Date ______________

Dear Family,

In Chapter 12 your child will learn about sequences and series.

A **sequence** is an ordered list of numbers. Each number in the sequence is called a **term**. In an **infinite sequence**, the terms continue without end, while a **finite sequence** has a limited number of terms.

Terms are indicated by the variable a with a subscript that tells the term number. The nth term, or a_n, represents any unspecified term in the sequence.

3, 5, 7, 9, 11, ...

First term	Second term	Third term
$a_1 = 3$	$a_2 = 5$	$a_3 = 7$

The pattern in a sequence can often be generalized with a rule. A **recursive formula** is a rule that tells you how to use one or more previous terms to generate the next term. For example, the sequence above begins with 3 and then 2 is added to create the next term, and the next, and so on.

Recursive Formula: $a_1 = 3,\ a_n = a_{n-1} + 2$

The first term is 3.

You find each next term by adding 2 to the previous term.

You may also be able to write an **explicit formula** that tells you how to find the nth term based on the value of n. In the sequence above, the first term is 3 with 2 added zero times; the second term is 3 with 2 added once; the third term is 3 with 2 added twice. In general, you add 2 one fewer times than the term number.

Explicit Formula: $a_n = 3 + 2(n - 1)$ or $a_n = 2n + 1$

A **series** is a sum of terms in a sequence. When you add only a few of the terms, you have a **partial sum**, indicated by S_n. If you know the explicit formula for the sequence, you can also represent the partial sum with **summation notation**, which uses the capital Greek letter sigma ($\sum$).

$$S_5 = 3 + 5 + 7 + 9 + 11 = 35 \quad \text{or} \quad \sum_{k=1}^{5}(2k + 1) = 3 + 5 + 7 + 9 + 11 = 35$$

S_5 represents the partial sum of the first five terms.

The summation notation says to add the first ($k = 1$) through fifth (5) terms of the sequence defined by $2k + 1$.

An **arithmetic sequence** is a special sequence in which there is a **common difference** d between terms. The following sequence is an arithmetic sequence with a common difference of 2.

3, 5, 7, 9, 11, ...

differences: $5 - 3 = 2$ $7 - 5 = 2$ $9 - 7 = 2$ $11 - 9 = 2$

All arithmetic sequences can be generalized by the same explicit formula:

Arithmetic Sequence: $a_n = a_1 + (n - 1)d$

A partial sum of an arithmetic series can also be generalized by a formula:

Sum of the First *n* Terms of an Arithmetic Series: $S_n = n\left(\dfrac{a_1 + a_n}{2}\right)$

In short, this formula says to multiply the number of terms by the average of the first and last term.

A **geometric sequence** is another special sequence in which there is a **common ratio** r between terms.

5, 20, 80, 320, 1280, ...

ratios: $\dfrac{20}{5} = 4$ $\dfrac{80}{20} = 4$ $\dfrac{320}{80} = 4$ $\dfrac{1280}{320} = 4$

All geometric sequences can be generalized by the same explicit formula:

Geometric Sequence: $a_n = a_1 \cdot r^{n-1}$

A partial sum of a geometric series can also be generalized by a formula:

Sum of the First *n* Terms of a Geometric Series: $S_n = a_1\left(\dfrac{1 - r^n}{1 - r}\right)$

For example, the sum of the first eight terms of the series $5 + 20 + 80 + \cdots$ is $S_8 = 5\left(\dfrac{1 - 4^8}{1 - 4}\right) = 5\left(\dfrac{-65{,}535}{-3}\right) = 109{,}225$.

Some geometric series **converge**, which means the partial sums approach a **limit** as n increases. This happens only when $|r| < 1$. If the series converges, then you can also find the sum of an infinite number of terms, indicated by S with no subscript.

Sum of an Infinite Geometric Series: $S = \dfrac{a_1}{1 - r}$

In addition to using the formulas for series, your child will prove these formulas by using a process called **mathematical induction**. In mathematical induction, you first prove the formula is true for the first term, $n = 1$. Then you assume the formula is true for all terms up to $n = k$. Finally you prove that the formula is true for the next term, $n = k + 1$.

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