Name _____ Date _____ Class _____

LESSON 9-1

Challenge

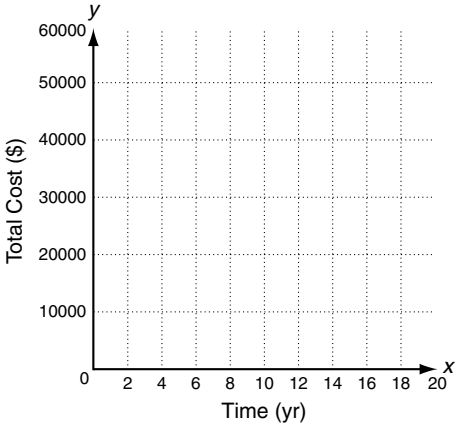
Find a Function to Fit

Some real-world problems can be modeled using functions. Consider a home owner in Maine faced with deciding which type of heating system to install in a new home. A solar system will cost $20,000 to install, but the annual heating costs will be $1000. A geothermal heat pump will cost $15,000 to install with annual heating costs of $1500. A conventional oil-fired furnace will cost $6000 to install with annual heating costs of $2400.

1. Write a function to represent the total cost of each system over t years.

_____

2. Graph each function over 20 years.



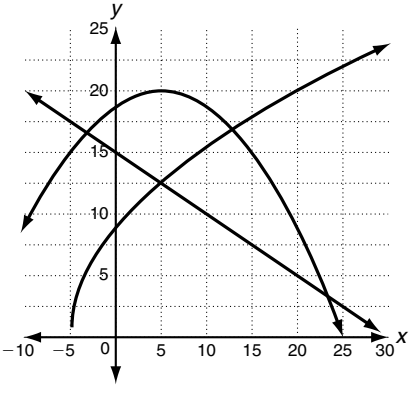
3. If the home owner plans to move within 7 years, which heating system is most economical?

_____

4. Which system is better for a home owner who is planning to live in the house for 30 or more years? How do you know?

_____

Consider the graph below. Three functions are shown: a linear function, $f(x)$, a quadratic function, $g(x)$, and a square root function, $h(x)$.



5. Identify three points on each graph.

a. $f(x)$ _____

b. $g(x)$ _____

c. $h(x)$ _____

6. Find an equation that models $f(t)$.

_____

7. Find an equation that models $g(t)$. Remember, every quadratic function is of the form $y = ax^2 + bx + c$. Set up a system of equations and solve for a, b, and c.

_____

8. Find an equation that models $h(t)$. A square root function can be modeled with $y = a\sqrt{x - h} + k$.

_____

LESSON 9-1

Reteach

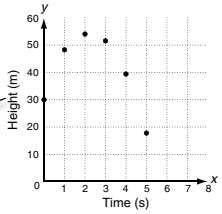
Multiple Representations of Functions (continued)

Different representations can be used to solve problems about nonlinear functions.

The table shows the height of a rocket after it is fired. When does the rocket reach its maximum height?

Time (s)	Height (m)
0	30.0
1	47.1
2	54.4
3	51.9
4	39.6
5	17.5

Step 1 Make a scatter plot.



The graph suggests the data are not linear.

Step 2 Check finite differences.

First differences: 17.1 7.3 −2.5 −12.3 −22.1

Second differences: −9.8 −9.8 −9.8 −9.8

Step 3 Second differences are constant. Use a graphing calculator to find the quadratic regression equation: $y = -4.9x^2 + 22x + 30$.

$y = ax^2 + bx + c$
$a = -4.9$
$b = 22$
$c = 30$
$r^2 = 1$

Step 4 Use TRACE to find the maximum height.

The rocket has a maximum height of about 54.7 m at about 2.2s.

The table shows the height of a rocket after it is fired. Solve.

Time (s)	Height (m)
0	45
1	58.1
2	61.4
3	54.9
4	38.6
5	12.5

4. Are the first differences constant? No

5. Are the second differences constant? Yes

6. Find an equation to model the data.

$y = -4.9x^2 + 18x + 45$

7. When does the rocket reach its maximum height?

About 61.5 m at about 1.9 s

LESSON 9-1

Challenge

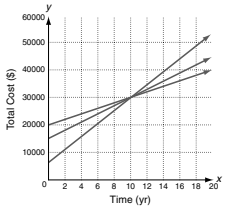
Find a Function to Fit

Some real-world problems can be modeled using functions. Consider a home owner in Maine faced with deciding which type of heating system to install in a new home. A solar system will cost $20,000 to install, but the annual heating costs will be $1000. A geothermal heat pump will cost $15,000 to install with annual heating costs of $1500. A conventional oil-fired furnace will cost $6000 to install with annual heating costs of $2400.

1. Write a function to represent the total cost of each system over t years.

$f(t) = 20{,}000 + 1000t$, $g(t) = 15{,}000 + 1500t$, $h(t) = 6000 + 2400t$

2. Graph each function over 20 years.



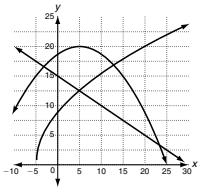
3. If the home owner plans to move within 7 years, which heating system is most economical?

Oil-fired furnace

4. Which system is better for a home owner who is planning to live in the house for 30 or more years? How do you know?

Solar system; after 10 years, this system is least expensive.

Consider the graph below. Three functions are shown: a linear function, $f(x)$, a quadratic function, $g(x)$, and a square root function, $h(x)$.



5. Identify three points on each graph.

a. $f(x)$ $f(x)$: (0, 15), (10, 10), (20, 5)

b. $g(x)$ $g(x)$: (5, 20), (15, 15), (25, 0)

c. $h(x)$ $h(x)$: (−5, 0), (4, 12), (20, 20)

6. Find an equation that models $f(t)$.

$f(x) = -0.5x + 15$

7. Find an equation that models $g(t)$. Remember, every quadratic function is of the form $y = ax^2 + bx + c$. Set up a system of equations and solve for a, b, and c.

$g(x) = -0.05(x - 5)^2 + 20 = -0.05x^2 + 0.5x + 18.75$

8. Find an equation that models $h(t)$. A square root function can be modeled with $y = a\sqrt{x - h} + k$.

$h(x) = 4\sqrt{x + 5}$

LESSON 9-1

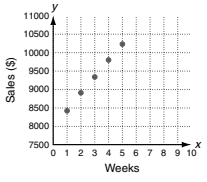
Problem Solving

Multiple Representations of Functions

Yvonne opened a new video game store. The table shows a record of her sales for the first five weeks. To break even, she needs to sell at least $12,500 worth of merchandise each week. She assumes the sales trend will continue and wants to know what to expect over the next weeks.

Weekly Store Sales	
Week	Sales ($)
1	8470
2	8920
3	9360
4	9790
5	10,210

1. Graph the data using weeks as the independent variable and sales as the dependent variable.



2. Yvonne thinks she can model her sales data using a quadratic function. Is she correct? How do you know?

Yes, because second differences are constant. For a quadratic function, second differences are constant.

3. Use a graphing calculator to perform the appropriate regression on the data. Write the equation that models the data.

$y = -5x^2 + 465x + 8010$

4. What sales can Yvonne expect in week 6?

$10,620

5. When will her sales exceed $11,000 per week?

Week 7

6. When will she break even?

Week 11

7. When will sales be twice the sales of week 1?

Week 28

Choose the letter for the best answer.

8. Which equation represents a steady increase of $420 per week in sales from week 5 on?

A $y = -420x + 10{,}210$

(B) $y = 420x + 10{,}210$

C $y = -5x^2 + 420x + 10{,}210$

D $y = -420x^2 + 10{,}210x$

9. During which week will Yvonne break even if the sales pattern changes and sales in week 6 and week 7 are $10,640 and $11,080, respectively?

F Week 9

G Week 10

(H) Week 11

J Week 12

LESSON 9-1

Reading Strategies

Identify Relationships

You can represent functions in several ways. You can use an equation, a table, or a graph. Each representation gives the same information.

Equation

$y = 2x$

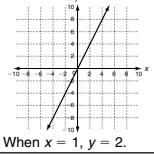
When $x = 1$, $y = 2$.

Table

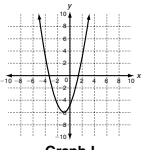
x	y
1	2
4	8
7	14
10	20

When $x = 1$, $y = 2$.

Graph



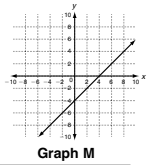
When $x = 1$, $y = 2$.



Graph L

$y = \frac{x}{3}$

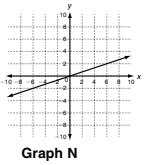
Equation A



Graph M

$y = x^2 + 2x - 5$

Equation B



Graph N

$y = x - 4$

Equation C

x	−4	−1	0	2
y	3	−6	−5	3

Table R

x	6	9	12	15
y	2	3	4	5

Table S

x	−5	−1	2	10
y	−9	−5	−2	6

Table T

Identify the relationships between the graphs, tables, and equations shown above. Explain your reasoning.

1. Which table and graph have a relationship with Equation A?

Table S and Graph N; possible answer: All the x-values are divided by 3 to get the y-values. The graph is linear and passes through the origin.

2. Which graph and equation have a relationship with Table R?

Graph L and Equation B; possible answer: The y-values alternate positive, negative, positive, so it is not linear.

3. Which equation and table have a relationship with Graph M?

Equation C and Table T; possible answer: The graph is linear and passes through the point (0, −4). In Table T the y-values are always four less than the x-values.