

SECTION 11A **Ready To Go On? Skills Intervention**
11-1 Permutations and Combinations

Find these vocabulary words in Lesson 11-1 and the Multilingual Glossary.

Vocabulary		
Fundamental Counting Principle	permutation	factorial
combination		

Finding Permutations

A county fair has 12 turkey projects. In how many ways can the judge select first, second, and third place?

A _____ is a selection of a group of objects in which order is important.

This problem is equivalent of selecting and arranging 3 items from _____.

Complete the formula: ${}_nP_r = \frac{\boxed{}!}{(\boxed{} - r)!}$

Substitute _____ for n and _____ for r .

$$\begin{aligned} {}_{12}P_3 &= \frac{\boxed{}!}{(\boxed{} - 3)!} = \frac{12}{\boxed{}} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}}{9 \cdot 8 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}} \\ &= 12 \cdot \boxed{} \cdot \boxed{} \quad \text{Cancel out common terms.} \\ &= \boxed{} \end{aligned}$$

Finding Combinations

Nicholas is going to select 4 baseballs from a group of 12. How many ways can he choose a group of 4 baseballs?

A _____ is a grouping of items in which order does not matter.

Complete the formula: ${}_nC_r = \frac{\boxed{}!}{r!(\boxed{} - r)!}$

$$\begin{aligned} {}_{12}C_4 &= \frac{\boxed{}!}{4!(\boxed{} - 4)!} = \frac{12!}{4! \boxed{}!} \quad \text{Substitute _____ for } n \text{ and _____ for } r. \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}} \\ &= \frac{12 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}}{4 \cdot \boxed{} \cdot \boxed{} \cdot \boxed{}} \quad \text{Cancel out common terms.} \\ &= \frac{\boxed{}}{24} = \boxed{} \end{aligned}$$

SECTION 11A **Ready To Go On? Skills Intervention**
11-2 Theoretical and Experimental Probability

Find these vocabulary words in Lesson 11-2 and the Multilingual Glossary.

Vocabulary		
probability	sample space	event
equally likely outcomes	favorable outcomes	complement
theoretical probability	geometric probability	experiment
trial	experimental probability	outcome

Finding Theoretical Probability

A box contains 15 pens: 4 blue, 6 red, 3 black, and 2 green. What is the probability that Judith selects a red pen?

There are _____ possible outcomes and 6 favorable outcomes.

$$P(\text{red}) = \frac{\boxed{}}{15} = \frac{\boxed{}}{5}$$

$$= 0.40$$

Finding Geometric Probability

Find the probability that a point chosen at random inside the figure shown is in the shaded area.

What is the formula for the area of a square? $A = \underline{\hspace{2cm}}$

What is the length of one side of the square? _____

Find the area of the square: $A = \underline{\hspace{2cm}}$

What is the formula for the area of a triangle? $A = \underline{\hspace{2cm}}$

What is the height of the triangle? _____

What is the length of the base of the triangle? _____

$$\text{Find the area of the triangle. } A = \frac{1}{2}(\underline{\hspace{1cm}}) \cdot (\underline{\hspace{1cm}})$$

$$= 48 \text{ in.}^2$$

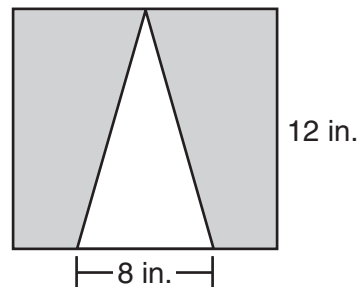
Do you add or subtract to find the shaded area? _____

The area of the shaded region is _____ - 48 = _____.

The ratio of the shaded region to total area is: $\frac{\boxed{}}{144} = \underline{\hspace{1cm}}$

The probability of a point chosen at random being in the shaded area is

_____.



SECTION 11A **Ready To Go On? Skills Intervention**
11-3 Independent and Dependent Events

Find these vocabulary words in Lesson 11-3 and the Multilingual Glossary.

Vocabulary		
independent events	dependent events	conditional probability

Using a Table to Find Conditional Probability

The table shows the breakdown of middle school students in the foreign language club. Find the probability that a Spanish Club member is in 7th grade.

Foreign Language Club		
	Spanish	French
6th	7	2
7th	14	8
8th	11	21

What is the total number of students in the

Spanish Club? _____

How many 7th graders are in the Spanish Club? _____

$$P\left(\frac{\text{7th graders who are in Spanish Club}}{\text{Total students in Spanish Club}}\right) = \frac{14}{\boxed{}} = \frac{\boxed{}}{\boxed{}}$$

Determining Whether Events Are Independent or Dependent

A change purse contains 35 coins – 20 dimes and 15 nickels. Determine whether the events “a dime is selected, not replaced, and then a nickel is selected” are independent or dependent, and find the probability.

After the first coin is selected, is it replaced back into the purse? _____

Is the selection of the second coin affected by the first selection? _____

Are the events independent or dependent? _____

Not replacing the coin after the first selection means there is one _____ coin to choose from.

How many total coins are there? _____

How many are dimes? _____

If one dime is selected and not replaced, how many coins are left? _____

How many nickels are in the purse? _____

Find $P(\text{dime}) \cdot P(\text{nickel/first coin was a dime})$.

$$= \frac{20}{\boxed{}} \cdot \frac{\boxed{}}{34} = \frac{300}{\boxed{}} = \underline{\hspace{2cm}}$$

SECTION 11A **Ready To Go On? Skills Intervention**
11A 11-4 Compound Events

Find these vocabulary words in Lesson 11-4 and the Multilingual Glossary.

Vocabulary	
simple events	compound events
mutually exclusive events	inclusive events

Finding Probabilities of Compound Events

Each letter of the alphabet is written on a card and placed in a bag. One card is drawn. Find each probability.

- A.** Drawing a vowel (A, E, I, O, U) or the letter M.

How many cards are in the bag? _____

How many vowels are there? _____

$$P(\text{M or a vowel}) = P(\text{M}) + P(\text{vowel})$$

$$= \frac{\boxed{}}{26} + \frac{5}{\boxed{}}$$

$$= \frac{\boxed{}}{26}$$

$$= \underline{\hspace{2cm}}$$

- B.** Drawing a letter in the word “Cat” or “Mouse”.

How many cards are in the bag? _____

How many letters are in the word Cat? _____

How many letters are in the word Mouse? _____

$$P(\text{Cat or Mouse}) = P(\text{Cat}) + P(\text{Mouse})$$

$$= \frac{3}{\boxed{}} + \frac{\boxed{}}{26}$$

$$= \frac{\boxed{}}{26}$$

$$= \underline{\hspace{2cm}}$$

- C.** Of a first grade class’s 28 students, 20 ride the bus and 15 buy their lunch. Half of the bus riders buy their lunch. What is the probability that a first grade student does not ride the bus or does not buy their lunch?

What is the total number of first grade students? _____

How many first grade students do not ride the bus? _____

How many first grade students do not buy their lunch? _____

How many bus riders do not buy their lunch? _____

Complete:

$$P(\text{non bus riders}) + P(\text{students not buying lunch}) - P(\text{bus riders not buying lunch})$$

$$\frac{\boxed{}}{28} + \frac{13}{28} - \underline{\hspace{2cm}}$$

$$= \underline{\hspace{2cm}}$$

SECTION
11A**Ready To Go On? Quiz****11-1 Permutations and Combinations**

1. A coach has 9 softball players on his team. In how many ways can the players be placed in a lineup if he does not place them according to ability and each player can only hit once?

2. A department manager is placing flashlights on the shelf. He has a box containing 9 flashlights of all different colors. In how many ways can the manager choose 4 flashlights?

3. A librarian is arranging 7 books in a display. There are places for 8 books. How many ways can the librarian arrange the books?

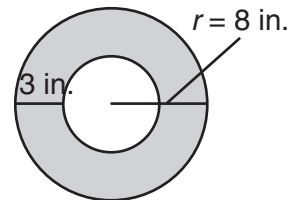
11-2 Theoretical and Experimental Probability

4. A box contains 21 pieces of fruit: 6 oranges, 3 kiwi, 9 red apples and 3 green apples. Katrina selects a fruit without looking. What is the probability that Katrina selects a red apple?

5. Penny has 6 flashlights in a box; 2 do not contain batteries. If her brother selects 2 flashlights from the box, what is the probability that both do not contain batteries?

6. Find the probability that a point chosen at random inside the figure shown is in the shaded area.

7. An odd-shaped die is rolled 60 times, and a 4 is rolled 18 times. Find the experimental probability of not rolling a 4.



SECTION 11A **Ready To Go On? Quiz** continued

11-3 Independent and Dependent Events

8. Explain why the events “selecting a card that is not a face card and selecting a face card if you replace the first card before selecting the second from a deck of 52 cards”, are independent and find the probability.

9. A bag contains 3 blue chips and 2 green chips. Explain why the events “choosing a blue chip then choosing a green chip without replacing the blue chip” are dependent, and find the probability.

10. The table shows the breakdown of chemistry students for one school year. Find the probability that a chemistry student is in the 11th grade.

Science Students by Grade		
	Biology	Chemistry
9th grade	4	0
10th grade	87	14
11th grade	23	84

11. A bag contains 25 slips of paper—12 with letters and 13 with numbers. Determine whether the events “a numbered paper is selected, replaced, and then a lettered paper is selected” are independent or dependent, and find the probability.

11-4 Compound Events

The numbers 1–20 are written on ping pong balls and placed in a bin. One ping pong ball is drawn. Find each probability.

12. drawing an even number or a 3 13. drawing an even number that is a multiple of 4

14. Of 120 students surveyed, 75 were male and 23 were taking algebra. Only 10 of the algebra students were female. What is the probability that a student surveyed was male taking algebra?

SECTION
11A**Ready To Go On? Enrichment****Probability****Solve each problem.**

- There are five roads from town A to town B, four roads from town B to town C and two roads directly from town A to town C.
 - How many ways can one make a trip from town A to town C? _____
 - How many ways can one make a trip from town A to town C by way of town B? _____
 - If a route from town A to town C is chosen at random, what is the probability that it goes through town B? _____
- On a surprise quiz with two questions, an instructor gave one multiple choice question with three choices a, b and c and one true-false question. A student is completely unprepared for the quiz, so she decides to answer at random. If the correct answer is (b, T), find the probability that:
 - both questions are answered correctly. _____
 - the first question is answered correctly. _____
- A child's set of blocks consists of two red, four blue and five yellow cubes. The blocks can be distinguished only by color. If a child lines the blocks in a row at random what is the probability that:
 - a red block comes at both ends? _____
 - the five yellow blocks are together? _____
 - a blue block comes at both ends? _____
- A sample survey was taken to investigate which papers *The Times*, *The Blade* or *The Daily Gazette* people read. In a sample of 100 people the following results were obtained:

60 read <i>The Times</i>	32 read <i>The Times and The Blade</i>	30 read all three
40 read <i>The Blade</i>	45 read <i>The Times and The Daily Gazette</i>	
70 read <i>The Daily Gazette</i>	38 read <i>The Bland and The Daily Gazette</i>	

If a person is selected at random from the sample of 100 people, what is the probability that
 - the person reads only *The Times*. _____
 - the person reads at least two of the papers. _____
 - the person does not read any newspaper. _____

SECTION 11B **Ready To Go On? Skills Intervention**
11-5 Measures of Central Tendency and Variation

Find these vocabulary words in Lesson 11-5 and the Multilingual Glossary.

Vocabulary					
expected value	probability distribution	variance	standard deviation	outlier	

Making a Box-and-Whisker Plot and Finding the Interquartile Range

Math test scores: 67, 99, 73, 75, 95, 92, 80, 84, 89, 85, 85, 86

Step 1 Order the data from least to greatest.

67, 73, _____, _____, _____, _____, _____, _____, _____, _____, _____, _____

Step 2 Find the minimum, maximum, median, and quartiles.

The minimum value is _____. The maximum value is _____.

The median is the middle value of the data. The median is _____.

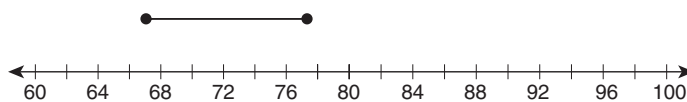
Quartiles are the _____ of the lower and upper halves of the data set.

The first quartile ($Q1$) is: _____. The third quartile ($Q3$) is: _____.

The interquartile range is $Q3 - Q1$: _____.

Step 3 Draw a box-and-whisker plot.

- Draw a number line.
- Plot a point above each of the five values.
- Draw a box from the first quartile to the third quartile with a line segment through the median.
- Draw whiskers from the box to the minimum and maximum values.



Finding the Mean and Standard Deviation

Number of hours worked: 8, 3, 4, 3, 3, 5, 2

Step 1 Find the mean: $\bar{x} = \frac{8 + 3 + \square + \square + \square + \square + \square}{7} = \underline{\hspace{2cm}}$

Step 2 Find the difference between the mean and each data value, and square it.

Data Value x	8	3	4	3	3	5	2
$x - \bar{x}$	4	-1	_____	_____	_____	_____	_____
$(x - \bar{x})^2$	16	1	0	_____	_____	_____	_____

Step 3 Find the variance: $\sigma^2 = \frac{16 + 1 + 0 + \square + \square + \square + \square}{7} = \underline{\hspace{2cm}}$

Step 4 Find the standard deviation: $\sigma = \sqrt{3.43} = \underline{\hspace{2cm}}$

SECTION 11B **Ready To Go On? Skills Intervention**
11-6 Binomial Distributions

Find these vocabulary words in Lesson 11-6 and the Multilingual Glossary.

Vocabulary		
Binomial Theorem	binomial experiment	binomial probability

Expanding Binomials

Use the Binomial Theorem to expand $(4y + 3)^3$.

The sum of the exponents for each term is _____.

$$\begin{aligned} (4y + 3)^3 &= {}_3C_0(4y)^33^0 + \text{_____}(4y)^23^1 + \text{_____}(4y)^13^2 + \text{_____}(4y)^03^3 \\ &= 1 \cdot (4y)^33^0 + \text{_____} \cdot 16y^23^1 + \text{_____} \cdot 4y^13^2 + \text{_____} \cdot (4y)^03^3 \\ &= 64y^3 + \text{_____}y^2 + \text{_____}y + 27 \end{aligned}$$

Finding Binomial Probabilities

One in five cars passing through an intersection will make a left turn. Three cars are stopped at the traffic light and will pass through the intersection.

A. What is the probability that exactly 2 cars will turn left?

What is the probability that a car will turn left? _____

The general form for determining a binomial probability is $P(r) = {}_nC_r \text{_____} q^{n-r}$.

What does n equal? _____ What does r equal? _____

What does p equal? _____ What does q equal? _____

$$\begin{aligned} \text{Substitute known values into the formula: } P(2) &= {}_3C_2 \text{_____} 0.8^{3-2} \\ &= 3(\text{_____})(\text{_____}) \\ &= \text{_____} \end{aligned}$$

The probability that exactly two of the cars will turn left is about _____%.

B. What is the probability that at least two cars will turn left?

At least two left turns is the same as exactly 2 or 3 cars turning left.

$$\begin{aligned} P(2) + P(3) \\ &= 0.096 + {}_3C_3 \text{_____} 0.8^{3-3} \\ &= 0.096 + \text{_____} \\ &= \text{_____} \end{aligned}$$

The probability that at least two cars turn left is about _____%.

SECTION
11B

Ready To Go On? Quiz

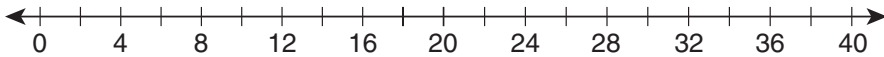
11-5 Measures of Central Tendency and Dispersion

1. The weights of the championship pen of rabbits at the local county fair were recorded as 19, 21, 18, 17, 19, 23, and 20 pounds. Find the mean, median, and mode of the data set.
- _____

2. The probability distribution for the number of defects in a shipment of computer chips, based on past data, is given below. Find the expected number of defects in a shipment of computer chips.

Number of Defects, n	0	1	2	3	4
Probability of n Defects	0.72	0.16	0.09	0.02	0.01

3. Make a box-and-whisker plot of the data. Find the interquartile range.
Number of birds at a bird feeder: 7, 13, 8, 12, 35, 18, 11, 15



4. The number of minutes a commuter waited for a train during the past week are given. Find the commuter wait time within 1 standard deviation of the mean.
Commuter wait time: 9, 5, 2, 14, 8, 4, 7
- _____

The data set shows the number of words spelled incorrectly on a test by 20 students.

6, 8, 4, 7, 5, 9, 3, 2, 0, 4, 2, 6, 8, 9, 3, 27, 12, 11, 9, 5

5. Find the mean and standard deviation of the data.
- _____
6. Identify the outlier, and describe how it affects the mean and standard deviation.
- _____
- _____

SECTION
11B

Ready To Go On? Quiz continued

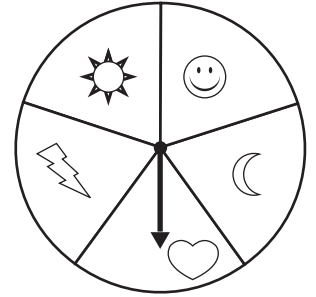
11-6 Binomial Distributions

7. Use the Binomial Theorem to expand $(2x - 3y)^3$.

The spinner shown is spun 10 times.

8. What is the probability that the spinner will land in the heart area exactly 4 times?

9. What is the probability that the spinner will land in the heart area at least 2 times.



A multiple-choice quiz has 7 questions. Each question has 4 possible answers. A student guesses the answer to each question. Find each probability.

10. The student answers all 7 questions correctly.

11. The student answers exactly 1 question correctly.

12. The student answers all 7 questions incorrectly.

13. The student answers at least 1 question correctly.

**SECTION
11B****Ready To Go On? Enrichment****Z-scores**

Standard scores, also known as z-scores enable statisticians to compare values more easily. A z-score is the number of standard deviations that a given value x is above or below the mean. It is found by using the formula: $z = \frac{x - \bar{x}}{s}$ where x is a particular data value, \bar{x} is the mean and s is the standard deviation. The use of z-scores in statistics is extremely important because they can be used to differentiate between ordinary and unusual values. A z-score less than -2.00 or greater than 2.00 is considered to be unusual.

Example: Heights of all adult males have a mean of 69 inches and a standard deviation of 2.8 inches. What is the z-score for a male that is 78 inches tall?

$$z = \frac{x - \bar{x}}{s} = \frac{78 - 69}{2.8} = 3.21$$

You can interpret this result by stating that an individual 78 inches tall is 3.21 standard deviations above the mean. This would be an unusual height.

Solve

1. A mathematics teacher gives two different tests to two different sections of Algebra classes. The statistics are shown below. Which score is better: an 82 on the section 1 test, or a 46 on the section 2 test?

Section 1: mean = 75 and standard deviation = 14

Section 2: mean = 40 and standard deviation = 8

2. The Bean Pole club is open to men and women who are very tall. The minimum height requirement for women is 70 in. If women's heights have a mean of 63.6 inches and a standard deviation of 2.5 inches, find the z-score corresponding to a height of exactly 70 inches. Is this height unusual?
-

3. Three potential employees take a required mathematics test in which three different areas are tested. Which of the following scores has the highest relative position?

Test 1: Score of 37 on a test with a mean of 28 and standard deviation of 6

Test 2: Score of 398 on a test with a mean of 312 and standard deviation of 56

Test 3: Score of 4.10 on a test with a mean of 2.75 and standard deviation of 0.92

SECTION 10B **Ready To Go On? Enrichment**

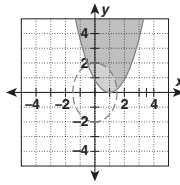
Nonlinear Systems of Inequalities

If two or more inequalities are considered at the same time, a **system of inequalities** is formed. To find the solution set of the system, locate the intersection of the graphs.

For example:
$$\begin{cases} y \geq x^2 - 2x + 1 \\ 2x^2 + y^2 > 4 \end{cases}$$

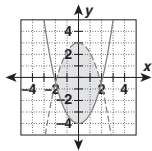
The graph of $y \geq x^2 - 2x + 1$ is a parabola with vertex at (1, 0). The points above or in the interior of the parabola satisfy the condition.

The graph of $2x^2 + y^2 > 4$ is an ellipse. It is drawn with a dashed line. To satisfy the inequality, a point must lie outside of the ellipse.

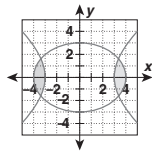


Graph each system of nonlinear inequalities.

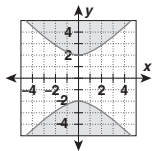
1.
$$\begin{cases} y \geq x^2 - 4 \\ y < -x^2 + 3 \end{cases}$$



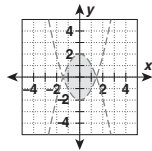
2.
$$\begin{cases} x^2 - y^2 \geq 9 \\ x^2 + y^2 \leq 1 \end{cases}$$



3.
$$\begin{cases} y^2 - x^2 \geq 4 \\ y > -5 \\ y < 5 \end{cases}$$



4.
$$\begin{cases} y < 2 - x^2 \\ y > x^2 - 2 \end{cases}$$



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SECTION 11A **Ready To Go On? Skills Intervention**

11-1 Permutations and Combinations

Find these vocabulary words in Lesson 11-1 and the Multilingual Glossary.

Vocabulary

Fundamental Counting Principle permutation factorial
combination

Finding Permutations

A county fair has 12 turkey projects. In how many ways can the judge select first, second, and third place?

A **permutation** is a selection of a group of objects in which order is important. This problem is equivalent of selecting and arranging 3 items from 12.

Complete the formula: ${}_nP_r = \frac{n!}{(n-r)!}$

Substitute 12 for n and 3 for r .

$$\begin{aligned} {}_{12}P_3 &= \frac{12!}{(12-3)!} = \frac{12!}{9!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= 12 \cdot 11 \cdot 10 \quad \text{Cancel out common terms.} \\ &= \underline{1320} \end{aligned}$$

Finding Combinations

Nicholas is going to select 4 baseballs from a group of 12. How many ways can he choose a group of 4 baseballs?

A **combination** is a grouping of items in which order does not matter.

Complete the formula: ${}_nC_r = \frac{n!}{r!(n-r)!}$

Substitute 12 for n and 4 for r .

$$\begin{aligned} {}_{12}C_4 &= \frac{12!}{4!(12-4)!} = \frac{12!}{4!8!} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} \\ &= \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2 \cdot 1} \quad \text{Cancel out common terms.} \\ &= \frac{11,880}{24} = \underline{495} \end{aligned}$$

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Holt Algebra 2

SECTION 11A **Ready To Go On? Skills Intervention**

11-2 Theoretical and Experimental Probability

Find these vocabulary words in Lesson 11-2 and the Multilingual Glossary.

Vocabulary

probability sample space event
equally likely outcomes favorable outcomes complement
theoretical probability geometric probability experiment
trial experimental probability outcome

Finding Theoretical Probability

A box contains 15 pens: 4 blue, 6 red, 3 black, and 2 green. What is the probability that Judith selects a red pen?

There are 15 possible outcomes and 6 favorable outcomes.

$$P(\text{red}) = \frac{6}{15} = \frac{2}{5} = 0.40$$

Finding Geometric Probability

Find the probability that a point chosen at random inside the figure shown is in the shaded area.

What is the formula for the area of a square? $A = s^2$ or $\ell \cdot w$

What is the length of one side of the square? 12 in.

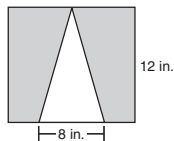
Find the area of the square: $A =$ 144 in.^2

What is the formula for the area of a triangle? $A = \frac{1}{2}bh$

What is the height of the triangle? 12 in.

What is the length of the base of the triangle? 8 in.

$$\text{Find the area of the triangle. } A = \frac{1}{2}(12) \cdot (8) = 48 \text{ in.}^2$$



Do you add or subtract to find the shaded area? Subtract

The area of the shaded region is $144 - 48 = 96 \text{ in.}^2$

$$\text{The ratio of the shaded region to total area is: } \frac{96}{144} = \frac{2}{3}$$

The probability of a point chosen at random being in the shaded area is $\frac{2}{3}$ or $66.\bar{6}\%$

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SECTION 11A **Ready To Go On? Skills Intervention**

11-3 Independent and Dependent Events

Find these vocabulary words in Lesson 11-3 and the Multilingual Glossary.

Vocabulary

independent events dependent events conditional probability

Using a Table to Find Conditional Probability

The table shows the breakdown of middle school students in the foreign language club.

Find the probability that a Spanish Club member is in 7th grade.

	Foreign Language Club	
	Spanish	French
6th	7	2
7th	14	8
8th	11	21

What is the total number of students in the Spanish Club? 32

How many 7th graders are in the Spanish Club? 14

$$P\left(\frac{7\text{th graders who are in Spanish Club}}{\text{Total students in Spanish Club}}\right) = \frac{14}{32} = \frac{7}{16}$$

Determining Whether Events Are Independent or Dependent

A change purse contains 35 coins – 20 dimes and 15 nickels. Determine whether the events “a dime is selected, not replaced, and then a nickel is selected” are independent or dependent, and find the probability.

After the first coin is selected, is it replaced back into the purse? No

Is the selection of the second coin affected by the first selection? Yes

Are the events independent or dependent? Dependent

Not replacing the coin after the first selection means there is one fewer coin to choose from.

How many total coins are there? 35

How many are dimes? 20

If one dime is selected and not replaced, how many coins are left? 34

How many nickels are in the purse? 15

Find $P(\text{dime}) \cdot P(\text{nickel/first coin was a dime})$.

$$= \frac{20}{35} \cdot \frac{15}{34} = \frac{300}{1190} = \frac{30}{119}$$

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SECTION 11A Ready To Go On? Skills Intervention

11A 11-4 Compound Events

Find these vocabulary words in Lesson 11-4 and the Multilingual Glossary.

Vocabulary	
simple events	compound events
mutually exclusive events	inclusive events

Finding Probabilities of Compound Events

Each letter of the alphabet is written on a card and placed in a bag. One card is drawn. Find each probability.

- A.** Drawing a vowel (A, E, I, O, U) or the letter M.
 How many cards are in the bag? $\frac{26}{26}$
 How many vowels are there? $\frac{5}{26}$
 $P(M \text{ or a vowel}) = P(M) + P(\text{vowel})$
 $= \frac{1}{26} + \frac{5}{26}$
 $= \frac{6}{26}$
 $= \frac{3}{13}$
- B.** Drawing a letter in the word "Cat" or "Mouse".
 How many cards are in the bag? $\frac{26}{26}$
 How many letters are in the word Cat? $\frac{3}{26}$
 How many letters are in the word Mouse? $\frac{5}{26}$
 $P(\text{Cat or Mouse}) = P(\text{Cat}) + P(\text{Mouse})$
 $= \frac{3}{26} + \frac{5}{26}$
 $= \frac{8}{26}$
 $= \frac{4}{13}$

- C.** Of a first grade class's 28 students, 20 ride the bus and 15 buy their lunch. Half of the bus riders buy their lunch. What is the probability that a first grade student does not ride the bus or does not buy their lunch?

What is the total number of first grade students? $\frac{28}{28}$
 How many first grade students do not ride the bus? $\frac{8}{28}$
 How many first grade students do not buy their lunch? $\frac{13}{28}$
 How many bus riders do not buy their lunch? $\frac{10}{28}$
 Complete:
 $P(\text{non bus riders}) + P(\text{students not buying lunch}) - P(\text{bus riders not buying lunch})$
 $= \frac{8}{28} + \frac{13}{28} - \frac{10}{28}$
 $= \frac{11}{28}$

SECTION 11A Ready To Go On? Quiz

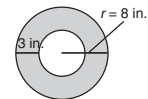
11A

11-1 Permutations and Combinations

1. A coach has 9 softball players on his team. In how many ways can the players be placed in a lineup if he does not place them according to ability and each player can only hit once?
 $362,880$
2. A department manager is placing flashlights on the shelf. He has a box containing 9 flashlights of all different colors. In how many ways can the manager choose 4 flashlights?
 ${}^9C_4 = 126$
3. A librarian is arranging 7 books in a display. There are places for 8 books. How many ways can the librarian arrange the books?
 $7! = 5040$

11-2 Theoretical and Experimental Probability

4. A box contains 21 pieces of fruit: 6 oranges, 3 kiwi, 9 red apples and 3 green apples. Katrina selects a fruit without looking. What is the probability that Katrina selects a red apple?
 $\frac{9}{21} = \frac{3}{7}$
5. Penny has 6 flashlights in a box; 2 do not contain batteries. If her brother selects 2 flashlights from the box, what is the probability that both do not contain batteries?
 $\frac{2}{6} \cdot \frac{1}{5} = \frac{2}{30} = \frac{1}{15}$
6. Find the probability that a point chosen at random inside the figure shown is in the shaded area.
7. An odd-shaped die is rolled 60 times, and a 4 is rolled 18 times. Find the experimental probability of not rolling a 4.



SECTION 11A Ready To Go On? Quiz continued

11A

11-3 Independent and Dependent Events

8. Explain why the events "selecting a card that is not a face card and selecting a face card if you replace the first card before selecting the second from a deck of 52 cards", are independent and find the probability.

Replacing the first card means the first selection will not affect the probability of the second selection.

$$\frac{40}{52} \cdot \frac{12}{52} = \frac{30}{169} \approx 0.178$$

9. A bag contains 3 blue chips and 2 green chips. Explain why the events "choosing a blue chip then choosing a green chip without replacing the blue chip" are dependent, and find the probability.

Not replacing the first chip means that there will be fewer chips to choose from, affecting the probability of the second selection.

$$\frac{3}{5} \cdot \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$$

10. The table shows the breakdown of chemistry students for one school year. Find the probability that a chemistry student is in the 11th grade.

Science Students by Grade		
	Biology	Chemistry
9th grade	4	0
10th grade	87	14
11th grade	23	84

$$\frac{84}{107}$$

11. A bag contains 25 slips of paper—12 with letters and 13 with numbers. Determine whether the events "a numbered paper is selected, replaced, and then a lettered paper is selected" are independent or dependent, and find the probability.

Independent, $\frac{12}{25} \cdot \frac{13}{25} = \frac{156}{625}$

11-4 Compound Events

The numbers 1-20 are written on ping pong balls and placed in a bin. One ping pong ball is drawn. Find each probability.

12. drawing an even number or a 3 $\frac{11}{20}$
13. drawing an even number that is a multiple of 4 $\frac{5}{20} = \frac{1}{4}$

14. Of 120 students surveyed, 75 were male and 23 were taking algebra. Only 10 of the algebra students were female. What is the probability that a student surveyed was male taking algebra?
 $\frac{75}{120} + \frac{23}{120} - \frac{13}{120} = \frac{85}{120} \approx 0.708$

SECTION 11A Ready To Go On? Enrichment

11A

Probability

Solve each problem.

1. There are five roads from town A to town B, four roads from town B to town C and two roads directly from town A to town C.
 a) How many ways can one make a trip from town A to town C? $\frac{22}{22}$
 b) How many ways can one make a trip from town A to town C by way of town B? $\frac{20}{20}$
2. On a surprise quiz with two questions, an instructor gave one multiple choice question with three choices a, b and c and one true-false question. A student is completely unprepared for the quiz, so she decides to answer at random. If the correct answer is (b, T), find the probability that:
 a) both questions are answered correctly. $\frac{1}{6}$
 b) the first question is answered correctly. $\frac{1}{3}$
3. A child's set of blocks consists of two red, four blue and five yellow cubes. The blocks can be distinguished only by color. If a child lines the blocks in a row at random what is the probability that:
 a) a red block comes at both ends? $\frac{1}{55}$
 b) the five yellow blocks are together? $\frac{1}{66}$
 c) a blue block comes at both ends? $\frac{1}{55}$
4. A sample survey was taken to investigate which papers *The Times*, *The Blade* or *The Daily Gazette* people read. In a sample of 100 people the following results were obtained:
 60 read *The Times* 32 read *The Times* and *The Blade* 30 read all three
 40 read *The Blade* 45 read *The Times* and *The Daily Gazette*
 70 read *The Daily Gazette* 38 read *The Bland* and *The Daily Gazette*
 If a person is selected at random from the sample of 100 people, what is the probability that
 a) the person reads only *The Times*. $\frac{0.13}{0.13}$
 b) the person reads at least two of the papers. $\frac{0.55}{0.55}$
 c) the person does not read any newspaper. $\frac{0.15}{0.15}$

SECTION 11B Ready To Go On? Skills Intervention
11-5 Measures of Central Tendency and Variation

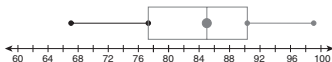
Find these vocabulary words in Lesson 11-5 and the Multilingual Glossary.

Vocabulary
expected value probability distribution variance standard deviation outlier

Making a Box-and-Whisker Plot and Finding the Interquartile Range
 Math test scores: 67, 99, 73, 75, 95, 92, 80, 84, 89, 85, 85, 86

- Step 1** Order the data from least to greatest.
 67, 73, 75, 80, 84, 85, 85, 86, 89, 92, 95, 99
- Step 2** Find the minimum, maximum, median, and quartiles.
 The minimum value is 67. The maximum value is 99.
 The median is the middle value of the data. The median is 85.
 Quartiles are the medians of the lower and upper halves of the data set.
 The first quartile (Q_1) is: 77.5. The third quartile (Q_3) is: 90.5.
 The interquartile range is $Q_3 - Q_1$: 13.

- Step 3** Draw a box-and-whisker plot.
 a) Draw a number line.
 b) Plot a point above each of the five values.
 c) Draw a box from the first quartile to the third quartile with a line segment through the median.
 d) Draw whiskers from the box to the minimum and maximum values.



Finding the Mean and Standard Deviation
 Number of hours worked: 8, 3, 4, 3, 3, 5, 2

- Step 1** Find the mean: $\bar{x} = \frac{8 + 3 + \boxed{4} + \boxed{3} + \boxed{3} + \boxed{5} + \boxed{2}}{7} = \underline{4}$
- Step 2** Find the difference between the mean and each data value, and square it.
- | | | | | | | | |
|-------------------|----|----|----------|-----------|-----------|----------|-----------|
| Data Value x | 8 | 3 | 4 | 3 | 3 | 5 | 2 |
| $x - \bar{x}$ | 4 | -1 | <u>0</u> | <u>-1</u> | <u>-1</u> | <u>1</u> | <u>-2</u> |
| $(x - \bar{x})^2$ | 16 | 1 | 0 | <u>1</u> | <u>1</u> | <u>1</u> | <u>4</u> |
- Step 3** Find the variance: $\sigma^2 = \frac{16 + 1 + 0 + \boxed{1} + \boxed{1} + \boxed{1} + \boxed{4}}{7} = \underline{3.43}$
- Step 4** Find the standard deviation: $\sigma = \sqrt{3.43} = \underline{1.85}$

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SECTION 11B Ready To Go On? Skills Intervention
11-6 Binomial Distributions

Find these vocabulary words in Lesson 11-6 and the Multilingual Glossary.

Vocabulary
Binomial Theorem binomial experiment binomial probability

Expanding Binomials

Use the Binomial Theorem to expand $(4y + 3)^3$.
 The sum of the exponents for each term is 3.
 $(4y + 3)^3 = {}_3C_0(4y)^33^0 + {}_3C_1(4y)^23^1 + {}_3C_2(4y)^13^2 + {}_3C_3(4y)^03^3$
 $= 1 \cdot (4y)^33^0 + \underline{3} \cdot 16y^23^1 + \underline{3} \cdot 4y13^2 + \underline{1} \cdot (4y)^03^3$
 $= 64y^3 + \underline{144}y^2 + \underline{108}y + 27$

Finding Binomial Probabilities

One in five cars passing through an intersection will make a left turn. Three cars are stopped at the traffic light and will pass through the intersection.

- A.** What is the probability that exactly 2 cars will turn left?
 What is the probability that a car will turn left? 0.2
 The general form for determining a binomial probability is $P(r) = {}_nC_r p^r q^{n-r}$.
 What does n equal? 3 What does r equal? 2
 What does p equal? 0.2 What does q equal? 0.8
 Substitute known values into the formula: $P(2) = {}_3C_2 \frac{0.2^2}{0.8^{3-2}}$
 $= 3(\underline{0.04})(\underline{0.8})$
 $= \underline{0.096}$
 The probability that exactly two of the cars will turn left is about 9.6 %.
- B.** What is the probability that at least two cars will turn left?
 At least two left turns is the same as exactly 2 or 3 cars turning left.
 $P(2) + P(3)$
 $= 0.096 + {}_3C_3 \frac{0.2^3}{0.8^{3-3}}$
 $= 0.096 + \underline{0.008}$
 $= \underline{0.104}$
 The probability that at least two cars turn left is about 10.4 %.

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SECTION 11B Ready To Go On? Quiz

11-5 Measures of Central Tendency and Dispersion

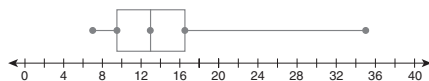
1. The weights of the championship pen of rabbits at the local county fair were recorded as 19, 21, 18, 17, 19, 23, and 20 pounds. Find the mean, median, and mode of the data set.
mean = 19.6, median = 19, mode = 19

2. The probability distribution for the number of defects in a shipment of computer chips, based on past data, is given below. Find the expected number of defects in a shipment of computer chips.

Number of Defects, n	0	1	2	3	4
Probability of n Defects	0.72	0.16	0.09	0.02	0.01

0.044

3. Make a box-and-whisker plot of the data. Find the interquartile range.
 Number of birds at a bird feeder: 7, 13, 8, 12, 35, 18, 11, 15



Interquartile range = 7

4. The number of minutes a commuter waited for a train during the past week are given. Find the commuter wait time within 1 standard deviation of the mean.
 Commuter wait time: 9, 5, 2, 14, 8, 4, 7
3.9 min to 10.63 min

The data set shows the number of words spelled incorrectly on a test by 20 students.

- 6, 8, 4, 7, 5, 9, 3, 2, 0, 4, 2, 6, 8, 9, 3, 27, 12, 11, 9, 5
5. Find the mean and standard deviation of the data.
mean = 7, standard deviation = 5.54
6. Identify the outlier, and describe how it affects the mean and standard deviation.
Outlier: 27; The mean changes from 7 to 5.9, and the standard deviation changes from 5.54 to 3.19.

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SECTION 11B Ready To Go On? Quiz continued

11-6 Binomial Distributions

7. Use the Binomial Theorem to expand $(2x - 3y)^3$.
 $8x^3 - 36x^2y + 54xy^2 - 27y^3$

The spinner shown is spun 10 times.

8. What is the probability that the spinner will land in the heart area exactly 4 times?
 $(\frac{1}{5})^4 \cdot (\frac{4}{5})^6 \cdot {}_{10}C_4 = 0.088$
9. What is the probability that the spinner will land in the heart area at least 2 times.
 $1 - ((\frac{1}{5})^0 \cdot (\frac{4}{5})^{10} \cdot {}_{10}C_0 + (\frac{1}{5})^1 \cdot (\frac{4}{5})^9 \cdot {}_{10}C_1) = 0.625$



A multiple-choice quiz has 7 questions. Each question has 4 possible answers. A student guesses the answer to each question. Find each probability.

10. The student answers all 7 questions correctly.
 $(\frac{1}{4})^7 \cdot (\frac{3}{4})^0 \cdot {}_7C_7 = 0.0000610$
11. The student answers exactly 1 question correctly.
 $(\frac{1}{4})^1 \cdot (\frac{3}{4})^6 \cdot {}_7C_1 = 0.31$
12. The student answers all 7 questions incorrectly.
 $(\frac{1}{4})^0 \cdot (\frac{3}{4})^7 \cdot {}_7C_0 = 0.13$
13. The student answers at least 1 question correctly.
 $1 - 0.13 = 0.87$

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SECTION 11B Ready To Go On? Enrichment

11B

Z-scores

Standard scores, also known as z-scores enable statisticians to compare values more easily. A z-score is the number of standard deviations that a given value x is above or below the mean. It is found by using the formula: $z = \frac{x - \bar{x}}{s}$, where x is a particular data value, \bar{x} is the mean and s is the standard deviation. The use of z-scores in statistics is extremely important because they can be used to differentiate between ordinary and unusual values. A z-score less than -2.00 or greater than 2.00 is considered to be unusual.

Example: Heights of all adult males have a mean of 69 inches and a standard deviation of 2.8 inches. What is the z-score for a male that is 78 inches tall?

$$z = \frac{x - \bar{x}}{s} = \frac{78 - 69}{2.8} = 3.21$$

You can interpret this result by stating that an individual 78 inches tall is 3.21 standard deviations above the mean. This would be an unusual height.

Solve

- A mathematics teacher gives two different tests to two different sections of Algebra classes. The statistics are shown below. Which score is better: an 82 on the section 1 test, or a 46 on the section 2 test?

Section 1: mean = 75 and standard deviation = 14

Section 2: mean = 40 and standard deviation = 8

z-score for section 1 = 0.50, z-score for section 2 = 0.75;

A 46 on the section 2 test is a better score.

- The Bean Pole club is open to men and women who are very tall. The minimum height requirement for women is 70 in. If women's heights have a mean of 63.6 inches and a standard deviation of 2.5 inches, find the z-score corresponding to a height of exactly 70 inches. Is this height unusual?
z-score is 2.56, the height would be considered unusual.

- Three potential employees take a required mathematics test in which three different areas are tested. Which of the following scores has the highest relative position?

Test 1: Score of 37 on a test with a mean of 28 and standard deviation of 6

Test 2: Score of 398 on a test with a mean of 312 and standard deviation of 56

Test 3: Score of 4.10 on a test with a mean of 2.75 and standard deviation of 0.92

z-scores of 1.5, 1.54 and 1.47; the score of 398 has the highest relative position.

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SECTION 12A Ready To Go On? Skills Intervention

12A 12-1 Introduction to Sequences

Find these vocabulary words in Lesson 12-1 and the Multilingual Glossary.

Vocabulary

sequence infinite sequence finite sequence

Finding Terms of a Sequence by Using a Recursive Formula

Find the first five terms of each sequence.

A. $a_n = a_{n-1} + 1$, where $n \geq 2$ and $a_1 = 1$

$$a_1 = 1$$

The first term is given.

$$a_2 = \frac{1}{1} + 1 = 2$$

Substitute $a_1 = 1$ into the rule $a_n = a_{n-1} + 1$ to find a_2 .

$$a_3 = \frac{2}{2} + 1 = 3$$

Repeat to find a_3 .

$$a_4 = \frac{3}{3} + 1 = 4$$

Repeat to find the next term.

$$a_5 = \frac{4}{4} + 1 = 5$$

Repeat to find the fifth term.

B. $a_n = 3a_{n-1} - 1$, where $n \geq 2$ and $a_1 = 1$

$$a_1 = 1$$

The first term is given.

$$a_2 = 3(1) - 1 = 2$$

Substitute $a_1 = 1$ into the rule $a_n = 3a_{n-1} - 1$ to find a_2 .

$$a_3 = 3(2) - 1 = 5$$

Repeat to find a_3 .

$$a_4 = 3(5) - 1 = 14$$

Repeat to find the next term.

$$a_5 = 3(14) - 1 = 41$$

Repeat to find the fifth term.

Finding Terms of a Sequence by Using the Explicit Formula

Find the first three terms of each sequence.

A. $a_n = 2n - 1$, where $n \geq 1$

$$a_1 = 2 \cdot 1 - 1 = 1$$

Let $n = 1$. Substitute this value into the formula $a_n = 2n - 1$ to find a_1 .

$$a_2 = 2(2) - 1 = 3$$

Let $n = 2$. Substitute this value into the formula $a_n = 2n - 1$ to find a_2 .

$$a_3 = 2(3) - 1 = 5$$

Let $n = 3$. Substitute this value into the formula $a_n = 2n - 1$ to find a_3 .

B. $a_n = 3^n - n$, where $n \geq 1$

$$a_1 = 3^1 - 1 = 2$$

Let $n = 1$. Substitute this value into the formula to find a_1 .

$$a_2 = 3^2 - 2 = 7$$

$$a_3 = 3^3 - 3 = 24$$

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SECTION 12A Ready To Go On? Problem Solving Intervention

12A 12-1 Introduction to Sequences

A recursive formula is a rule to describe a sequence where one or more previous terms are used to generate the next term.

Grace has an action figure from a popular science fiction movie still in its original packaging. It is currently worth \$75. Action figures from this movie typically increase in value by about 7% a year.

- Write a recursive rule predicting the value of the figure each year.
- Use the recursive formula to predict the value of the figure in 10 years.

Understand the Problem

- What is the initial value of the figure? \$75
- Will the value of the figure increase or decrease? Increase
- By how much will the value increase each year? By 7% of whatever the value was the previous year

Make a Plan

- What do you need to determine? A recursive rule that will predict the value of the action figure over the next few years.
- Let a_n represent the value of the figure in year n . What symbol represents the value in the previous year? a_{n-1}
- By how much did the value of the figure increase in year $n - 1$? $0.07a_{n-1}$
- Write a recursive rule to model the value of the figure.

$$a_n = a_{n-1} + 0.07a_{n-1}$$

Solve

- Use the recursive rule to predict the value of the figure in 10 years. \$147.54

Look Back

- You can check your solution by using an explicit formula. The explicit rule for this pattern is the same as the formula for compound interest.

$$a_n = a_0(1 + r)^t = 75(1 + 0.07)^{10} = 147.54$$

Does your answer check? Yes

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SECTION 12A Ready To Go On? Skills Intervention

12A 12-2 Series and Summation Notation

Find these vocabulary words in Lesson 12-2 and the Multilingual Glossary.

Vocabulary

series summation notation infinite series

Using Summation Notation

Write this series using summation notation. $2 + \frac{2}{5} + \frac{2}{25} + \frac{2}{125} + \frac{2}{625}$

Find a rule for the k th term of the sequence.

Look at the numerators:

Each numerator is 2.

Look at the denominators:

Each denominator is the base 5 to a power of 0, 1, 2, 3, and 4.

Write the rule for the sequence: $a_k = \frac{2}{5^k}$

Write the notation for the first five terms: $\sum_{k=0}^4 \frac{2}{5^k}$

Evaluating a Series

The value of π can be approximated by a partial sum of an infinite series.

Expand the series $\pi \approx 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1}$ and evaluate.

Write out the first five terms.

$$\pi \approx 4 \left(\frac{(-1)^0}{2 \cdot 0 + 1} + \frac{(-1)^1}{2 \cdot 1 + 1} + \frac{(-1)^2}{2 \cdot 2 + 1} + \frac{(-1)^3}{2 \cdot 3 + 1} + \frac{(-1)^4}{2 \cdot 4 + 1} \right)$$

Simplify each term.

$$\pi \approx 4 \left(\frac{1}{1} + \frac{-1}{3} + \frac{1}{5} + \frac{-1}{7} + \frac{1}{9} \right)$$

Simplify.

$$\pi \approx 3.33968$$

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