Name ______________________________________ Date ___________ Class ____________

SECTION 9A

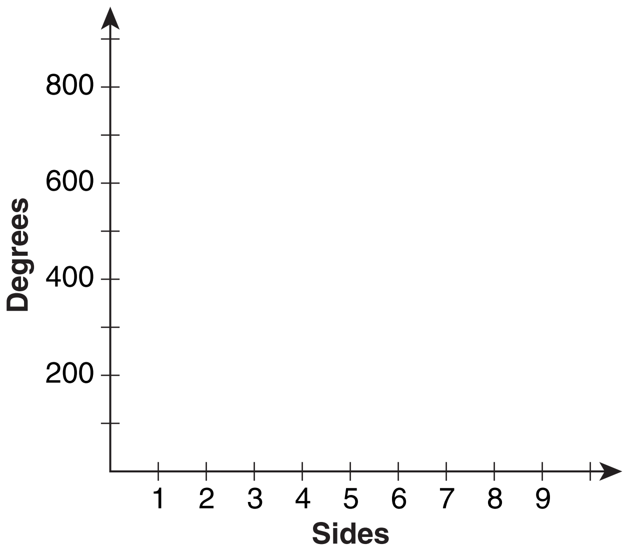
Ready To Go On? Skills Intervention

9-1 Multiple Representations of Functions

Using Multiple Representations to Solve Problems

The table shows the sum of the interior angles of polygons and the number of sides of the polygons. Use a graph and an equation to find the sum of the interior angles of a 24-gon.

Step 1 Graph the data.



Number of sides	Sum of the angles (degrees)
3	180
4	360
5	540
6	720
7	900

The data appears to be __________.

Step 2 Write an equation to represent the data. Let y = the sum of the interior angles and x = the number of sides of the polygon.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{360 - \square}{\square - 3} =$ ______ Find the slope using any two points.

$y - y_1 = m(x - x_1)$ Write point-slope form.

$y -$ ______ $=$ ______ $(x -$ ______$)$ Substitute values into point-slope form.

$y =$ ______$x -$ ______ Simplify.

Step 3 Evaluate the function for a polygon with 24 sides.

$y = 180($______$) - 360$ Substitute $x = 24$.

$y =$ ______ Simplify.

The sum of the interior angles of a polygon with 24 sides is ______°.

Name ______________________ Date __________ Class __________

SECTION 9A

Ready To Go On? Problem Solving Intervention

9-1 Multiple Representations of Functions

Often functions can be represented in a variety of ways. Use the method that is easiest for you.

A hot air balloon is descending from an altitude of 1000 feet at a rate of 8 feet per second. Create a table, equation, and graph to represent the hot air balloon's altitude, *a,* with relation to time, *t*. When will it reach the ground?

Understand the Problem

1. Describe the hot air balloon's descent. ______________________

____

Make a Plan

2. What do you need to determine? ______________________

____

Solve

3. Create a table. Let *t* equal time and *a* equal altitude.

***t* (seconds)**	0	1	2	3	4
***a* (feet)**	1000	______	______	______	______

First differences: −8 ______ ______ ______

The first differences are __________, so a __________ model is appropriate.

4. Write an equation to model: "Altitude is equal to 1000 minus ______ feet per second."

$a =$ ______ − ______

5. Graph the equation.

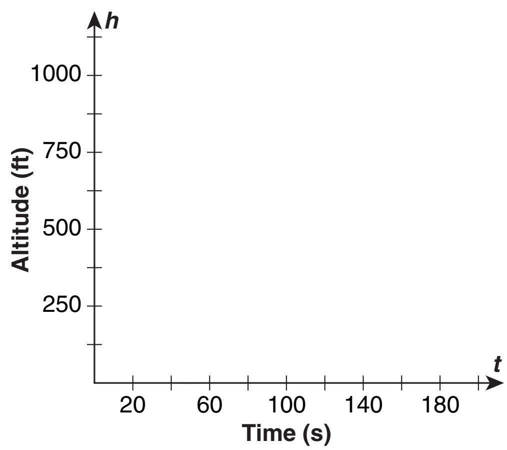
a-intercept: __________

Solve the equation for *t* when $a = 0$:

$0 = 1000 -$ ______

$t =$ ______

t-intercept: __________



6. The hot air balloon will reach the ground after ______ seconds.

Look Back

7. Check the intercepts by graphing the equation on your graphing calculator.

Name ______________________ Date ________ Class ________

SECTION 9A

Ready To Go On? Skills Intervention

9-2 Piecewise Functions

Find these vocabulary words in Lesson 9-2 and the Multilingual Glossary.

Vocabulary	
piecewise function	step function

Evaluating a Piecewise Function

Evaluate each piecewise function at $x = -2$ and $x = 6$.

A. $f(x) = \begin{cases} 4x - 2 & \text{if } x < 6 \\ x^3 + 1 & \text{if } x \geq 6 \end{cases}$

Evaluate the function at $x = -2$.

Because -2 ____ 6, use the rule for x ____ 6.

$f(-2) = 4(-2) -$ ____ $=$ ____

Evaluate the function at $x = 6$.

Because 6 ____ 6, use the rule for x ____ 6.

$f(6) = 6^3 +$ ____ $=$ ____

B. $f(x) = \begin{cases} 7 & \text{if } x \leq -2 \\ x^2 + 3x - 28 & \text{if } -2 < x \leq 6 \\ \sqrt{x + 25} & \text{if } x > 6 \end{cases}$

Evaluate the function at $x = -2$.

Because -2 ____ -2, use the rule for x _______.

$f(-2) =$ ____

Evaluate the function at $x = 6$.

Because 6 ____ 6, use the rule for ______________.

$f(6) =$ ____ $+ 3(6) -$ ____ $=$ ____

Graphing Piecewise Functions

Graph the piecewise function. $f(x) = \begin{cases} -1 & \text{if } x < 3 \\ 2x - 4 & \text{if } x \geq 3 \end{cases}$

Because the function is divided at $x =$ ____,

evaluate both branches of the function at $x =$ ____.

Plot the point $(3, -1)$ with a/an __________ circle

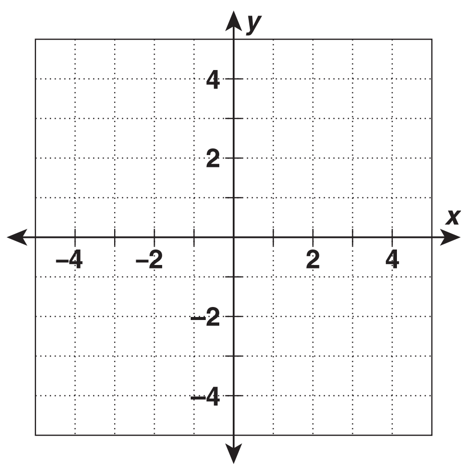
and draw a horizontal ray to the __________.

Substitute $x = 3$ into the function $f(x) = 2x - 4$.

$2(3) - 4 =$ ____

Plot the point (3, ____) with a/an __________ circle

and draw a ray to the __________ with a slope of ____.



Name ______________________________ Date __________ Class __________

SECTION **9A**

Ready To Go On? Problem Solving Intervention

9-2 Piecewise Functions

A piecewise function is a combination of one or more functions. If the function is constant for each interval in its domain, the function is called a step function.

Maureen drove from her house in the suburbs to her office in the city. She drove 10 minutes through town at an average speed of 30 mi/h, 30 minutes on the highway at an average speed of 65 mi/h, and 6 minutes in city traffic at an average speed of 15 mi/h. Write a piecewise function to represent Maureen's distance versus time. Then graph the function.

Understand the Problem

1. Maureen's trip can be described by how many steps? ______

2. What do you need to determine? ______________________________

____

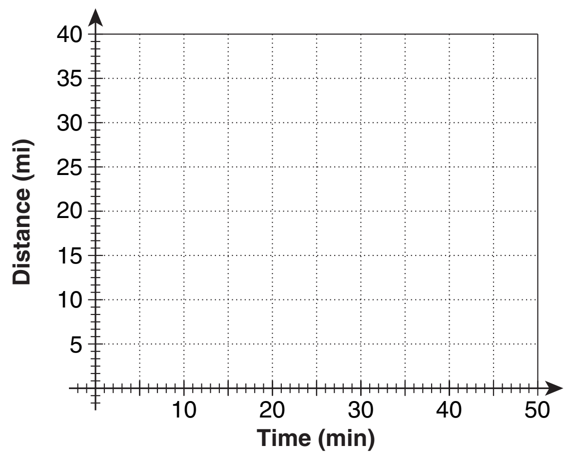
Make a Plan

3. Make a table to organize the data. Use the distance formula to find Maureen's rate for each leg of her drive.

	Rate (mi/h)	Time (h)	Distance (mi)
Through Town	30	______	______
On the Highway	______	$\frac{1}{2}$	______
In City Traffic	______	$\frac{1}{10}$	______

Solve

4. Graph the function.



5. Write a linear function for each leg.

Town: $d = 0.5$______

Highway: $d =$ ______$t - \frac{35}{6}$

City: $d =$ ______$t +$ ______

The function rule is:

$$d(t) = \begin{cases} \_\_\_\_ t & \text{if } 0 \le t < 10 \\ \frac{13}{12}t - \_\_\_\_ & \text{if } 10 < t \le 40 \\ \frac{1}{4}t + \_\_\_\_ & \text{if } 40 < t \le 46 \end{cases}$$

____

Look Back

6. Use your graph to find out how far Maureen has traveled after 5 min: __________

20 min: __________ 45 min: __________ Does your function make sense? __________

Name ______________________ Date __________ Class __________

SECTION 9A

Ready To Go On? Skills Intervention

9-3 Transforming Functions

Transforming Piecewise Functions

Given $f(x) = \begin{cases} 6x - 8 & \text{if } x \le 2 \\ x^2 & \text{if } x > 2 \end{cases}$, write the rule for $h(x)$, a horizontal shift of $f(x)$ 3 units left.

Replace every x in the function with $x +$ ______.

$$h(x) = f(x+3) = \begin{cases} 6(\_\_\_\_\_\_ + \_\_\_\_\_\_) - 8 & \text{if } (x+3) \le 2 \\ (\_\_\_\_\_\_ + 3)^2 & \text{if } (x+3) > 2 \end{cases}$$

$$h(x) = \begin{cases} \_\_\_\_\_\_ + 10 & \text{if } x \le \_\_\_\_\_\_ \\ x^2 + \_\_\_\_\_\_ + 9 & \text{if } x > \_\_\_\_\_\_ \end{cases}$$ Simplify.

Identifying Intercepts

Identify the x- and y-intercepts of $f(x)$. Without graphing $g(x)$, identify its x- and y-intercepts.

A. $f(x) = \frac{1}{4}x + 5$ and $g(x) = 2f(x)$

$f(0) = \frac{1}{4}(\_\_\_\_\_\_) + 5 = \_\_\_\_\_\_$ Find the y-intercept of the original function.

$\_\_\_\_\_\_ = \frac{1}{4}x + 5$ Find the x-intercept of the original function.

$-5 = \frac{1}{4}x$

$\_\_\_\_\_\_ = x$

The y-intercept of $f(x)$ is ______ and the x-intercept is ______.

$g(x)$ is a __________________ of $f(x)$ by a factor of ______.

The y-intercept of $g(x)$ is ______ and the x-intercept is ______.

Check your answer: Graph $f(x)$ and $g(x)$ to verify the intercepts.

B. $f(x) = x^2 - 8$ and $g(x) = -f(5x)$

$f(0) = \_\_\_\_\_\_^2 - 8 = \_\_\_\_\_\_$ Find the y-intercept of the original function.

$\_\_\_\_\_\_ = x^2 - 8$ Find the x-intercept of the original function.

$x^2 = 8$

$x = \sqrt{8}$

The y-intercept of $f(x)$ is ______ and the x-intercept(s) is/are _______.

$g(x)$ is a ________________________ of $f(x)$ by a factor of ______, and a reflection of $f(x)$ across the ______ axis.

The y-intercept of $g(x)$ is ______ and the x-intercept(s) is/are ______.

Check your answer: Graph $f(x)$ and $g(x)$ to verify their intercepts.

Name ______________________ Date __________ Class __________

SECTION 9A

Ready To Go On? Quiz

9-1 Multiple Representations of Functions

1. A plane is descending from an altitude of 32,000 feet at a rate of 25 feet per second. Create a table and an equation to represent the plane's altitude, *a,* with relation to time, *t*.

t **(s)**	0	1	2	3	4
a **(ft)**	______	______	______	______	______

____

2. The population of a certain bacteria at different times is shown in the table.

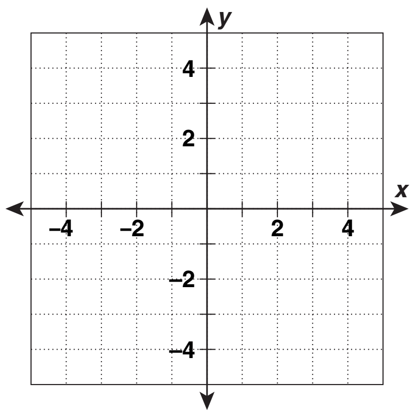
Time (hours)	0	1	2	3	4
Number of bacteria	200	600	1800	5400	16,200

a. Find an appropriate model for the population of bacteria. ______________________

b. Assuming the growth in population continues, when will the bacteria's population equal 1,312,200? ______________________

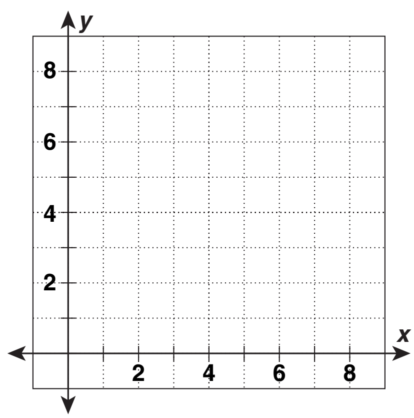
9-2 Piecewise Functions

3. Graph the function $f(x) = \begin{cases} 3x - 1 & \text{if } x \leq 1 \\ 1 & \text{if } x > 1 \end{cases}$.



4. The cost to park in a parking garage in Boston is $5 for the first 4 hours and $2 for each additional hour. Sketch a graph of the cost of parking in the garage for 0 to 6 hours, and write a piecewise function for the graph.

____

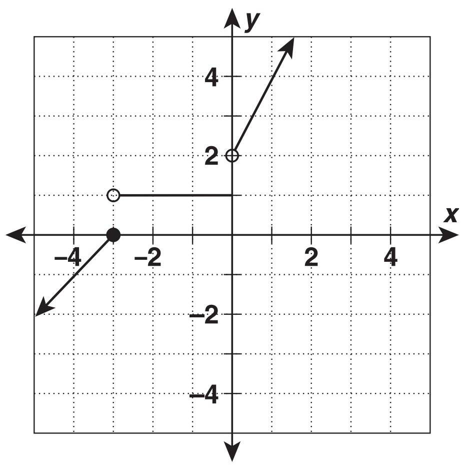


Name ______________________ Date __________ Class __________

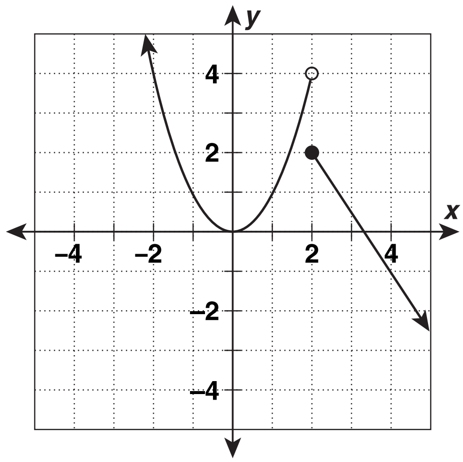
SECTION 9A Ready To Go On? Quiz continued

Write a piecewise function for each graph.

5. ______________________



6. ______________________



9-3 Transforming Functions

Identify the *x*- and *y*-intercepts of $f(x)$. Without graphing $g(x)$, identify its *x*- and *y*-intercepts.

7. $f(x) = -x + 4$ and $g(x) = -2f(x)$

x-intercept(s) of $f(x)$ __________

y-intercepts(s) of $f(x)$ __________

x-intercept(s) of $g(x)$ __________

y-intercepts(s) of $g(x)$ __________

8. $f(x) = x^2 - 1$ and $g(x) = f(-\frac{1}{4}x)$

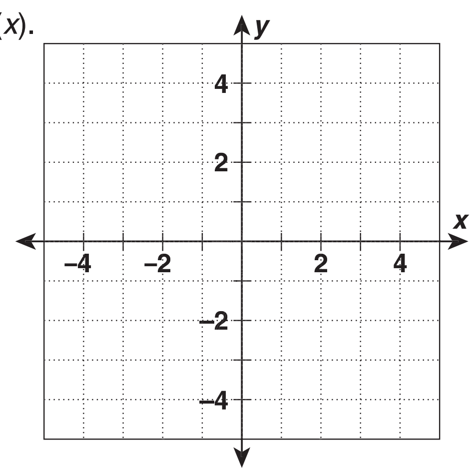
x-intercept(s) of $f(x)$ __________

y-intercepts(s) of $f(x)$ __________

x-intercept(s) of $g(x)$ __________

y-intercepts(s) of $g(x)$ __________

9. Given $f(x) = |2x|$ and $g(x) = -f(x) - 1$, graph $g(x)$.



Name ______________________________ Date __________ Class __________

SECTION 9A

Ready To Go On? Enrichment

Exploring Step Functions

A step function is a special type of piecewise function whose graph resembles steps, not lines or curves. One common step function is the *greatest integer function*, or rounding-down function. Given any number *x*, the greatest integer function is defined as:

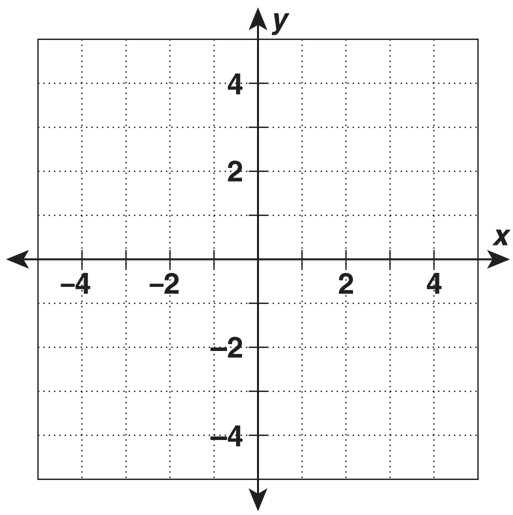
$f(x)$ = the greatest integer less than or equal to x.

Find *f*(x) for the following values of *x*.

1. 4 ____________
2. $8.\overline{7}$ ____________
3. $\sqrt{3}$ ____________
4. 2.25 ____________
5. The domain of the greatest integer function is the set of Real numbers. What is the range of the greatest integer function?

____

6. Graph the greatest integer function for all values of x such that $-5 \leq x < 5$.



7. The cost of a telephone call between Boston and Paris is \$0.90 for the first minute and \$0.42 for each additional minute or portion of a minute.

   a. Use the greatest integer function to model the cost of a call, *C*.

   ____

   b. How much does an 8 minute and 25 second phone call cost?

   ____

Name ________________________________ Date ___________ Class ___________

SECTION 9B

Ready To Go On? Skills Intervention

9-4 Operations with Functions

Find this vocabulary word in Lesson 9-4 and the Multilingual Glossary.

Vocabulary
composition of functions

Evaluating Functions

Given $f(x) = 6x - 5$, $g(x) = x^2 + 6x - 27$, and $h(x) = \frac{12}{x + 9}$, find each function or value.

A. $(g - f)(x)$

$(g - f)(x) = \_\_\_\_ + 6x - 27 - (\_\_\_\_ - 5)$ — Substitute function rules.

$= \_\_\_\_ - 22$ — Combine like terms.

B. $(fh)(-1)$

$(fh)(x) = (\_\_\_\_ - 5) \cdot \frac{\square}{x + 9}$ — Substitute function rules.

$(fh)(-1) = (6(\square) - 5) \cdot \frac{\square}{(\square) + 9}$ — Substitute -1 for x.

$= -\frac{33}{\square}$ — Simplify.

C. $\frac{h}{g}(x)$

$\frac{h}{g}(x) = \frac{h(x)}{\square(x)} = \frac{\frac{12}{x + 9}}{\square}$ — Substitute function rules.

$= \frac{\frac{12}{\square}}{(x + \square)(x - \square)}$ — Factor completely. Note that $x \neq -9$ or 3.

$= \frac{\frac{12}{x + 9}}{(x + \square)(x - \square)} \cdot \frac{x + 9}{x + \square}$ — Simplify the fraction.

$= \frac{\square}{(x + \square)^2 (x - \square)}$ where $x \neq -9$ or ____. — Divide out common factors and simplify.

D. $g(h(-3))$

$h(-3) = \frac{12}{\square + 9} = \_\_\_\_$ — Find $h(-3)$.

$g(2) = \_\_\_\_^2 + 6(\_\_\_\_) - 27 = \_\_\_\_$ — Substitute your result for $h(-3)$ into g.

Name ______________________________ Date __________ Class ___________

SECTION 9B

Ready To Go On? Problem Solving Intervention

9-4 Operations with Functions

You can multiply functions by following this rule: $(fg)(x) = f(x) \cdot g(x)$.

During a sale, a shoe store is selling boots for 25% off. Preferred customers also receive an additional 10% off. Write a composite function to represent the final cost of a pair of boots that originally cost *c* dollars. Then find the cost of a pair of boots originally priced at $92 that a preferred customer wants to buy.

Understand the Problem

1. Describe the shoe store's sale.

____

____

Make a Plan

2. What do you need to determine?

____

____

3. Write a function *P* for the final price of boots after the sale.

$P(c) =$ __________ c

4. Write a function *D* for the price after applying the preferred customer discount.

$D(c) =$ __________ c

Solve

5. Find the composition $P(D(c))$.

$P(D(c)) = P($__________ $c)$ Substitute the rule of *D* into *P*.

$=$ __________ $(0.90c)$ Apply the rule for *P*.

$=$ __________ c Simplify.

6. Find the cost of the boots that originally cost $92. ____________________

Look Back

7. To check your answer, find 25% of $92: Sale savings = 25%(92) = ______________

Then subtract the savings from $92 and take 10% of that value.

Preferred customer discount = 10%(92 − savings) = ______________

Cost = 92 − sale savings − preferred customer discount = ______________

8. Does your solution in Exercise 6 make sense? ______________

Name ______________________________ Date __________ Class __________

SECTION 9B

Ready To Go On? Skills Intervention

9-5 Functions and Their Inverses

Find these vocabulary words in Lesson 9-5 and the Multilingual Glossary.

Vocabulary

one-to-one function   horizontal-line test

Using the Horizontal-Line Test and Writing Rules for Inverses

Find the inverse of each function. Then state the domain and range of the inverse.

A. $f(x) = 7 - \sqrt{x + 4}$

Step 1 Graph the function on your graphing calculator.

Does it pass the horizontal-line test? __________ Therefore, the inverse __________ a function.

Step 2 Find the inverse.

$\_\_\_\_ = 7 - \sqrt{x + 4}$ Rewrite the function using y instead of $f(x)$.

$\_\_\_\_ = 7 - \sqrt{\_\_\_\_ + 4}$ Interchange x and y. Note the range restriction $y \geq \_\_\_\_$.

$\_\_\_\_\_\_ = -\sqrt{\_\_\_\_ + 4}$ Isolate the radical.

$\_\_\_\_\_\_^2 = y + \_\_\_\_$ Square both sides of the equation.

$\_\_\_\_ - 14x + \_\_\_\_ = y + 4$ Multiply.

$y = x^2 - 14x + \_\_\_\_$ Solve for y.

The domain of the inverse is the __________ of $f(x)$: __________.

The range of the inverse is the __________ of $f(x)$: __________.

B. $f(x) = \dfrac{8 - 6x}{5}$

Step 1 Graph the function on your graphing calculator.

Does it pass the horizontal-line test? __________ Therefore, the inverse __________ a function.

Step 2 Find the inverse.

$\_\_\_\_ = \dfrac{8 - 6x}{5}$ Rewrite the function using y instead of $f(x)$.

$\_\_\_\_ = \dfrac{8 - 6\boxed{}}{5}$ Interchange x and y.

$y = \_\_\_\_\_\_\_\_$ Solve for y.

The domain of the inverse is: $\{x \mid x \in \_\_\_\_\}$ and its range is: $\{y \mid y \in \_\_\_\_\}$.

Name ______________________________ Date __________ Class __________

SECTION 9B

Ready To Go On? Skills Intervention

9-6 Modeling Real-World Data

Identifying Models by Using Constant Ratios or Differences
Use finite differences or ratios to determine which parent function best models each set of data.

A.

x	1	2	3	4	5
y	11	14	9	−4	−25

Are the *x*-values evenly spaced? ______________

Check the first differences between *y*-values: ________, −5, ________, ________

Check the second differences between *y*-values: ________, ________, −8

Check the ratios between *y*-values: ______________

Because the ______________ differences are constant,

a/an ______________ model best models the data set.

B.

x	−3	0	3	6	9
y	56	7	0.875	0.109	0.014

Are the *x*-values evenly spaced? ______________

Check the first differences between *y*-values: ________, −6.125, ________, ________

Check the second differences between *y*-values: ________, 5.359, ________

Check the ratios between *y*-values: ________, 0.125, ________, ________

Because the ______________ are constant, a/an ______________ model best models the data set.

C.

x	6	7	8	9	10
y	42.3	44.1	45.9	47.7	49.5

Are the *x*-values evenly spaced? ______________

Check the first differences between *y*-values: 1.8, ______, ______, ______

Check the second differences between *y*-values: ______________

Because the ______________ are constant, a/an ______________ model best models the data set.

Name ______________________________ Date __________ Class __________

SECTION **9B**

Ready To Go On? Problem Solving Intervention

9-6 Modeling Real-World Data

To model data in a real-world application, look for a pattern.

The table shows the estimated population of a Southwestern town. Using time as the independent variable and 1980 as a reference year, find a model that best fits the data. Use your model to predict the town's population in the year 2030.

Year	1980	1985	1990	1995	2000	2005
Population	1840	2208	2650	3180	3815	4617

Understand the Problem

1. Describe the town's population. ______________________________

____

Make a Plan

2. What do you need to determine? ______________________________

____

Solve

3. Are the years evenly spaced? ____________

Check the first differences: ______________________

Check the second differences: ______________________

Check the ratios between population values: ______________________

Because the ____________ are constant, a/an ____________ model is best.

4. Use your graphing calculator to perform a/an ____________ regression.

5. A function that models the data is:

$$f(t) = 1837.17(\_\_\_\_\_\_)^t$$

6. Solve for the town's population in the year 2030. Let $t =$ ______.

$$f(t) = 1837.17(\_\_\_\_\_\_)^{50} \approx \_\_\_\_\_\_\_\_$$

Look Back

7. To check your model, let $t = 0, 5, 10, 15, 20,$ and 25 and see if your function values using the function in Exercise 5 match the population figures in the table.

____

8. Does your model seem reasonable? ____________

Name ______________________ Date __________ Class __________

SECTION 9B

Ready To Go On? Quiz

9-4 Operations with Functions

Given $f(x) = x + 1$, $g(x) = x^2 - 11x + 18$, and $h(x) = \frac{10}{x}$, find each function or value.

1. $(g - f)(4)$

____

2. $\left(\frac{h}{g}\right)(5)$

____

3. $(fg)(x)$

____

4. $h(f(x))$

____

5. a. Find $(g \circ f)(x)$. ______________

b. What is the domain of the composite function? ______________

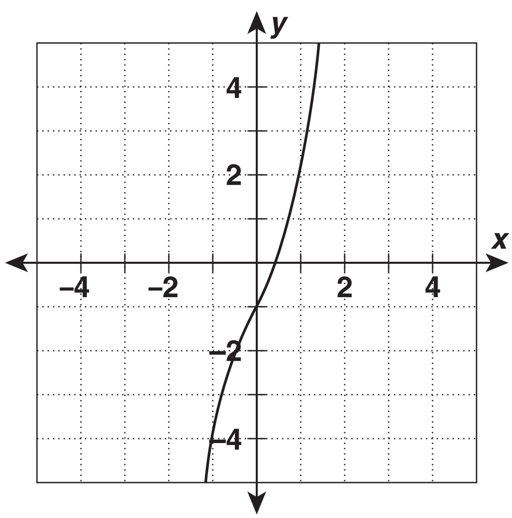
6. Mike imports lemon juice from Sicily. The cost of a case of lemon juice includes a 15% import tax and 50 euros for shipping. Given 1 dollar = 0.83 euros, write a composite function for the total cost of a case of lemon juice in dollars, if the cost of a case is c euros,.

____

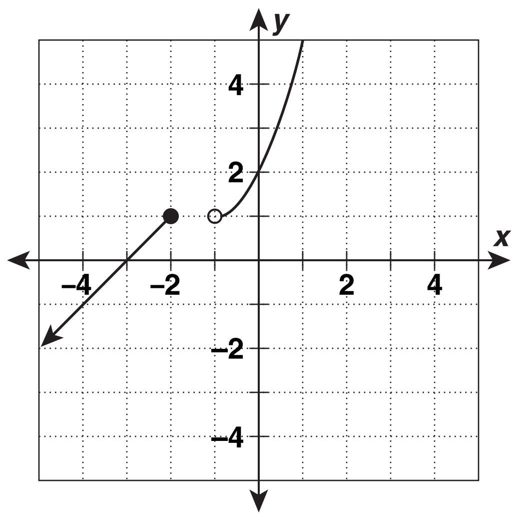
9-5 Functions and Their Inverses

State whether the inverse of each relation is a function.

7. ______________________



8. ______________________



Name ________________________ Date __________ Class __________

SECTION 9B

Ready To Go On? Quiz continued

Write the rule for the inverse of each function. Then state the domain and range of the inverse.

9. $f(x) = (8x + 4)^2$

____

domain: ______________________

range: ______________________

10. $g(x) = \dfrac{9}{x - 7}$

____

domain: ______________________

range: ______________________

9-6 Modeling Real-World Data

11. Use finite differences or ratios to determine which parent function would best model this set of data.

t	0	2	4	6	8
a	−5	5	31	73	131

____

12. The table shows the years that a park ranger has been monitoring the giraffe population in a game park in Namibia and the size of the giraffe population. Using time as the independent variable, find a model for the size of the giraffe population.

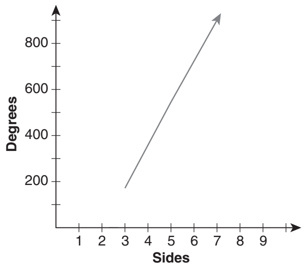
Time (year)	Giraffe
5	72
6	83
7	96
8	110
9	127

____

SECTION 9A Ready To Go On? Skills Intervention
9-1 Multiple Representations of Functions

Using Multiple Representations to Solve Problems
The table shows the sum of the interior angles of polygons and the number of sides of the polygons. Use a graph and an equation to find the sum of the interior angles of a 24-gon.

Step 1 Graph the data.



Number of sides	Sum of the angles (degrees)
3	180
4	360
5	540
6	720
7	900

The data appears to be linear.

Step 2 Write an equation to represent the data. Let y = the sum of the interior angles and x = the number of sides of the polygon.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{360 - 180}{4 - 3} = 180$ Find the slope using any two points.

$y - y_1 = m(x - x_1)$ Write point-slope form.

$y - 180 = 180(x - 3)$ Substitute values into point-slope form.

$y = 180x - 360$ Simplify.

Step 3 Evaluate the function for a polygon with 24 sides.

$y = 180(24) - 360$ Substitute $x = 24$.

$y = 3960$ Simplify.

The sum of the interior angles of a polygon with 24 sides is 3960°.

SECTION 9A Ready To Go On? Problem Solving Intervention
9-1 Multiple Representations of Functions

Often functions can be represented in a variety of ways. Use the method that is easiest for you.

A hot air balloon is descending from an altitude of 1000 feet at a rate of 8 feet per second. Create a table, equation, and graph to represent the hot air balloon's altitude, *a*, with relation to time, *t*. When will it reach the ground?

Understand the Problem

1. Describe the hot air balloon's descent. The balloon begins at an altitude of 1000 ft, and its altitude decreases by 8 feet per second.

Make a Plan

2. What do you need to determine? How to represent the situation using a table, an equation, and a graph, and the time it will take for the balloon to reach the ground

Solve

3. Create a table. Let *t* equal time and *a* equal altitude.

t (seconds)	0	1	2	3	4
a (feet)	1000	992	984	976	968

First differences: −8 −8 −8 −8

The first differences are constant, so a linear model is appropriate.

4. Write an equation to model: "Altitude is equal to 1000 minus 8 feet per second."

$a = 1000 - 8t$

5. Graph the equation.

a-intercept: (0, 1000)

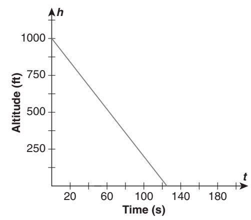
Solve the equation for *t* when $a = 0$:

$0 = 1000 - 8t$

$t = 125$

t-intercept: (125, 0)

6. The hot air balloon will reach the ground after 125 seconds.



Look Back

7. Check the intercepts by graphing the equation on your graphing calculator.

SECTION 9A Ready To Go On? Skills Intervention
9-2 Piecewise Functions

Find these vocabulary words in Lesson 9-2 and the Multilingual Glossary.

Vocabulary
piecewise function step function

Evaluating a Piecewise Function
Evaluate each piecewise function at $x = -2$ and $x = 6$.

A. $f(x) = \begin{cases} 4x - 2 & \text{if } x < 6 \\ x^3 + 1 & \text{if } x \geq 6 \end{cases}$

Evaluate the function at $x = -2$.

Because $-2 < 6$, use the rule for $x < 6$.

$f(-2) = 4(-2) - 2 = -10$

Evaluate the function at $x = 6$.

Because $6 \geq 6$, use the rule for $x \geq 6$.

$f(6) = 6^3 + 1 = 217$

B. $f(x) = \begin{cases} 7 & \text{if } x \leq -2 \\ x^2 + 3x - 28 & \text{if } -2 < x \leq 6 \\ \sqrt{x + 25} & \text{if } x > 6 \end{cases}$

Evaluate the function at $x = -2$.

Because $-2 \leq -2$, use the rule for $x \leq -2$.

$f(-2) = 7$

Evaluate the function at $x = 6$.

Because $6 \leq 6$, use the rule for $-2 < x \leq 6$.

$f(6) = 6^2 + 3(6) - 28 = 26$

Graphing Piecewise Functions
Graph the piecewise function. $f(x) = \begin{cases} -1 & \text{if } x < 3 \\ 2x - 4 & \text{if } x \geq 3 \end{cases}$

Because the function is divided at $x = 3$,

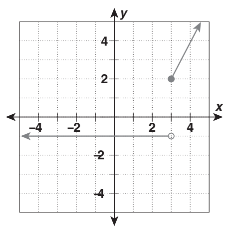
evaluate both branches of the function at $x = 3$.

Plot the point $(3, -1)$ with a/an open circle and draw a horizontal ray to the left.

Substitute $x = 3$ into the function $f(x) = 2x - 4$.

$2(3) - 4 = 2$

Plot the point $(3, 2)$ with a/an solid circle and draw a ray to the right with a slope of 2.



SECTION 9A Ready To Go On? Problem Solving Intervention
9-2 Piecewise Functions

A piecewise function is a combination of one or more functions. If the function is constant for each interval in its domain, the function is called a step function.

Maureen drove from her house in the suburbs to her office in the city. She drove 10 minutes through town at an average speed of 30 mi/h, 30 minutes on the highway at an average speed of 65 mi/h, and 6 minutes in city traffic at an average speed of 15 mi/h. Write a piecewise function to represent Maureen's distance versus time. Then graph the function.

Understand the Problem

1. Maureen's trip can be described by how many steps? 3
2. What do you need to determine? A piecewise function to represent the distance Maureen traveled on the 3 intervals.

Make a Plan

3. Make a table to organize the data. Use the distance formula to find Maureen's rate for each leg of her drive.

	Rate (mi/h)	Time (h)	Distance (mi)
Through Town	30	$\frac{1}{6}$	5
On the Highway	65	$\frac{1}{2}$	32.5
In City Traffic	15	$\frac{1}{10}$	1.5

Solve

4. Graph the function.
5. Write a linear function for each leg.

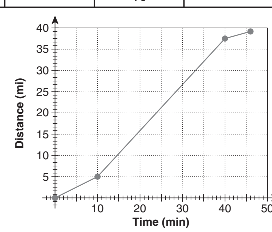
Town: $d = 0.5t$

Highway: $d = \frac{13}{12}t - \frac{35}{6}$

City: $d = 0.25t + \frac{55}{2}$

The function rule is:

$d(t) = \begin{cases} \frac{1}{2}t & \text{if } 0 \leq t < 10 \\ \frac{13}{12}t - \frac{35}{6} & \text{if } 10 < t \leq 40 \\ \frac{1}{4}t + \frac{55}{2} & \text{if } 40 < t \leq 46 \end{cases}$



Look Back

6. Use your graph to find out how far Maureen has traveled after 5 min: 2.5 mi
20 min: 16 mi 45 min: 38 mi Does your function make sense? Yes

SECTION 9A

Ready To Go On? Skills Intervention

9-3 Transforming Functions

Transforming Piecewise Functions

Given $f(x) = \begin{cases} 6x - 8 & \text{if } x \le 2 \\ x^2 & \text{if } x > 2 \end{cases}$, write the rule for $h(x)$, a horizontal shift of $f(x)$ 3 units left.

Replace every x in the function with $x +$ <u>3</u>.

$h(x) = f(x + 3) = \begin{cases} 6(\underline{x} + \underline{3}) - 8 & \text{if } (x + 3) \le 2 \\ (\underline{x} + 3)^2 & \text{if } (x + 3) > 2 \end{cases}$

$h(x) = \begin{cases} \underline{6x} + 10 & \text{if } x \le \underline{-1} \\ x^2 + \underline{6x} + 9 & \text{if } x > \underline{-1} \end{cases}$ Simplify.

Identifying Intercepts

Identify the x- and y-intercepts of $f(x)$. Without graphing $g(x)$, identify its x- and y-intercepts.

A. $f(x) = \frac{1}{4}x + 5$ and $g(x) = 2f(x)$

$f(0) = \frac{1}{4}(\underline{0}) + 5 = \underline{5}$ Find the y-intercept of the original function.

$\underline{0} = \frac{1}{4}x + 5$ Find the x-intercept of the original function.

$-5 = \frac{1}{4}x$

$\underline{-20} = x$

The y-intercept of $f(x)$ is <u>5</u> and the x-intercept is <u>−20</u>.

$g(x)$ is a <u>vertical stretch</u> of $f(x)$ by a factor of <u>2</u>.

The y-intercept of $g(x)$ is <u>10</u> and the x-intercept is <u>−20</u>.

Check your answer: Graph $f(x)$ and $g(x)$ to verify the intercepts.

B. $f(x) = x^2 - 8$ and $g(x) = -f(5x)$

$f(0) = \underline{0}^2 - 8 = \underline{-8}$ Find the y-intercept of the original function.

$\underline{0} = x^2 - 8$ Find the x-intercept of the original function.

$x^2 = 8$

$x = \sqrt{8}$

The y-intercept of $f(x)$ is <u>−8</u> and the x-intercept(s) is/are <u>$\pm 2\sqrt{2}$</u>.

$g(x)$ is a <u>horizontal compression</u> of $f(x)$ by a factor of <u>$\frac{1}{5}$</u>, and a reflection of $f(x)$ across the <u>x</u> axis.

The y-intercept of $g(x)$ is <u>8</u> and the x-intercept(s) is/are <u>$\pm\frac{2\sqrt{2}}{5}$</u>.

Check your answer: Graph $f(x)$ and $g(x)$ to verify their intercepts.

SECTION 9A

Ready To Go On? Quiz

9-1 Multiple Representations of Functions

1. A plane is descending from an altitude of 32,000 feet at a rate of 25 feet per second. Create a table and an equation to represent the plane's altitude, a, with relation to time, t.

t (s)	0	1	2	3	4
a (ft)	32,000	31,975	31,950	31,925	31,900

<u>$a = 32{,}000 - 25t$</u>

2. The population of a certain bacteria at different times is shown in the table.

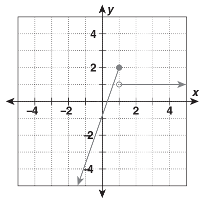
Time (hours)	0	1	2	3	4
Number of bacteria	200	600	1800	5400	16,200

a. Find an appropriate model for the population of bacteria. <u>$f(t) = 200(3)^t$</u>

b. Assuming the growth in population continues, when will the bacteria's population equal 1,312,200? <u>in 8 hours</u>

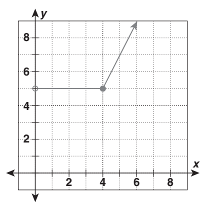
9-2 Piecewise Functions

3. Graph the function $f(x) = \begin{cases} 3x - 1 & \text{if } x \le 1 \\ 1 & \text{if } x > 1 \end{cases}$.



4. The cost to park in a parking garage in Boston is \$5 for the first 4 hours and \$2 for each additional hour. Sketch a graph of the cost of parking in the garage for 0 to 6 hours, and write a piecewise function for the graph.

$f(x) = \begin{cases} 5 & \text{if } 0 < x \le 4 \\ 5 + 2(x - 4) & \text{if } x > 4 \end{cases}$

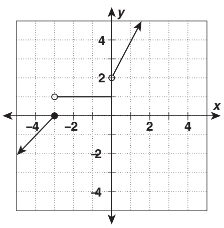


SECTION 9A

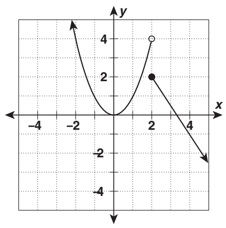
Ready To Go On? Quiz continued

Write a piecewise function for each graph.

5. $f(x) = \begin{cases} x + 3 & \text{if } x \le -3 \\ 1 & \text{if } -3 < x \le 0 \\ 2x + 2 & \text{if } x > 0 \end{cases}$



6. $f(x) = \begin{cases} x^2 & \text{if } x < 2 \\ -\frac{3}{2}x + 5 & \text{if } x \ge 2 \end{cases}$



9-3 Transforming Functions

Identify the x- and y-intercepts of $f(x)$. Without graphing $g(x)$, identify its x- and y-intercepts.

7. $f(x) = -x + 4$ and $g(x) = -2f(x)$

x-intercept(s) of $f(x)$ <u>4</u>

y-intercepts(s) of $f(x)$ <u>4</u>

x-intercept(s) of $g(x)$ <u>4</u>

y-intercepts(s) of $g(x)$ <u>−8</u>

8. $f(x) = x^2 - 1$ and $g(x) = f(-\frac{1}{4}x)$

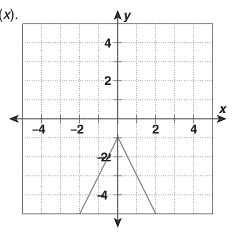
x-intercept(s) of $f(x)$ <u>±1</u>

y-intercepts(s) of $f(x)$ <u>−1</u>

x-intercept(s) of $g(x)$ <u>±4</u>

y-intercepts(s) of $g(x)$ <u>−1</u>

9. Given $f(x) = |2x|$ and $g(x) = -f(x) - 1$, graph $g(x)$.



SECTION 9A

Ready To Go On? Enrichment

Exploring Step Functions

A step function is a special type of piecewise function whose graph resembles steps, not lines or curves. One common step function is the *greatest integer function*, or rounding-down function. Given any number x, the greatest integer function is defined as:

$f(x) =$ the greatest integer less than or equal to x.

Find $f(x)$ for the following values of x.

1. 4 <u>4</u>

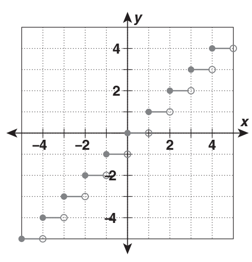
2. $8.\overline{7}$ <u>8</u>

3. $\sqrt{3}$ <u>1</u>

4. 2.25 <u>2</u>

5. The domain of the greatest integer function is the set of Real numbers. What is the range of the greatest integer function? <u>The set of all integers</u>

6. Graph the greatest integer function for all values of x such that $-5 \le x < 5$.



7. The cost of a telephone call between Boston and Paris is \$0.90 for the first minute and \$0.42 for each additional minute or portion of a minute.

a. Use the greatest integer function to model the cost of a call, C.

<u>$C = 0.90 + 0.42(f(x))$ where $f(x) > 1$</u>

b. How much does an 8 minute and 25 second phone call cost?

<u>$C = 0.90 + 0.42(7) = \$3.84$</u>

SECTION 9B

Ready To Go On? Skills Intervention

9-4 Operations with Functions

Find this vocabulary word in Lesson 9-4 and the Multilingual Glossary.

Vocabulary
composition of functions

Evaluating Functions

Given $f(x) = 6x - 5$, $g(x) = x^2 + 6x - 27$, and $h(x) = \frac{12}{x + 9}$, find each function or value.

A. $(g - f)(x)$

$(g - f)(x) = x^2 + 6x - 27 - (6x - 5)$ — Substitute function rules.

$= x^2 - 22$ — Combine like terms.

B. $(fh)(-1)$

$(fh)(x) = (6x - 5) \cdot \frac{12}{x + 9}$ — Substitute function rules.

$(fh)(-1) = (6(-1) - 5) \cdot \frac{12}{(-1) + 9}$ — Substitute -1 for x.

$= -\frac{33}{2}$ — Simplify.

C. $\frac{h}{g}(x)$

$\frac{h}{g}(x) = \frac{h(x)}{g(x)} = \frac{\frac{12}{x+9}}{x^2 + 6x - 27}$ — Substitute function rules.

$= \frac{\frac{12}{x + 9}}{(x + 9)(x - 3)}$ — Factor completely. Note that $x \neq -9$ or 3.

$= \frac{\frac{12}{x + 9}}{(x + 9)(x - 3)} \cdot \frac{x + 9}{x + 9}$ — Simplify the fraction.

$= \frac{12}{(x + 9)^2 (x - 3)}$ where $x \neq -9$ or 3. — Divide out common factors and simplify.

D. $g(h(-3))$

$h(-3) = \frac{12}{-3 + 9} = 2$ — Find $h(-3)$.

$g(2) = 2^2 + 6(2) - 27 = -11$ — Substitute your result for $h(-3)$ into g.

SECTION 9B

Ready To Go On? Problem Solving Intervention

9-4 Operations with Functions

You can multiply functions by following this rule: $(fg)(x) = f(x) \cdot g(x)$.

During a sale, a shoe store is selling boots for 25% off. Preferred customers also receive an additional 10% off. Write a composite function to represent the final cost of a pair of boots that originally cost c dollars. Then find the cost of a pair of boots originally priced at $92 that a preferred customer wants to buy.

Understand the Problem

1. Describe the shoe store's sale.
   Boots are 25% off the original price and preferred customers receive an additional 10% off.

Make a Plan

2. What do you need to determine?
   A composite function that applies both the 25% off sale and the preferred customer's 10% discount; the cost of a $92 pair of boots.
3. Write a function P for the final price of boots after the sale.
   $P(c) = 0.75c$
4. Write a function D for the price after applying the preferred customer discount.
   $D(c) = 0.90c$

Solve

5. Find the composition $P(D(c))$.
   $P(D(c)) = P(0.90c)$ — Substitute the rule of D into P.
   $= 0.75(0.90c)$ — Apply the rule for P.
   $= 0.675c$ — Simplify.
6. Find the cost of the boots that originally cost $92. $P(92) = 0.675(92) = \$62.10$

Look Back

7. To check your answer, find 25% of $92: Sale savings = 25%(92) = $23
   Then subtract the savings from $92 and take 10% of that value.
   Preferred customer discount = 10%(92 − savings) = 10%(69) = $6.9
   Cost = 92 − sale savings − preferred customer discount = $62.10
8. Does your solution in Exercise 6 make sense? Yes

SECTION 9B

Ready To Go On? Skills Intervention

9-5 Functions and Their Inverses

Find these vocabulary words in Lesson 9-5 and the Multilingual Glossary.

Vocabulary	
one-to-one function	horizontal-line test

Using the Horizontal-Line Test and Writing Rules for Inverses

Find the inverse of each function. Then state the domain and range of the inverse.

A. $f(x) = 7 - \sqrt{x + 4}$

Step 1 Graph the function on your graphing calculator.

Does it pass the horizontal-line test? Yes Therefore, the inverse is a function.

Step 2 Find the inverse.

$y = 7 - \sqrt{x + 4}$ — Rewrite the function using y instead of $f(x)$.

$x = 7 - \sqrt{y + 4}$ — Interchange x and y. Note the range restriction $y \geq -4$.

$x - 7 = -\sqrt{y + 4}$ — Isolate the radical.

$(x - 7)^2 = y + 4$ — Square both sides of the equation.

$x^2 - 14x + 49 = y + 4$ — Multiply.

$y = x^2 - 14x + 45$ — Solve for y.

The domain of the inverse is the range of $f(x)$: $\{x \mid x \leq 7\}$.

The range of the inverse is the domain of $f(x)$: $\{y \mid y \geq -4\}$.

B. $f(x) = \frac{8 - 6x}{5}$

Step 1 Graph the function on your graphing calculator.

Does it pass the horizontal-line test? Yes Therefore, the inverse is a function.

Step 2 Find the inverse.

$y = \frac{8 - 6x}{5}$ — Rewrite the function using y instead of $f(x)$.

$x = \frac{8 - 6y}{5}$ — Interchange x and y.

$y = -\frac{5x}{6} + \frac{4}{3}$ — Solve for y.

The domain of the inverse is: $\{x \mid x \in \mathbb{R}\}$ and its range is: $\{y \mid y \in \mathbb{R}\}$.

SECTION 9B

Ready To Go On? Skills Intervention

9-6 Modeling Real-World Data

Identifying Models by Using Constant Ratios or Differences

Use finite differences or ratios to determine which parent function best models each set of data.

A.

x	1	2	3	4	5
y	11	14	9	−4	−25

Are the x-values evenly spaced? Yes

Check the first differences between y-values: 3, −5, −13, −21

Check the second differences between y-values: −8, −8, −8

Check the ratios between y-values: Not applicable

Because the second differences are constant, a/an quadratic model best models the data set.

B.

x	−3	0	3	6	9
y	56	7	0.875	0.109	0.014

Are the x-values evenly spaced? Yes

Check the first differences between y-values: −49, −6.125, −0.766, −0.095

Check the second differences between y-values: 42.875, 5.359, 0.671

Check the ratios between y-values: 0.125, 0.125, 0.125, 0.125

Because the ratios are constant, a/an exponential model best models the data set.

C.

x	6	7	8	9	10
y	42.3	44.1	45.9	47.7	49.5

Are the x-values evenly spaced? Yes

Check the first differences between y-values: 1.8, 1.8, 1.8, 1.8

Check the second differences between y-values: Not applicable

Because the first differences are constant, a/an linear model best models the data set.

SECTION 9B

Ready To Go On? Problem Solving Intervention

9-6 Modeling Real-World Data

To model data in a real-world application, look for a pattern.

The table shows the estimated population of a Southwestern town. Using time as the independent variable and 1980 as a reference year, find a model that best fits the data. Use your model to predict the town's population in the year 2030.

Year	1980	1985	1990	1995	2000	2005
Population	1840	2208	2650	3180	3815	4617

Understand the Problem

1. Describe the town's population. The town's population is growing. In 1980, the population was 1840 and in 2005 it was 4617.

Make a Plan

2. What do you need to determine? Determine if the population can be modeled by a linear, quadratic, exponential, or square root function. Then predict the population in 2030.

Solve

3. Are the years evenly spaced? Yes

   Check the first differences: 368 442 530 635 802

   Check the second differences: 74 88 105 167

   Check the ratios between population values: 1.20 1.20 1.20 1.20 1.21

   Because the ratios are constant, a/an exponential model is best.
4. Use your graphing calculator to perform a/an exponential regression.
5. A function that models the data is:

   $f(t) = 1837.17(1.0374)^t$
6. Solve for the town's population in the year 2030. Let $t =$ 50.

   $f(t) = 1837.17(1.0374)^{50} \approx$ 11,520

Look Back

7. To check your model, let $t = 0, 5, 10, 15, 20,$ and 25 and see if your function values using the function in Exercise 5 match the population figures in the table.

   1837; 2207; 2652; 3187; 3829; 4601
8. Does your model seem reasonable? Yes

SECTION 9B

Ready To Go On? Quiz

9-4 Operations with Functions

Given $f(x) = x + 1$, $g(x) = x^2 - 11x + 18$, and $h(x) = \frac{10}{x}$, find each function or value.

1. $(g - f)(4)$ -15
2. $\left(\frac{h}{g}\right)(5)$ $-\frac{1}{6}$
3. $(fg)(x)$ $x^3 - 10x^2 + 7x + 18$
4. $h(f(x))$ $\frac{10}{x+1}$ where $x \neq -1$
5. a. Find $(g \circ f)(x)$. $x^2 - 9x + 8$

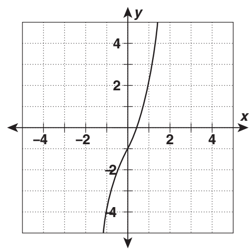
   b. What is the domain of the composite function? $\{x \mid x \in R\}$
6. Mike imports lemon juice from Sicily. The cost of a case of lemon juice includes a 15% import tax and 50 euros for shipping. Given 1 dollar = 0.83 euros, write a composite function for the total cost of a case of lemon juice in dollars, if the cost of a case is c euros,.

   $E(c) = 1.15c + 50$; $D(c) = \frac{c}{0.83}$; $D(E(c)) = \frac{1.15c + 50}{0.83}$

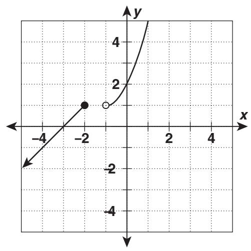
9-5 Functions and Their Inverses

State whether the inverse of each relation is a function.

7. Function



8. Function



SECTION 9B

Ready To Go On? Quiz continued

Write the rule for the inverse of each function. Then state the domain and range of the inverse.

9. $f(x) = (8x + 4)^2$

   $y = \pm\frac{1}{8}\sqrt{x} - \frac{1}{2}$; not a function

   domain: $\{x \mid x \geq 0\}$

   range: $\{y \mid y \in R\}$
10. $g(x) = \frac{9}{x - 7}$

   $f^{-1}(x) = \frac{9}{x} + 7$

   domain: $\{x \mid x \neq 0\}$

   range: $\{y \mid y \neq 7\}$

9-6 Modeling Real-World Data

11. Use finite differences or ratios to determine which parent function would best model this set of data.

t	0	2	4	6	8
a	−5	5	31	73	131

Quadratic

12. The table shows the years that a park ranger has been monitoring the giraffe population in a game park in Namibia and the size of the giraffe population. Using time as the independent variable, find a model for the size of the giraffe population.

Time (year)	Giraffe
5	72
6	83
7	96
8	110
9	127

$f(x) = 36(1.15)^x$

LESSON 9B

Ready To Go On? Enrichment

Composition of Functions

Recall that the composition of functions f and g is notated:

$$(f \circ g)(x) = f(g(x))$$

The domain of $(f \circ g)(x)$ is all values of x in the domain of g such that $g(x)$ is in the domain of f.

In Exercises 1-4, find two functions f and g such that $(f \circ g)(x)$ equals the rule shown. (There is more than one correct answer. Find multiple answers if you can.)

1. $(f \circ g)(x) = \frac{1}{x - 4}$

   $f(x) = \frac{1}{x}$;

   $g(x) = x - 4$
2. $(f \circ g)(x) = \sqrt[3]{2(x^2 - 1)}$

   $f(x) = \sqrt[3]{2x}$;

   $g(x) = x^2 - 1$
3. $(f \circ g)(x) = (x + 2)^2$

   $f(x) = x^2$;

   $g(x) = x + 2$
4. $(f \circ g)(x) = 162x^2$

   $f(x) = \frac{x^4}{8}$;

   $g(x) = -6\sqrt{x}$

