Name _____ Date _____ Class _____

CHAPTER 12

Performance Assessment Teacher Support

Sequences and Series

Purpose
This performance task assesses the student's ability to write rules for sequences, find the terms of a geometric sequence, and find the sums of geometric series and infinite geometric series.

Time
30−45 minutes

Grouping
Pairs or small groups

Preparation Hints
Ask students in what ways they earn money and how they, in turn, spend that money. Ask whether they have ever thought about what happens to their money after they have spent it in a store. How much do they think is kept as profit; where do they think the rest is spent? Connect the two examples to the idea of reinvestment in the economy.

Overview
Students use a geometric sequence and a geometric series to investigate the principle of *economic multipliers.* They eventually see that the economic multiplier is related to the percent at which money is reinvested in the economy; in so doing, students also need to revisit their skills at simplifying rational expressions.

Introduce the Task
Go through the introductory paragraph and make sure all students understand how money introduced into the economy can have a far-reaching and long-term "ripple" effect.

Performance Indicators

_____ Identifies monthly reinvestment as a situation involving a geometric sequence and writes terms of the sequence.

_____ Writes a rule for the nth term of the geometric sequence.

_____ Identifies total economic impact as a situation involving a geometric series and calculates partial sums and the infinite sum.

_____ Determines the relationship between the economic multiplier and the common ratio r, and uses the relationship to solve problems.

Scoring Rubric

Level 4: Student solves problems correctly and gives good explanations.

Level 3: Student solves problems but does not give satisfactory explanations.

Level 2: Student solves some problems but does not give satisfactory explanations.

Level 1: Student is not able to solve any of the problems.

Name _____ Date _____ Class _____

CHAPTER 12

Performance Assessment

Sequences and Series

When you spend $1 at the grocery store, the store doesn't keep all of your money as profit. Some of it is reinvested in the economy when the grocery store buys more merchandise from suppliers. In turn, the suppliers don't keep all of that money as profit, either; they reinvest it in the economy when they buy more raw materials from other suppliers. The end result is that your $1 may contribute tens or hundreds of dollars to the economy.

In January, a factory invests $400,000 into the economy in the form of employee salaries. During February, assume that the employees reinvest in the economy by spending 80% of their salaries. Then, during March, 80% of what was spent by the employees is again reinvested in the economy by the merchants. And so on.

1. Find the amount of money that is invested during the first four months.

_____	_____	_____	_____
Month 1 (Jan.)	Month 2 (Feb.)	Month 3 (Mar.)	Month 4 (Apr.)

2. Starting with January, the amount of money that is invested in the economy forms what kind of sequence? _____

3. Write a rule for the amount of money a_n that is invested during Month n. _____

The *total economic impact* of the factory's initial investment is the sum of all the investments and reinvestments over the months.

4. What is the total economic impact after the first four months? _____

5. If the money is infinitely reinvested in the economy, the total economic impact is modeled by what kind of series? _____

6. Calculate the total economic impact of the factory's initial investment. _____

The ratio of the total economic impact to the initial investment is called the *economic multiplier.*

7. What is the economic multiplier for this situation? _____

8. How is the economic multiplier related to the percent of money that is reinvested each month? (*Hint:* Write the ratio with generic variables, such as a_1 and r, and simplify.) _____

_____

9. What is the economic multiplier if you assume that only 50% of the initial investment is reinvested each month? _____

10. Another economist predicts that the economic multiplier is 4. What percent does she think will be reinvested each month? _____

9. 10

10. $20

11. 45.5

12. 0.5

13. 170.5

14. $8908.64

15. ± 30

16. 2

17. 153

18. 11.43

19. 21.875

20. diverges

21. 24

22. 7.5

23. any $n \geq 2$

24. 40

Chapter Test Form C

1. 0, 3.5, -7, 24.5, -70

2. $a_n = 150 - (3.5)(n - 1)$

3. 2326

4. a_4: 24 squares; a_5: 25 squares

5. $\sum_{k=1}^{3} 45\left(\frac{1}{9}\right)^{k-1}$

6. 4.9 inches

7. 501

8. 127 minutes

9. -434.18

10. 8:35

11. 462

12. 0.82

13. -7272

14. $28.81

15. $\pm 2\sqrt{95}$

16. $\frac{4}{5}$

17. 89 square feet

18. -1

19. 14.76 hours

20. diverges

21. 17 cm

22. $\frac{1}{2}$

23. $0 < n < 1$

24. 25

Performance Assessment

1. $400,000; $320,000; $256,000; $204,800

2. geometric sequence

3. $a_n = a_{n-1} \cdot (0.8)$ or $a_n = 400{,}000 \cdot (0.8)^{n-1}$

4. $1,180,800

5. geometric series

6. $S = \frac{a_1}{1 - r} = \frac{\$400{,}000}{1 - 0.8} = \$2{,}000{,}000$

7. $\frac{\$2{,}000{,}000}{\$400{,}000} = 5$

8. The economic multiplier is $\frac{1}{1 - r}$;

e.m. $= \frac{S}{a_1} = \frac{\frac{a_1}{1 - r}}{a_1} = \frac{\cancel{a_1}}{1 - r} \cdot \frac{1}{\cancel{a_1}} = \frac{1}{1 - r}$.

9. $\frac{1}{1 - 0.5} = 2$

10. $4 = \frac{1}{1 - r}$; $r = 0.75 = 75\%$

Cumulative Test

1. B

2. H

3. A

4. G

5. B

6. F

7. D