply this value by 10. ____ Add this value to 22. ____ Does your answer in Exercise 10 make sense? _____

Name ____ Date ___________ Class ____________

SECTION **12B**

Ready To Go On? Skills Intervention

12-5 Mathematical Induction and Infinite Geometric Series

Find these vocabulary words in Lesson 12-5 and the Multilingual Glossary.

Vocabulary		
infinite geometric series	converge	limit

Finding the Sums of Infinite Geometric Series

Determine if the following infinite series converges. If so, find its sum.

$$2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625} + \ldots$$

For an infinite geometric series, if $r < 1$, the series ___________. If $r \geq$ ____, the series does not converge and does not have a sum.

Calculate the ratio between successive terms:

$$r = \frac{2/5}{\square} = \frac{1}{\square},\ r = \frac{2/\square}{\square/5} = \frac{1}{\square}$$

Since $r < 1$, the series ___________.

Calculate the sum of the series: $S = \dfrac{a_1}{1 - r} = \dfrac{\square}{1 - 1/\square} = \dfrac{2}{\square/5} = \dfrac{\square}{4} =$ ____

Writing Repeating Decimals as Fractions

Write 0.09090909 . . . as a fraction in simplest form.

Repeating decimals are __________ numbers. Infinite geometric series can be used to convert a repeating decimal to a fraction.

$0.090909\ldots =$ ______ $+ 0.0009 +$ __________	Write the repeating decimal as an infinite series.
$\dfrac{0.0009}{0.09} =$ _______	Calculate the common ratio.
$a_1 =$ _______	Identify the first term.
$S = \dfrac{a_1}{1 - \square}$	Write the formula.
$= \dfrac{\square}{1 - \square}$	Substitute known values and evaluate.
$= \dfrac{\square}{0.99}$	
$= \dfrac{1}{\square}$	

So, $0.09090909 = \dfrac{1}{\square}$.

Name ______________________________ Date ___________ Class ____________

SECTION 12B

Ready To Go On? Problem Solving Intervention

12-5 Mathematical Induction and Infinite Geometric Series

You can find the sum of an infinite geometric series, if it converges.

Bacteria reproduce by fission results in one cell dividing into two cells. One particular species of bacteria reproduces about once every hour. A researcher swabs a plate of growth medium leaving 100 bacteria. Assuming inexhaustible space and unlimited food, how many bacteria would there be in 12 hours?

Understand the Problem

1. Is this a geometric series? Explain how you know.

   ____

2. What is the common ratio? ____
3. What is the first term? _____
4. What is the formula for a partial sum of an infinite series? __________________

Make a Plan

5. In the formula for the partial sum of an infinite series, what are the known values? ____
6. In the formula for the partial sum of an infinite series, what is the unknown value? ____

Solve

7. Substitute the known values into the formula and evaluate.

$$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$= \_\_\_\_\_\left(\frac{1 - 2^{\square}}{1 - \square}\right)$$

$$= 100\left(\frac{\square}{-1}\right)$$

$$= \_\_\_\_\_\_\_\_\_\_$$

8. How many bacteria were there in 12 hours? __________

Look Back

9. Graph the function $y = \_\_\_\_\_\left(\frac{1 - 2^{\square}}{1 - \square}\right)$ on your graphing calculator.

   Does your graph support your answer to Exercise 8? ______

Name ______________________ Date __________ Class __________

SECTION 12B

Ready To Go On? Quiz

12-4 Geometric Sequences and Series

Write the first six terms of each sequence.

1. 27, 135, 675, 3375, . . .

____

2. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

____

Find the 10th term of each sequence.

3. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

____

4. $\sqrt{2}, 2, 2\sqrt{2}, 4, \ldots$

____

5. $a_1 = 0.625$, $r = 2$

____

6. $a_4 = 4$, $r = 2$

____

7. A piece of paper is 0.03 mm thick. It is torn in two equal pieces and stacked. The stack is torn in again into two equal pieces and stacked again, so it has four pieces of paper. This process is repeated many times, with the number of sheets of paper doubling each time. Write a formula for the height of the paper stack after *n* times. Assuming it is possible, calculate the height of the stack after 11 tears.

____

Find the geometric mean of each pair of numbers.

8. $\frac{1}{4}$ and 4

____

9. 4 and 64

____

Find the geometric means to complete each sequence.

10. 8, ____, ____, 27, . . .

____

11. −2, ____, ____, 54, . . .

____

SECTION 12B

Ready To Go On? Quiz continued

Find the indicated sums.

12. S_6 for $1 + 5 + 25 + 125 + \ldots$

13. S_8 for $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots$

12–5 Mathematical Induction and Infinite Geometric Series

Determine if the series converges. If it does, find the sum.

14. $\frac{1}{25} + \frac{1}{250} + \frac{1}{2500} + \ldots$

15. $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \ldots$

16. $\sum_{k=1}^{\infty} \left(\frac{1}{9}\right)^{k-1}$

17. $\frac{2}{7} + \frac{4}{7} + \frac{8}{7} + \frac{16}{7} + \ldots$

18. $\sum_{k=1}^{\infty} 2\left(\frac{1}{3}\right)^{k-1}$

19. $\sum_{k=1}^{\infty} 4(3)^{k-1}$

20. Prove by mathematical induction that the nth partial sum of the series

$1 + 3 + 5 + 7 + \ldots + (2n - 1)$ is equal to n^2.

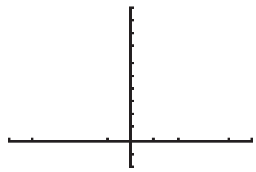
Name ______________________ Date __________ Class __________

SECTION 12B

Ready To Go On? Enrichment

Mathematical Induction

Mathematical Induction is a powerful means of proving many mathematical conjectures. Graph the two functions $f(x) = x^2$ and $g(x) = 2^x$ on the same axes.



1. What is the relationship between $f(x)$ and $g(x)$ for large values of x?

____

Mathematical Induction can be used to prove the conjecture in Exercise 1 for integers. Let the variable be n, where $n > 4$ and can have only integer values.

2. Rewrite this conjecture using the new variable.

____

3. Prove the conjecture for the first case.

____

4. Assume the statement is true for for $n = k$ and show that it is true for $n = k + 1$.

____

____

____

Alternating Series

Does the series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$ have a sum?
To find out, calculate the first few partial sums.

S1 = ____ S3 = ____

S2 = ____ S4 = ____

It is clear that these terms do not converge to a limit, so this series does not have a sum. To further test the idea, apply the rule for the sum of a convergent series,

$S = \frac{a_1}{1 - r} = \frac{\square}{1 - (-1)} = \frac{1}{\square}$. This answer makes no sense.

Many alternating series do converge.

5. Suggest an example of an alternating series that converges.

____

SECTION 11B

Ready To Go On? Enrichment

***Z*-scores**

Standard scores, also known as *z*-scores enable statisticians to compare values more easily. A *z*-score is the number of standard deviations that a given value x is above or below the mean. It is found by using the formula: $z = \frac{x - \bar{x}}{s}$ where x is a particular data value, $\bar{x}$ is the mean and s is the standard deviation. The use of *z*-scores in statistics is extremely important because they can be used to differentiate between ordinary and unusual values. A *z*-score less than −2.00 or greater than 2.00 is considered to be unusual.

Example: Heights of all adult males have a mean of 69 inches and a standard deviation of 2.8 inches. What is the *z*-score for a male that is 78 inches tall?

$z = \frac{x - \bar{x}}{s} = \frac{78 - 69}{2.8} = 3.21$

You can interpret this result by stating that an individual 78 inches tall is 3.21 standard deviations above the mean. This would be an unusual height.

Solve

1. A mathematics teacher gives two different tests to two different sections of Algebra classes. The statistics are shown below. Which score is better: an 82 on the section 1 test, or a 46 on the section 2 test?

   Section 1: mean = 75 and standard deviation = 14

   Section 2: mean = 40 and standard deviation = 8

   z-score for section 1 = 0.50, *z*-score for section 2 = 0.75; A 46 on the section 2 test is a better score.

2. The Bean Pole club is open to men and women who are very tall. The minimum height requirement for women is 70 in. If women's heights have a mean of 63.6 inches and a standard deviation of 2.5 inches, find the *z*-score corresponding to a height of exactly 70 inches. Is this height unusual?

   z-score is 2.56, the height would be considered unusual.

3. Three potential employees take a required mathematics test in which three different areas are tested. Which of the following scores has the highest relative position?

   Test 1: Score of 37 on a test with a mean of 28 and standard deviation of 6

   Test 2: Score of 398 on a test with a mean of 312 and standard deviation of 56

   Test 3: Score of 4.10 on a test with a mean of 2.75 and standard deviation of 0.92

   z-scores of 1.5, 1.54 and 1.47; the score of 398 has the highest relative position.

SECTION 12A

Ready To Go On? Skills Intervention

12-1 Introduction to Sequences

Find these vocabulary words in Lesson 12-1 and the Multilingual Glossary.

Vocabulary

sequence   infinite sequence   finite sequence

Finding Terms of a Sequence by Using a Recursive Formula
Find the first five terms of each sequence.

A. $a_n = a_{n-1} + 1$, where $n \geq 2$ and $a_1 = 1$

$a_1 = 1$	The first term is given.
$a_2 = 1 + 1 = 2$	Substitute $a_1 = 1$ into the rule $a_n = a_{n-1} + 1$ to find a_2.
$a_3 = 2 + 1 = 3$	Repeat to find a_3.
$a_4 = 3 + 1 = 4$	Repeat to find the next term.
$a_5 = 4 + 1 = 5$	Repeat to find the fifth term.

B. $a_n = 3a_{n-1} - 1$, where $n \geq 2$ and $a_1 = 1$

$a_1 = 1$	The first term is given.
$a_2 = 3(1) - 1 = 2$	Substitute $a_1 = 1$ into the rule $a_n = 3a_{n-1} - 1$ to find a_2.
$a_3 = 3(2) - 1 = 5$	Repeat to find a_3.
$a_4 = 3(5) - 1 = 14$	Repeat to find the next term.
$a_5 = 3(14) - 1 = 41$	Repeat to find the fifth term.

Finding Terms of a Sequence by Using the Explicit Formula
Find the first three terms of each sequence.

A. $a_n = 2n - 1$, where $n \geq 1$

$a_1 = 2 \cdot 1 - 1 = 1$	Let $n = 1$. Substitute this value into the formula $a_n = 2n - 1$ to find a_1.
$a_2 = 2(2) - 1 = 3$	Let $n = 2$. Substitute this value into the formula $a_n = 2n - 1$ to find a_2.
$a_3 = 2(3) - 1 = 5$	Let $n = 3$. Substitute this value into the formula $a_n = 2n - 1$ to find a_3.

B. $a_n = 3^n - n$, where $n \geq 1$

$a_1 = 3^1 - 1 = 2$ Let $n = 1$. Substitute this value into the formula to find a_1.

Repeat to find the remaining terms: $a_2 = 3^2 - 2 = 7$; $a_3 = 3^3 - 3 = 24$

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-1 Introduction to Sequences

A recursive formula is a rule to describe a sequence where one or more previous terms are used to generate the next term.

Grace has an action figure from a popular science fiction movie still in its original packaging. It is currently worth \$75. Action figures from this movie typically increase in value by about 7% a year.

a. Write a recursive rule predicting the value of the figure each year.

b. Use the recursive formula to predict the value of the figure in 10 years.

Understand the Problem

1. What is the initial value of the figure? \$75
2. Will the value of the figure increase or decrease? Increase
3. By how much will the value increase each year? By 7% of whatever the value was the previous year

Make a Plan

4. What do you need to determine? A recursive rule that will predict the value of the action figure over the next few years.
5. Let a_n represent the value of the figure in year n. What symbol represents the value in the previous year? a_{n-1}
6. By how much did the value of the figure increase in year $n - 1$? $0.07a_{n-1}$
7. Write a recursive rule to model the value of the figure.

   $a_n = a_{n-1} + 0.07a_{n-1}$

Solve

8. Use the recursive rule to predict the value of the figure in 10 years. \$147.54

Look Back

9. You can check your solution by using an explicit formula. The explicit rule for this pattern is the same as the formula for compound interest.

   $a_n = a_0(1 + r)^t = 75(1 + 0.07)^{10} = 147.54$

   Does your answer check? Yes

SECTION 12A

Ready To Go On? Skills Intervention

12-2 Series and Summation Notation

Find these vocabulary words in Lesson 12-2 and the Multilingual Glossary.

Vocabulary

series   summation notation   infinite series

Using Summation Notation

Write this series using summation notation. $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625}$

Find a rule for the kth term of the sequence.

Look at the numerators:

Each numerator is 2.

Look at the denominators:

Each denominator is the base 5 to a power of 0, 1, 2, 3, and 4.

Write the rule for the sequence: $a_k = \frac{2}{5^k}$

Write the notation for the first five terms: $\sum_{k=0}^{4} \frac{2}{5^k}$

Evaluating a Series

The value of π can be approximated by a partial sum of an infinite series.

Expand the series $\pi \approx 4\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ **and evaluate.**

Write out the first five terms.

$$\pi \approx 4\left(\frac{(-1)^0}{2 \cdot 0 + 1} + \frac{(-1)^1}{2 \cdot 1 + 1} + \frac{(-1)^2}{2 \cdot 2 + 1} + \frac{(-1)^3}{2 \cdot 3 + 1} + \frac{(-1)^4}{2 \cdot 4 + 1}\right)$$

Simplify each term.

$$\pi \approx 4\left(\frac{1}{1} + \frac{-1}{3} + \frac{1}{5} + \frac{-1}{7} + \frac{1}{9}\right)$$

Simplify.

$\pi \approx 3.33968$

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-2 Series and Summation Notation

A series can be represented by summation notation, indicated by Σ, and then evaluated to find the sum of the series.

A hot air balloon rises 30 m in the first minute. In each minute thereafter, the balloon rises 70% as far as it did the previous minute. How far does the balloon rise in 5 minutes?

Understand the Problem

1. What is the initial height? 30 m
2. What is the height gain during the second minute? $0.70 \times 30 \text{ m} = 21 \text{ m}$
3. What is the total height after the second minute? $30 \text{ m} + 21 \text{ m} = 51 \text{ m}$
4. What is the height gain during the third minute? $0.70 \times 0.70 \times 30 = 0.70 \times 21 \text{ m} = 14.7 \text{ m}$
5. What is the total height after the third minute? $30 \text{ m} + 21 \text{ m} + 14.7 \text{ m} = 65.7 \text{ m}$

Make a Plan

6. Write a rule that represents the height gain during the kth minute. $(0.70)^{k-1}(30)$
7. Write a summation that represents the total height after n minutes $\sum_{k=1}^{n}(0.70)^{k-1}(30)$

Solve

8. Use the summation to calculate the total height of the balloon after 5 minutes.

$$h = \sum_{k=1}^{5}(0.70)^{k-1}(30) \text{ m}$$

$$h = (0.70)^0(30) + (0.70)^1(30) + (0.70)^2(30) + (0.70)^3(30) + (0.70)^4(30)$$

$$30 \text{ m} + 21 \text{ m} + 14.7 \text{ m} + 10.3 \text{ m} + 7.2 \text{ m} = 83.2 \text{ m}$$

Look Back

9. You can check your solution by using the formula for the sum of an infinite geometric series. For this series, $|r| = 0.70$, which is less than, so it is a convergent series. The sum is: $S_n = \frac{a_1}{1 - r} = \frac{\boxed{30}}{1 - 0.70} = \frac{\boxed{30}}{\boxed{0.3}} = \boxed{100}$

Since the result in Exercise 8 is less than the sum of the series, is your answer reasonable? Yes

SECTION 12A

Ready To Go On? Skills Intervention

12-3 Arithmetic Sequences and Series

Find this vocabulary word in Lesson 12-3 and the Multilingual Glossary.

Vocabulary
arithmetic series

Identifying Arithmetic Sequences

Determine whether the sequence, 5, 9, 13, 17, ... , is arithmetic. If so, determine the common difference and the next term.

Find the difference of the first two terms: 9 − 5 = 4

Find the difference of the next two terms: 13 − 9 = 4

Find the difference of the next two terms: 17 − 13 = 4

Is the difference constant? Yes; If so, what is the common difference? 4

Find the next term: 17 + 4 = 21

Finding the nth Term Given an Arithmetic Sequence

Find the ninth term of the sequence: $\frac{7}{2}$, 4, $\frac{9}{2}$, 5, ...

The formula for the nth term in a sequence is $a_n = a_1 + (n - 1)d$, where a_1 is the first term.

What is a_1? $\frac{7}{2}$ What is n? 9

Find the common difference: $4 - \frac{7}{2} = \frac{1}{2}$

Substitute $d = \frac{1}{2}$; $a_1 = \frac{7}{\boxed{2}}$; and $n = 9$ into the formula and solve.

$a_9 = \frac{7}{\boxed{2}} + (\boxed{9} - 1)\frac{1}{\boxed{2}} = \frac{7}{2} + (8)\frac{1}{\boxed{2}} = \frac{7}{\boxed{2}} + \frac{\boxed{8}}{2} = \frac{\boxed{15}}{2}$

Finding the Sum of an Arithmetic Sequence

Find the sum of the first 10 terms of the sequence: $\frac{7}{2}$, 4, $\frac{9}{2}$, 5, ...

First find the 10th term of the sequence.

What is a_1? $\frac{7}{2}$ What is n? 10 What is the common difference: $4 - \frac{7}{2} = \frac{1}{2}$

Substitute known values into the formula to find the 10th term: $a_{10} = \frac{7}{2} + (10 - 1)\frac{1}{2} = 8$

The formula for the sum of the first n terms in an arithmetic sequence is $S_n = n\left(\frac{a_1 + a_n}{2}\right)$.

Find S_{10}: $S_n = n\left(\frac{a_1 + a_n}{2}\right) = \boxed{10}\left(\frac{\frac{7}{2} + \boxed{8}}{2}\right) = 57.5$

SECTION 12A

Ready To Go On? Problem Solving Intervention

12-3 Arithmetic Sequences and Series

Thanh has a college fund of $1200. He withdraws $10 from this fund the first week of college and increases the withdrawal amount by $10 each week as his expenses increase. If Thanh then withdraws $150 from his account and his statement shows an account balance of $0, how many weeks have passed?

Understand the Problem

1. Is this an arithmetic series? Explain. Yes, since $d = \$10$.
2. What is the formula for the sum of an arithmetic series? $S_n = n\left(\frac{a_1 + a_n}{2}\right)$
3. What is the unknown in this problem? The number of weeks, n

Make a Plan

4. What is the first term in the series? $a_1 = \$10$
5. What is the arithmetic difference? $d = \$10$
6. What is the last term in the series? $a_n = \$150$
7. Solve the formula for the sum of an arithmetic series for n. $n = \frac{2S_n}{a_1 + a_n}$

Solve

8. Substitute the known values into the formula and solve for n.

$n = \frac{2S_n}{a_1 + a_n} = \frac{2\boxed{\$1200}}{\boxed{\$10} + \boxed{\$150}} = \frac{\boxed{\$2400}}{\boxed{\$10} + \boxed{\$150}} = \boxed{15}$

9. How many weeks have passed for Thanh to now have an account balance of $0? 15 weeks

Look Back

10. Use another method to check your solution: Write out the first 15 terms of the series, then find the sum of all 15 terms.

10 + 20 + 30 + 40 + 50 + 60 + 70 + 80 + 90 + 100 + 110
120 + 130 + 140 + 150 = 1200

11. Is your solution the same using both methods: the series formula in Exercise 8 and adding up the terms of the series in Exercise 11? Yes

SECTION 12A

Ready To Go On? Quiz

12-1 Introduction to Sequences

Find the first five terms of each sequence.

1. $a_n = (-2)^n$ — $-2, 4, -8, 16, -32$
2. $a_n = -n - 5$ — $-6, -7, -8, -9, -10$
3. $a_n = \frac{n}{n+1}$ — $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}$
4. $a_n = a_{n-1} + 1$, $a_1 = 1$ — 1, 2, 3, 4, 5

5. A piece of paper is 0.03 mm thick. It is torn in two equal pieces and stacked. The stack is torn in two and stacked again. The process is repeated 5 times. How thick is the pile? 1.86 mm

Write an explicit rule for the nth term of each sequence.

6. 1, 2, 3, 4, 5, . . . — $a_n = n$
7. 1, 6, 13, 22, 33, — $a_n = a_{n-1} + (2n + 1)$; $a_1 = 1$; or $a_n = n^2 - 3$; $n \geq 2$
8. −4, −1, 4, 11, 20, . . . — $a_n = n^2 - 5$
9. $\frac{3}{16}, \frac{4}{25}, \frac{5}{36}, \frac{6}{49}, \ldots$ — $a_n = \frac{2 + n}{(3 + n)^2}$

12–2 Series and Summation Notation

Write the following sums using summation notation.

10. 1 + 6 + 13 + 22 + 33 — $\sum_{n=2}^{6} n^2 - 3$
11. $\frac{2}{9} + \frac{3}{11} + \frac{4}{13} + \frac{5}{15}$ — $\sum_{n=2}^{5} \frac{n}{2n + 5}$

SECTION 12A

Ready To Go On? Quiz *continued*

Expand and evaluate each series

12. $\sum_{n=1}^{5}\left(\frac{n}{2n+3}\right)$

$\frac{1}{5}+\frac{2}{7}+\frac{3}{9}+\frac{4}{11}+\frac{5}{13}=1.567$

13. $\sum_{n=3}^{6}\left(-\frac{1}{2}\right)^n$

$-\frac{1}{8}+\frac{1}{16}-\frac{1}{32}+\frac{1}{64}=-\frac{5}{64}$

14. Bowling pins are set up so that the first row has one pin and each subsequent row has one more pin than the previous. There are four rows. Use summation notation to write an expression for the number of bowling pins. Evaluate the expression.

$\sum_{n=1}^{4} n;\ 1+2+3+4=10$

12-3 Arithmetic Sequences and Series

Find the eighth term of each arithmetic sequence.

15. 13, 21.5, 30, 38.5, 47, . . .

72.5

16. −5, 7, 19, 31, . . .

79

Find the missing terms in each arithmetic sequence.

17. −11, −6, ____, ____, ____, 14, 19, . . .

−1, 4, 9

18. 9, 18, 27, ____, ____, 54, 63, . . .

36, 45

Find the indicated sum for each series.

19. 5, 9, 13, 17, . . .

S_{50} = 5150

20. $\sum_{n=1}^{5}(5n+2)$

85

21. Honus loves to collect baseball cards. His younger brother Ty decides to give him baseball cards for his birthday. As a surprise, Ty plans to give Honus only two cards today, but then give four cards tomorrow, six cards the day after, and continue the sequence for one week. What is the total number of cards Ty gives to Honus?

$\sum_{n=1}^{7} 2n$; 56 cards

SECTION 12A

Ready To Go On? Enrichment

Physics and Sequences

It is easy to make a simple device that will demonstrate Galileo's rule for falling objects. If small weights are tied to a cord so that the distances between the weights has the following pattern, 1, 3, 5, 7, 9, then when the string is dropped, the time interval between when the weights hit the ground will be the same for all weights. To see why, complete this table.

Column 1	Column 2	Column 3
Weight Number	**Distance to Next Weight**	**Total Distance**
1	1 unit	1 unit
2	3 units	4 units
3	5 units	9 units
4	7 units	16 units
5	9 units	25 units

Use the table to answer each question.

1. Are the numbers in Column 2 an arithmetic series? If so, calculate *d*.

Yes, because *d* is constant, $d = 2$.

2. Are the numbers in Column 3 an arithmetic series? If so, calculate *d*.

No, because *d* is not constant.

3. Write an explicit rule for the numbers in Column 2.

$a_n = 2n - 1$

4. Write an explicit rule for the numbers in Column 3.

$a_n = n^2$

5. The formula for distance traveled as a function of time, according to Galileo (using modern notation), is $d = \frac{1}{2}at^2$, where *a* is a constant quantity called acceleration and *t* is time. In the table, which column of numbers represents time?

Column 1, weight number

SECTION 12B

Ready To Go On? Skills Intervention

12-4 Geometric Sequences and Series

Find these vocabulary words in Lesson 12-4 and the Multilingual Glossary.

Vocabulary		
geometric sequence	geometric series	geometric means

Identifying Geometric Sequences

Determine if each sequence is arithmetic or geometric. If it is arithmetic, find the common difference. If it is geometric, find the common ratio.

A. −7, −3, 1, 5, 9, 13, 17

$5 - 1 = 4$, $9 - 5 = 4$ — Find the difference between two pairs of successive terms.

$\frac{5}{1} = 5$, $\frac{9}{5} = 1.8$ — Find the ratio of two pairs of successive terms.

Is the sequence arithmetic or geometric? Arithmetic

Is there a common difference or common ratio? Common difference

What is it? 4

B. $\frac{1}{3}, \frac{1}{12}, \frac{1}{48}, \frac{1}{192}$

$\frac{1}{12} - \frac{1}{3} = -\frac{1}{4}$, $\frac{1}{48} - \frac{1}{12} = -\frac{1}{16}$ — Find the difference between two pairs of successive terms.

$\frac{1/12}{1/3} = \frac{1}{4}$, $\frac{1/48}{1/12} = \frac{1}{4}$ — Find the ratio of two pairs of successive terms.

Is the sequence arithmetic or geometric? Geometric

Is there a common difference or common ratio? Common ratio

What is it? $\frac{1}{4}$

Finding the *n*th term Given a Geometric Sequence

Find the 8th term of a geometric sequence if $a_1 = 16$ and $r = 0.5$.

$a_n = a_1r^{n-1}$ — Write the formula for geometric sequence.

$a_n = 16(0.5)^{7}$ — Substitute known values and evaluate.

$= 0.125$

$= \frac{1}{8}$

The 8^{th} term in the sequence is $\frac{1}{8}$.

SECTION 12B

Ready To Go On? Problem Solving Intervention

12-4 Geometric Sequences and Series

Each term in a geometric sequence is the product of the previous term and the common ratio.

Carlos planted three tomato plants. He exposed the plants to artificial light for 12 hours per day and measured their height each week. His measurements showed that the plants increased in height by an average of 8% per week. The average initial height of the three plants was 22 cm. Predict the average height of the plants at the end of 10 weeks.

Understand the Problem

1. Does the measured height of the plants form a geometric series? Explain.

Yes, because there is a common ratio.

2. What formula gives the *n*th term of a geometric sequence? $a_n = a_1r^{n-1}$

3. What is the first term of the sequence? 22 cm

4. What is the second term of the sequence? $22 + 0.08 \times 22 = 23.76$

5. What is the common ratio? $\frac{23.76}{22} = 1.08$

Make a Plan

6. In the expression for the *n*th term of a geometric sequence, what are the known values? $a_1 = 22$, $r = 1.08$

7. In the expression for the *n*th term of a geometric sequence, what is the unknown value? a_n

Solve

8. Substitute the appropriate values into the geometric sequence formula.

$a_n = a_1r^{n-1} = 22(1.08)^9$

9. Evaluate the formula.

$a_n = a_1r^{n-1} = 22(1.08)^9 \approx 44$

10. What is the average height of the plants after 10 weeks? About 44 cm

Look Back

11. Use estimation to determine if your answer is reasonable. 8% can be rounded up to 10%. What is 10% of 22 cm? 2.2 cm Multiply this value by 10. 22 Add this value to 22. 44 Does your answer in Exercise 10 make sense? Yes

SECTION 12B

Ready To Go On? Skills Intervention

12-5 Mathematical Induction and Infinite Geometric Series

Find these vocabulary words in Lesson 12-5 and the Multilingual Glossary.

Vocabulary		
infinite geometric series	converge	limit

Finding the Sums of Infinite Geometric Series

Determine if the following infinite series converges. If so, find its sum.

$2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625} + \ldots$

For an infinite geometric series, if $r < 1$, the series converges. If $r \geq$ 1, the series does not converge and does not have a sum.

Calculate the ratio between successive terms:

$r = \frac{2/5}{2} = \frac{1}{5}$, $r = \frac{2/25}{2/5} = \frac{1}{5}$

Since $r < 1$, the series converges.

Calculate the sum of the series: $S = \frac{a_1}{1 - r} = \frac{2}{1 - 1/5} = \frac{2}{4/5} = \frac{10}{4} = 2.5$

Writing Repeating Decimals as Fractions

Write 0.09090909 . . . as a fraction in simplest form.

Repeating decimals are rational numbers. Infinite geometric series can be used to convert a repeating decimal to a fraction.

$0.090909\ldots = 0.09 + 0.0009 + 0.000009$ — Write the repeating decimal as an infinite series.

$\frac{0.0009}{0.09} = 0.01$ — Calculate the common ratio.

$a_1 = 0.09$ — Identify the first term.

$S = \frac{a_1}{1 - r}$ — Write the formula.

$= \frac{0.09}{1 - 0.01}$ — Substitute known values and evaluate.

$= \frac{0.09}{0.99}$

$= \frac{1}{11}$

So, $0.09090909 = \frac{1}{11}$.

SECTION 12B

Ready To Go On? Problem Solving Intervention

12-5 Mathematical Induction and Infinite Geometric Series

You can find the sum of an infinite geometric series, if it converges.

Bacteria reproduce by fission results in one cell dividing into two cells. One particular species of bacteria reproduces about once every hour. A researcher swabs a plate of growth medium leaving 100 bacteria. Assuming inexhaustible space and unlimited food, how many bacteria would there be in 12 hours?

Understand the Problem

1. Is this a geometric series? Explain how you know. Yes, because the ratio is constant.
2. What is the common ratio? 2
3. What is the first term? 100
4. What is the formula for a partial sum of an infinite series? $S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

Make a Plan

5. In the formula for the partial sum of an infinite series, what are the known values? $a_1 = 100$, $r = 2$, $n = 12$
6. In the formula for the partial sum of an infinite series, what is the unknown value? S_n

Solve

7. Substitute the known values into the formula and evaluate.

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right)$

$= 100\left(\frac{1 - 2^{12}}{1 - 2}\right)$

$= 100\left(\frac{-4095}{-1}\right)$

$= 409{,}500$

8. How many bacteria were there in 12 hours? 409,500

Look Back

9. Graph the function $y = 100\left(\frac{1 - 2^{12}}{1 - 2}\right)$ on your graphing calculator.

Does your graph support your answer to Exercise 8? Yes

SECTION 12B

Ready To Go On? Quiz

12-4 Geometric Sequences and Series

Write the first six terms of each sequence.

1. 27, 135, 675, 3375, . . .

27, 135, 675, 3375, 16,875, 84,375

2. $1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \ldots$

$1, \frac{1}{3}, \frac{1}{9}, \frac{1}{27}, \frac{1}{81}, \frac{1}{243}$

Find the 10th term of each sequence.

3. $2, 1, \frac{1}{2}, \frac{1}{4}, \ldots$

$\frac{1}{256}$

4. $\sqrt{2}, 2, 2\sqrt{2}, 4, \ldots$

32

5. $a_1 = 0.625$, $r = 2$

320

6. $a_4 = 4$, $r = 2$

256

7. A piece of paper is 0.03 mm thick. It is torn in two equal pieces and stacked. The stack is torn in again into two equal pieces and stacked again, so it has four pieces of paper. This process is repeated many times, with the number of sheets of paper doubling each time. Write a formula for the height of the paper stack after n times. Assuming it is possible, calculate the height of the stack after 11 tears.

$0.03(2)^{n-1} = 0.03(2)^{10} = 30.72$ mm

Find the geometric mean of each pair of numbers.

8. $\frac{1}{4}$ and 4

1

9. 4 and 64

16

Find the geometric means to complete each sequence.

10. 8, ____, ____, 27, . . .

12, 18

11. −2, ____, ____, 54, . . .

6, −18

SECTION 12B

Ready To Go On? Quiz continued

Find the indicated sums.

12. S_6 for $1 + 5 + 25 + 125 + \ldots$

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) = 3906$

13. S_8 for $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots$

$S_n = a_1\left(\frac{1 - r^n}{1 - r}\right) \approx 1.33$

12–5 Mathematical Induction and Infinite Geometric Series

Determine if the series converges. If it does, find the sum.

14. $\frac{1}{25} + \frac{1}{250} + \frac{1}{2500} + \ldots$

$r = 0.1 < 1;\ S = \frac{a_1}{1 - r} = 0.0\overline{4}$

15. $5 + \frac{5}{2} + \frac{5}{4} + \frac{5}{8} + \ldots$

$r = \frac{1}{2} < 1;\ S = \frac{5}{1 - \frac{1}{2}} = 10$

16. $\sum_{k=1}^{\infty}\left(\frac{1}{9}\right)^{k-1}$

$r = \frac{1}{9} < 1;\ S = \frac{1}{1 - \frac{1}{9}} = 1.125$

17. $\frac{2}{7} + \frac{4}{7} + \frac{8}{7} + \frac{16}{7} + \ldots$

$r = 2$, does not converge

18. $\sum_{k=1}^{\infty} 2\left(\frac{1}{3}\right)^{k-1}$

$r = \frac{1}{3} < 1;\ S = \frac{2}{1 - \frac{1}{3}} = 3$

19. $\sum_{k=1}^{\infty} 4(3)^{k-1}$

$r = 3$, does not converge

20. Prove by mathematical induction that the nth partial sum of the series $1 + 3 + 5 + 7 + \ldots + (2n - 1)$ is equal to n^2.

If $n = 1$, $S_1 = 1^2 = 1$. Assume that for $n = k$, $S_k = k^2$

Given $S_k = k^2$; $\sum_{n=1}^{k}(2n - 1) = k^2$

$\sum_{n=1}^{k+1}(2n - 1) = \sum_{n=1}^{k}(2n - 1) + (2(k + 1) - 1)$

$= \sum_{n=1}^{k}(2n - 1) + 2k + 1$

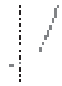
$= k^2 + 2k + 1 = (k + 1)^2$

SECTION 12B

Ready To Go On? Enrichment

Mathematical Induction

Mathematical Induction is a powerful means of proving many mathematical conjectures. Graph the two functions $f(x) = x^2$ and $g(x) = 2^x$ on the same axes.



1. What is the relationship between $f(x)$ and $g(x)$ for large values of x?
   For $x > 4$, $x^2 < 2^x$.

Mathematical Induction can be used to prove the conjecture in Exercise 1 for integers. Let the variable be n, where $n > 4$ and can have only integer values.

2. Rewrite this conjecture using the new variable.
   $n^2 < 2^n$, $n > 4$
3. Prove the conjecture for the first case.
   $n^2 < 2^n$; $5^2 < 2^5$; $25 < 32$
4. Assume the statement is true for for $n = k$ and show that it is true for $n = k + 1$.
   $(k + 1)^2 = k^2 + 2k + 1$; $(k + 1)^2 < k^2 + (k - 1)k + 1$ since $2 < k - 1$; $(k + 1)^2 < 2k^2 - k + 1$; $(k + 1)^2 < 2(2^k)$ since $k^2 < 2^k$; $(k + 1)^2 < 2^{k+1}$

Alternating Series

Does the series $1 - 1 + 1 - 1 + 1 - 1 + \ldots$ have a sum?
To find out, calculate the first few partial sums.

S1 = 1 S3 = 1
S2 = 0 S4 = 0

It is clear that these terms do not converge to a limit, so this series does not have a sum. To further test the idea, apply the rule for the sum of a convergent series,

$S = \frac{a_1}{1 - r} = \frac{1}{1 - (-1)} = \frac{1}{2}$. This answer makes no sense.

Many alternating series do converge.

5. Suggest an example of an alternating series that converges.
   Sample answer: $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{16} + \ldots$

SECTION 13A

Ready To Go On? Skills Intervention

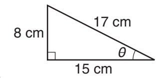
13-1 Right-Angle Trigonometry

Find these vocabulary words in Lesson 13-1 and the Multilingual Glossary.

Vocabulary			
trigonometric function	sine	cosine	tangent
cosecant	secant	cotangent	

Finding Values of Trigonometric Functions

In a right triangle, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$, $\tan\theta = \frac{\text{opp}}{\text{adj}}$



A. Find the value of the sine function for the triangle shown.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{\text{hyp}}$ The side opposite is the side that does not touch the angle.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = \frac{8}{17}$ The hypotenuse is the side opposite from the right angle.

$\sin\theta = \frac{\text{opp}}{\text{hyp}} = 0.47$ Solve.

B. Find the value of the cosine function for the triangle shown.

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{\text{hyp}}$ The side adjacent is the side that touches the angle.

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = \frac{15}{17}$ The hypotenuse is the side opposite from the right angle.

$\cos\theta = \frac{\text{adj}}{\text{hyp}} = 0.88$ Solve.

C. Find the value of the tangent function for the triangle shown.

$\tan\theta = \frac{\text{opp}}{\text{adj}} = \frac{8}{15}$ The opposite and the adjacent are the two shortest sides.

$\tan\theta = \frac{\text{opp}}{\text{adj}} = 0.53$ x The hypotenuse is the side opposite from the right angle.

D. Find the value of the cosecant function for the triangle shown.

$\csc\theta = \frac{1}{\sin\theta}$

$\csc\theta = \frac{1}{0.47}$

$\csc\theta = 2.13$

SECTION 13A

Ready To Go On? Problem Solving Intervention

13-1 Right-Angle Trigonometry

A trigonometric ratio compares the length of two sides of right triangles.

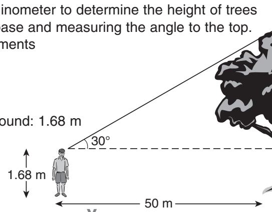
Foresters use a simple device called a clinometer to determine the height of trees by standing a certain distance from the base and measuring the angle to the top. A forester obtains the following measurements

a. Angle to top of tree: 30°

b. Distance from base of tree: 50 m

c. Height of the forester's eyes above ground: 1.68 m

What is the height of the tree?



Understand the Problem

Examine the figure.

1. Is there a right triangle? Yes
2. What are the sides of the triangle? Distance to the base, height, distance to top
3. Which side is unknown? Height
4. What should you do with the height of the forester's eyes above ground?
   Add it to the height of the tree.

Make a Plan

5. Is the unknown the opposite side, adjacent side, or hypotenuse?
   Opposite side
6. Is the known side the opposite side, adjacent side, or hypotenuse?
   Adjacent side
7. Which trig function includes the knowns and the unknowns from Exercise 5 and 6? tangent

Solve

8. Substitute known values into the tangent ratio and solve: $\tan 30° = \frac{\text{opp}}{50 \text{ m}}$
9. Add the height of eye level to find the height of the tree.
   30.6 m
   opp = 50 m × tan 30°
   = 28.9 m

Look Back

10. You can check your solution by using the formula for a 30-60-90 right triangle. The sides are in proportion as shown. Complete: $\frac{1}{\sqrt{3}} = \frac{x}{50}$. Solve for x. $x = \frac{50}{\sqrt{3}} = 28.9$

SECTION 13A

Ready To Go On? Skills Intervention

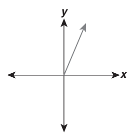
13-2 Angles of Rotation

Find these vocabulary words in Lesson 13-2 and the Multilingual Glossary.

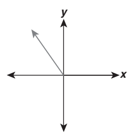
Vocabulary			
standard position	initial side	terminal side	angle of rotation
coterminal angles	reference angle		

Drawing Angles in Standard Position
Draw an angle with the given measure in standard position.

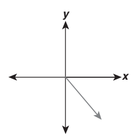
A. 67°
The angle is positive so rotate the terminal side counterclockwise.



B. 125°
The angle is positive so rotate the terminal side counterclockwise.



C. −50°
The angle is negative so rotate the terminal side clockwise.



Finding Coterminal Angles
Find a positive angle coterminal with each given angle.

A. 35°
To find a positive coterminal angle, add 360°.
35° + 360° = 395°

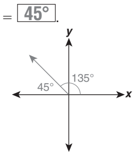
B. 135°
To find a positive coterminal angle, add 360°.
135° + 360° = 495°

C. −35°
To find a positive coterminal angle, add 360°.
−35° + 360° = 325°

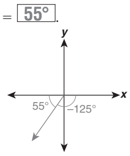
Finding Reference Angles
Find the measure of the reference angle for each given angle.

The reference angle is the positive acute angle formed by the terminal side of θ and the x-axis.

A. 135°
The reference angle = 45°.



B. −125°
The reference angle = 55°.



C. 250°
The reference angle = 70°.