Name ______________________________ Date __________ Class __________

LESSON 9-6

Reteach

Modeling Real-World Data

A pattern in data can suggest a model to fit the data.

If *x*-values are evenly spaced and first differences of *y*-values are constant, a **linear model** fits the data.

x	1	2	3	4	5
y	12	27	42	57	72

First differences: 15 15 15 15

Linear model: first differences are constant.

If *x*-values are evenly spaced and second differences of *y*-values are constant, a **quadratic model** fits the data.

x	4	5	6	7	8
y	9	15	23	33	45

First differences: 6 8 10 12
Second differences: 2 2 2

If first differences are not constant, try second differences.

If *x*-values are evenly spaced and ratios of *y*-values are constant, an **exponential model** fits the data.

x	10	11	12	13
y	40	100	250	625

First differences: 60 150 375
Second differences: 90 225

Ratios: $\frac{100}{40} = 2.5$ $\frac{250}{100} = 2.5$ $\frac{625}{250} = 2.5$

If first and second differences are not constant, try ratios of *y*-values.

If *y*-values are evenly spaced and second differences of *x*-values are constant, a **square root model** fits the data.

x	42	45	52	63	78
y	3	4	5	6	7

First differences: 3 7 11 15
Second differences: 4 4 4

For evenly spaced *y*-values, try first differences of *x*-values.

Determine which parent function would best model the data.

1.

x	3	4	5	6	7
y	22.3	26.6	30.9	35.2	39.5

____

2.

x	32	41	56	77	104
y	1	2	3	4	5

____

Name ______________________ Date __________ Class __________

LESSON 9-6

Reteach

Modeling Real-World Data (continued)

After determining a parent function to model a data set, use the regression feature on a graphing calculator to find a function that models the data.

Write a function that models the data.

x	4	5	6	7	8
y	71	93	121	157	204

Make sure the x-values are evenly spaced.

Step 1 Find first differences.
First differences: 22 28 36 47

$204 - 157 = 47$

Step 2 Since first differences are not constant, find second differences.
Second differences: 6 8 11

Step 3 Since second differences are not constant, analyze ratios.
$\frac{93}{71} = 1.310$, $\frac{121}{93} = 1.301$, $\frac{157}{121} = 1.298$, $\frac{204}{157} = 1.299$

Ratios are all close to 1.3.

Step 4 An exponential model best fits the data since the ratios are almost constant. Use a graphing calculator. Perform exponential regression. Select ExpReg from the STAT CALC menu.

ExpReg
$y = a*b^x$
$a = 24.8379125$
$b = 1.301415677$
$r^2 = .999953961$
$r = .9999769803$

An exponential model that fits the data is $f(x) = 24.8(1.3^x)$.

Complete to write a function that models the given data.

x	3	4	5	6	7
y	33	56	86	123	167

3. Are the x-values evenly spaced? ____________

4. Are the first differences constant? ____________

5. Are the second differences constant? ____________

6. What is an appropriate model for the data? ____________

7. Find a function that models the data. ____________

LESSON 9-6

Practice A

Modeling Real-World Data

Determine which parent function would best model the given data set. Choose among linear, quadratic, exponential, and square root.

1. a. Look at the table at right. Are the data for one variable evenly spaced?

Yes, the y-values

b. Look at the data for the other variable. Which differences, if any, are constant?

Second differences

c. Which parent function best models the data?

Square root function

x	y
5	1
8	2
13	3
20	4
29	5
40	6

2.

x	y
2	84
4	72
6	52
8	24
10	−12
12	−56

Quadratic

3.

x	y
8	−26
16	−2
24	22
32	46
40	70

Linear

4.

x	y
1	−2
2	4
3	−8
4	16
5	−32
6	64

Exponential

Write a function that models the given data.

5. Use a graphing calculator to make a scatter plot. Then use the regression feature to find the function that best represents the data.

x	−2	0	2	4	6
y	8	10	8	2	−8

$f(x) = -0.5x^2 + 10$

Solve.

6. The table shows the number of sport utility vehicles sold in the United States from 1997 to 2003. Write a function that models the data.

Years after 1996	1	2	3	4	5	6	7
SUVs (millions)	2.3	2.8	3.1	3.2	3.8	4.0	4.3

$f(x) = 0.33x + 2.06$

LESSON 9-6

Practice B

Modeling Real-World Data

Use constant differences or ratios to determine which parent function would best model the given data set.

1.

x	12	16	20	24	28
y	0.8	3.6	16.2	72.9	328.05

Exponential

2.

x	13	19	25	31	37	43
y	−1	17	35	53	71	89

Linear

3.

x	2	7	12	17	22
y	−100	−55	40	185	380

Quadratic

4.

x	0.10	0.37	0.82	1.45	2.26
y	0.3	0.6	0.9	1.2	1.5

Square root

Write a function that models the data set.

5.

x	2.2	2.6	3.0	3.4	3.8
y	0.68	4.52	9.0	14.12	19.88

$f(x) = 2x^2 - 9$

6.

x	−5	0	5	10	15	20
y	8	6	4	2	0	−2

$f(x) = -0.4x + 6$

7.

x	0.3	0.7	1.1	1.5	1.9
y	2.5	3	3.6	4.32	5.184

$f(x) = 2.18(1.577)^x$

8.

x	0.06	0.375	0.96	1.815	2.94
y	0.2	0.5	0.8	1.1	1.4

$f(x) = 0.816\sqrt{x}$

9.

x	−6	1	8	15	22
y	15	1	30.12	102.36	217.72

$f(x) = 0.44x^2 + 0.2x + 0.36$

10.

x	0.32	2.07	4.8	8.51	13.2
y	0.9	1.6	2.3	3.0	3.7

$f(x) = 1.318x^{0.378}$

Solve.

11. The table shows the population growth of a small town.

Years after 1974	1	6	11	16	21	26	31
Population	662	740	825	908	1003	1095	1200

a. Write a function that models the data. $f(x) = 657.3(1.02)^x$

b. Use your model to predict the population in 2020. 1634 people

LESSON 9-6

Practice C

Modeling Real-World Data

Use constant differences or ratios to determine which parent function would best model the given data set.

1.

x	−0.2	0	0.2	0.4	0.6
y	2.2	1.0	0.2	−0.2	−0.2

Quadratic

2.

x	6	12	18	24	30
y	8000	1200	180	27	4.05

Exponential

Write a function that models the data set.

3.

x	−7	−4	−1	2	5
y	512	64	8	1	0.125

$f(x) = 4(0.5)^x$

4.

x	−6	−3	0	3	6
y	7.1	4.7	2.3	−0.1	−2.5

$f(x) = -0.8x + 2.3$

5.

x	0.75	17	45.75	87	140.75
y	2	4.5	7	9.5	12

$f(x) = 2.045x^{0.336}$

6.

x	1.3	1.35	2.9	4.95	7.5
y	0.8	1.3	1.8	2.3	2.8

$f(x) = 0.88x^{0.597}$

7.

x	0.4	0.7	1.0	1.3	1.6
y	4	249.11	141	79.81	45.17

$f(x) = 940(0.15)^x$

8.

x	−0.6	−0.2	0.2	0.6	1.0
y	0.23	0.69	0.83	0.65	0.15

$f(x) = -x^2 + 0.35x + 0.8$

Solve.

9. The table shows the number of shares of stock listed at the New York Stock Exchange since 1950.

Years since 1949	1	11	21	31	41	51
Shares (in billions)	2.4	6.5	16.1	33.7	90.7	313.9

a. Write a function that models the data. $f(x) = 2.15(1.1)^x$

b. Use your model to predict the number of shares that will be listed in 2010. 720 billion shares

c. Use your model to determine the year in which the number of shares of stock listed first exceeded 10 billion. 1966

LESSON 9-6

Reteach

Modeling Real-World Data

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First differences: 15 15 15 15

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First differences: 6 8 10 12
Second differences: 2 2 2

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If x-values are evenly spaced and ratios of y-values are constant, an **exponential model** fits the data.

x	10	11	12	13
y	40	100	250	625

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Second differences: 90 225
Ratios: $\frac{100}{40} = 2.5$ $\frac{250}{100} = 2.5$ $\frac{625}{250} = 2.5$

If first and second differences are not constant, try ratios of y-values.

If y-values are evenly spaced and second differences of x-values are constant, a **square root model** fits the data.

x	42	45	52	63	78
y	3	4	5	6	7

First differences: 3 7 11 15
Second differences: 4 4 4

For evenly spaced y-values, try first differences of x-values.

Determine which parent function would best model the data.

1.

x	3	4	5	6	7
y	22.3	26.6	30.9	35.2	39.5

Linear model

2.

x	32	41	56	77	104
y	1	2	3	4	5

Square root model

LESSON **9-6**

Reteach

Modeling Real-World Data (continued)

After determining a parent function to model a data set, use the regression feature on a graphing calculator to find a function that models the data.

Write a function that models the data.

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Complete to write a function that models the given data.

x	3	4	5	6	7
y	33	56	86	123	167

3. Are the *x*-values evenly spaced? **Yes**
4. Are the first differences constant? **No**
5. Are the second differences constant? **Yes**
6. What is an appropriate model for the data? **Quadratic model**
7. Find a function that models the data. $f(x) = 3.5x^2 - 1.5x + 6$

LESSON **9-6**

Challenge

Polynomials by Interpolation

Constant differences of the dependent variables can also be used to determine cubic, quartic, and higher degree polynomial functions.

Constant third differences indicate a cubic polynomial. Constant fourth differences indicate a quartic polynomial, and so on.

Once the degree of the polynomial is determined, polynomial interpolation can be used. For a cubic model an equation of the form $f(x) = ax^3 + bx^2 + cx + d$ will be appropriate. Substitute any four points from the data set to identify the constants *a*, *b*, *c*, and *d* by solving the appropriate system of linear equations.

For each data, determine the degree of the polynomial that is the best fit and then find the polynomial by interpolation.

1.

x	−4	−3	−2	−1	0	1	2	3	4
y	21	5	−5	−9	−7	1	15	35	61

Quadratic; $y = 3x^2 + 5x - 7$

2.

x	−4	−3	−2	−1	0	1	2	3	4
y	−795	−284	−61	12	25	20	−9	−116	−403

Quartic; $y = -2x^4 + 3x^3 - 7x^2 + x + 25$

3.

x	−4	−3	−2	−1	0	1	2	3	4
y	−348	−154	−52	−12	−4	2	36	128	308

Cubic; $y = 5x^3 - x^2 + 2x - 4$

4.

x	−4	−3	−2	−1	0	1	2	3	4
y	−1003	−222	−11	20	21	22	53	264	1045

Quintic; $y = x^5 + 21$

5.

x	−4	−3	−2	−1	0	1	2	3	4
y	1268	442	152	98	100	98	152	442	1268

Quartic; $y = 5x^4 - 7x^2 + 100$

LESSON **9-6**

Problem Solving

Modeling Real-World Data

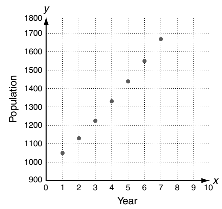
The table shows the population of Lincoln Valley over the last 7 years. The town council is developing long range plans and is considering how the population might grow in the future if the current trend continues.

Lincoln Valley Population 2000–2006							
Year	1	2	3	4	5	6	7
Population	1049	1137	1229	1326	1434	1542	1662

1. What is the independent variable? What is the dependent variable? Assign *x* or *y* to each variable.
The independent variable (x) is the year. The dependent variable (y) is the population.

2. Make a scatter plot of the data. Do the data form a linear pattern? For this to be true, explain what must be true about finite differences.
Possible answer: The first few points appear to be linear, but the later points start a curve upward. For the data to be linear, the first differences must be constant.

3. Use the table of data.



a. Find the first differences.
88, 92, 97, 108, 108, 120

b. Find the second differences.
4, 5, 11, 0, 12

c. Find the third differences.
1, 6, −11, 12

d. Find the ratios between *y*-values.
All ratios round to 1.08.

4. What kind of function will best describe the data? Justify your conclusion.
Exponential function, because the ratios between *y*-values are almost constant

Choose the letter for the best answer.

5. Which function best models the given data?
A $y = 101.9x + 932.1$
B $y = 3.1x^2 + 77.0x + 969.6$
C $y = 996.6x^{0.233}$
(D) $y = 974.9(1.08)^x$

6. Predict the population of Lincoln Valley in 2012.
F 2270
G 2450
(H) 2650
J 2860

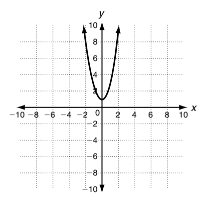
LESSON **9-6**

Reading Strategies

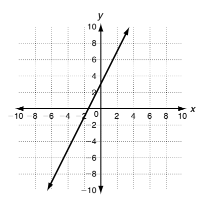
Draw Conclusions

Sometimes there is a pattern in real-world data that describes the relationship. Often we can use the pattern to draw conclusions about the function.

Function	Linear	Quadratic	Exponential	Square Root
Constant Differences/ Ratios	Constant first differences between *y*-values (*x*-values evenly spaced).	Constant second differences between *y*-values (*x*-values evenly spaced).	Constant ratios between *y*-values (*x*-values evenly spaced).	Constant second differences between *x*-values (*y*-values evenly spaced).



Graph A



Graph B

Use the graphs above for Exercises 1–2.

1. a. What type of function is represented by Graph A? **Quadratic function**
b. What conclusions can you draw about the data set for the function represented by Graph A?
The data set has constant second differences between *y*-values for evenly spaced *x*-values.

2. a. What type of function is represented by Graph B? **Linear function**
b. What conclusions can you draw about the data set for the function represented by Graph B?
The data set has constant first differences between *y*-values for evenly spaced *x*-values.

3. On the coordinate plane at right, sketch the graph of a function that has constant ratios between *y*-values with evenly spaced *x*-values.

4. On the coordinate plane at right, sketch the graph of a function that has constant first differences and includes the points $(-4, 6)$ and $(0, 0)$.