SKILL 58

Are You Ready?

Connect Words and Algebra

Teaching Skill 58

Objective Connect words and algebra.

Point out to students that an algebraic expression (or equation) is simply a mathematical way of writing a phrase or sentence.

Ask: **What is the difference between an expression and an equation?** (An expression does not have an equal sign while an equation does.)

Tell students that associating key words with the correct operation is critical to being able to make the connection between words and algebra.

Review with students the table of key words. Provide some real life examples. Start with the following two: **(1)If your age is 10 years more than your sister's, and your sister is 12, how do you find your age?** (Add 12 plus 10.) **If you receive $10 per week allowance for 8 weeks, how do you find the total amount you received?** (Multiply 10 times 8.) Have students provide additional examples.

Review the lawn cutting example with students and then have them complete the exercises.

PRACTICE ON YOUR OWN

In exercises 1–6, students write algebraic expressions and equations.

CHECK

Determine that students know how to write algebraic expressions and equations.

Students who successfully complete the **Practice on Your Own** and **Check** are ready to move on to the next skill.

COMMON ERRORS

Students may not know the key words associated with operations, and as a result may use the wrong operation.

Students who made more than 1 error in the **Practice on Your Own,** or who were not successful in the **Check** section, may benefit from the **Alternative Teaching Strategy.**

Alternative Teaching Strategy

Objective Connect words and algebra.

Materials needed: multiple copies of the flashcards shown below (index cards work nicely), blank index cards

Have students work in pairs. Distribute copies of the flashcards and have students shuffle the cards and divide them in half so that each student has six cards.

$3x + 1$	$n - 5$	$x + 7$
$\frac{1}{2}n - 2$	$C = 3(25)$	$t = 6(2)$
$C = 12 + 2.50$	$P = 2(8) + 2w$	$A = \frac{1}{2}(6)(h)$
$I = (500)(2)r$	$A = 50 + 10w$	$S = 600 - 50w$

Review with students key words and the operations with which the words are associated.

Tell students they are going to make up phrases or situations that match the algebraic expressions and equations on the cards. Give the following example: $5n + 3$. Ask: **How would you verbalize $5n$?** (5 times a number) **And + 3?** (increased by 3) Write the phrase on the board: "5 times a number increased by 3." Point out that variations of this phrase are also correct, such as 3 more than 5 times a number.

Next, write the equation $C = 40 - 5(d)$ on the board. Remind students that variables typically represent something that makes sense. For example, C might represent cost and d number of days. Or, provide the following scenario: There are 40 cookies in a bag and Tom eats 5 cookies per day. $C = 40 - 5(d)$ could be the number of cookies remaining at the end of any given day.

Instruct students to take turns making up situations to match the cards they have. As an extension of the exercise, have students make flashcards and continue.

Name _____ Date _____ Class _____

SKILL 58

Are You Ready?

Connect Words and Algebra

To connect words and algebra, you must understand the operations involved and how to represent them. Key words are helpful in determining the operations.

Key Words	Operation or Representation
a number; an unknown quantity	any variable, such as x or n
Twice, three times, etc.	multiplication ($2n$, $3n$, etc.)
sum; more than; increased by	addition ($+$)
difference; less than; decreased by	subtraction ($-$)
each; per	multiplication
is; equals	$=$

Example: Jared must cut 6 lawns over the weekend. Each of the lawns takes 2 hours to cut. Write an equation representing the total time t to cut all 6 lawns.

Answer: Since each lawn takes 2 hours, multiply 2 times the number of lawns to get the total time: $t = 6(2)$.

Practice on Your Own

1. Write an expression that represents the quantity 5 more than a number. _____
2. Write a phrase that could be modeled by the expression $x - 15$. _____
3. John bought 3 CDs and 2 DVDs. Each CD costs $9.95, and each DVD costs $14.98. Write an equation representing the total cost C. _____
4. A triangle has sides of length 7, 10, and s. Write an equation representing the perimeter P of the triangle. _____
5. The value of a painting begins at $12,000 and increases by $500 per year. Write an equation representing the value V of the painting at the end of any given year y. _____
6. David has 56 baseball cards of which he sells 3 cards per week. Write an equation representing the number of cards n he has left at the end of any given week w. _____

Check

7. Write an expression that represents a number decreased by 6. _____
8. Tina bought 6 plates and 2 glasses. Each plate costs $6.99, and each glass costs $22.98. Write an equation representing the total cost C. _____
9. Joseph opens a checking account with $400. Each month he adds $150 to the account. Write an equation representing the total amount A in the account at the end of any given month m. _____

SKILL 86

Are You Ready?

Line Graphs

Teaching Skill 86

Objective Read and understand information presented in a line graph.

Remind students that line graphs are created by plotting points on a coordinate plane and then connecting the points.

Point out that when written as x and $f(x)$, $f(x)$ represents the y-value of the point. Explain that it is possible to locate the y-value for any given x-value, and visa versa.

Have students look at the line graph in the example. Ask: **Which axis is the *x*-axis?** (the horizontal axis) **Which is the *y*-axis?** (the vertical axis) **In a coordinate pair, which value is written first?** (the x-value)

Next, direct students' attention to the questions in the example. Ask: **In the first and second questions, which value is given, *x* or *y*?** (x) Work through the first two questions. Ask: **In the third and fourth questions, which value is given?** (y) Work through the last two questions.

Have students complete the practice exercises.

PRACTICE ON YOUR OWN

In exercises 1–6, students find values on a line graph.

CHECK

Determine that students know how to read and understand information presented in a line graph.

Students who successfully complete the **Practice on Your Own** and **Check** are ready to move on to the next skill.

COMMON ERRORS

Students may not understand whether they should be looking for the x-value or the y-value on the graph.

Students who made more than 1 error in the **Practice on Your Own,** or who were not successful in the **Check** section, may benefit from the **Alternative Teaching Strategy.**

Alternative Teaching Strategy

Objective Create a line graph by plotting given points and connecting line segments.

Materials needed: rulers; multiple copies of a grid with values -2 to 22 on the x-axis and -2 to 22 on the y-axis. Grid should be labeled in increments of 2.

Some students may better understand how to read a line graph if they practice making one.

Tell students they are going to create a line graph using a few critical points and then locate other points on the graph.

Write the following points on the board: (0, 0); (10, 6); (14, 16); and (20, 22).

Instruct the students to plot these points on their grids. Be sure students understand which coordinate represents horizontal movement and which represents vertical movement. Before moving on, check students' points to make sure they are plotted correctly.

Next, instruct students to use a ruler to connect the points they plotted and then label the graph as $f(x)$. Explain that the point (14, 16) means the following: when x is 14, $f(x)$ is 16.

Then instruct students to use their graph to answer the following questions:

1) When x is 5, what is $f(5)$? (Answer: 3)
2) When x is 12, what is $f(12)$? (Answer: 11)
3) When $f(x)$ is 19, what is x? (Answer: 17)
4) When $f(x)$ is 6, what is x? (Answer: 10)

Repeat the exercise using the following points and questions.

(0, 2); (4, 6); (6, 14); and (20, 20)

1) When x is 2, what is $f(2)$? (Answer: 4)
2) When x is 13, what is $f(13)$? (Answer: 17)
3) When $f(x)$ is 19, what is x? (Answer: 17)
4) When $f(x)$ is 10, what is x? (Answer: 5)

Name _____ Date _____ Class _____

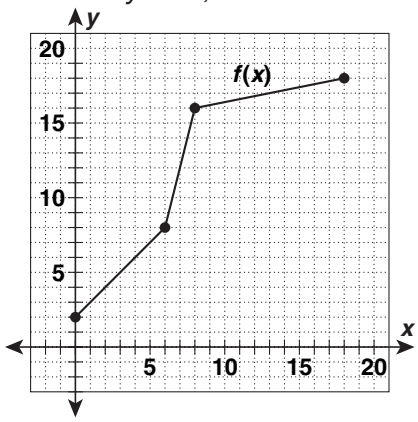
SKILL 86

Are You Ready?

Line Graphs

To read a line graph, remember that a function table generates coordinate pairs. So, $(x, f(x))$ is also a coordinate point (x, y).

To find $f(x)$ at a particular x, look for the y-value of the point with that x-coordinate. To find an x such that $f(x)$ is a specific value, look for the value on the y-axis, and then find the corresponding x-coordinate.



Example: Find each value for the graph of $f(x)$ shown.

What is $f(8)$?
Answer: 16 since $y = 16$ when $x = 8$

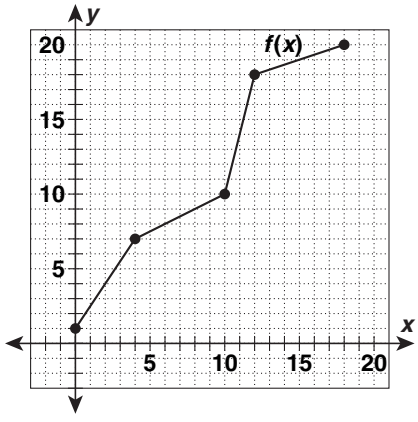
What is $f(4)$?
Answer: 6 since $y = 6$ when $x = 4$

What is x such that $f(x) = 8$?
Look for the x value where $y = 8$.
Answer: $x = 6$

What is x such that $f(x) = 17$
Look for the x value where $y = 17$.
Answer: $x = 13$

Practice on Your Own

Find each value for the graph of $f(x)$ shown.



1. $f(4) =$ _____

2. $f(0) =$ _____

3. $f(15) =$ _____

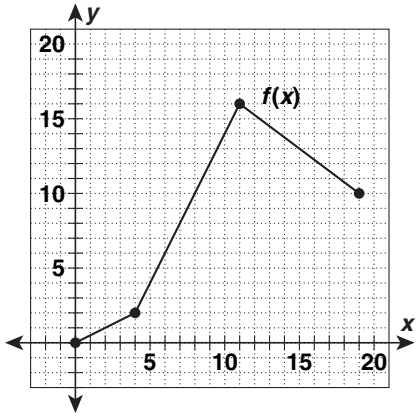
4. What is x such that $f(x) = 10$? _____

5. What is x such that $f(x) = 4$? _____

6. What is x such that $f(x) = 18$? _____

Check

Find each value for the graph of $f(x)$ shown.



7. $f(11) =$ _____

8. $f(7) =$ _____

9. What is x such that $f(x) = 16$? _____

10. What is x such that $f(x) = 4$? _____

SKILL 64

Are You Ready?

Multiply Binomials

Teaching Skill 64

Objective Multiply binomials.

Review with students the definition of a binomial. Ask: **Will a binomial always have two terms?** (Yes)

Point out that multiplying binomials is much like multiplying a polynomial by a monomial. The only difference is that you must use the Distributive Property twice.

Direct students' attention to the example. Explain that you will be distributing the x to both terms in the second set of parentheses and also the $+6$ to both terms in the second set of parentheses. Ask: **If you are multiplying two terms by two other terms, how many terms should you end up with before combining like terms?** (4) Work through the example.

If time permits, present and explain the FOIL method. Explain that FOIL reminds you to multiply all four sets of terms–the first terms, the outer terms, the inner terms, and the last terms. Remind students that they will still need to combine like terms.

Have students complete the practice exercises using either the Distributive Property or the FOIL method.

PRACTICE ON YOUR OWN

In exercises 1–10, students multiply binomials.

CHECK

Determine that students know how to multiply binomials.

Students who successfully complete the **Practice on Your Own** and **Check** are ready to move on to the next skill.

COMMON ERRORS

Students may multiply only the first terms and the last terms, rather than distributing.

Students who made more than 2 errors in the **Practice on Your Own,** or who were not successful in the **Check** section**,** may benefit from the **Alternative Teaching Strategy.**

Alternative Teaching Strategy

Objective Multiply binomials.

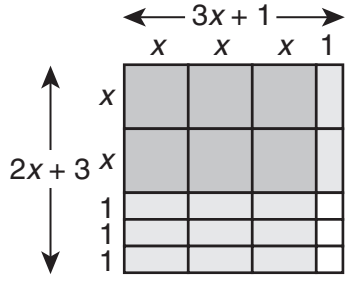
Materials needed: algebra tiles

Tell students that they are going to model the product of two binomials using algebra tiles.

Ask: **How do you find the area of a rectangle given its length and width?** (You multiply the length times the width.)

Explain that you are going to use algebra tiles to find the area of a rectangle that is $(2x + 3)$ units long and $(3x + 1)$ units wide by counting tiles. Write the equation $A = (2x + 3)(3x + 1)$ on the board.

Arrange the algebra tiles as shown below. Have the students draw a similar arrangement on their paper and label as shown.



Ask: **What is the area of each of the larger squares?** ($x \cdot x = x^2$) **How many are there?** (6) **What is the area of each of the smaller rectangles?** ($x \cdot 1 = x$) **How many are there?** (11) **What is the area of the smaller squares?** ($1 \cdot 1 = 1$) **How many are there?** (3) **What is the total area of the figure?** ($6x^2 + 11x + 3$)

Demonstrate how to find the same product using the Distributive Property (or FOIL) to confirm the answer.

Repeat this exercise to find the products of the following pairs of binomials:

$(2x + 2)(x + 3)$ and $(3x + 2)(2x + 1)$

Have students confirm their answers each time using the Distributive Property or FOIL.

Answers: $2x^2 + 8x + 6$ and $6x^2 + 7x + 2$

Name _____ Date _____ Class _____

SKILL 64

Are You Ready?

Multiply Binomials

Definition: A binomial is the sum or difference of two monomials.

To multiply a binomial by another binomial, use the Distributive Property twice.

Example: Multiply $(x + 6)(3x - 5)$.

$= x(3x - 5) + 6(3x - 5)$	First use of the Distributive Property.
$= x(3x) - x(5) + 6(3x) + 6(-5)$	Second use of the Distributive Property.
$= 3x^2 - 5x + 18x - 30$	Multiply using properties of exponents.
$= 3x^2 + 13x - 30$	Combine like terms $(-5x + 18x = 13x)$.

Practice on Your Own

Find each product.

1. $(n + 6)(n + 3)$

_____

2. $(c + 12)(c - 5)$

_____

3. $(10q + 3)(q + 4)$

_____

4. $(k + 7)(3k - 1)$

_____

5. $(u - 1)(u + 1)$

_____

6. $(r + 6)(r + 6)$

_____

7. $(5a - 4)(5a + 4)$

_____

8. $(3g + 1)(8g + 12)$

_____

9. $(5z + 8)(4z - 2)$

_____

10. $(4p - 9)(2p - 1)$

_____

Check

Find each product.

11. $(x + 4)(x + 7)$

_____

12. $(2w + 6)(w - 9)$

_____

13. $(p + 5)(p + 5)$

_____

14. $(2t + 7)(2t - 7)$

_____

15. $(7y - 3)(y - 1)$

_____

16. $(3m + 4)(9m + 2)$

_____

SKILL 63

Are You Ready?

Simplify Polynomial Expressions

Teaching Skill 63

Objective Simplify polynomial expressions.

Point out to students that simplifying polynomials works much the same way as combining like terms. The goal is to "put together" any terms that are similar.

Review with students the steps for simplifying a polynomial expression. Explain that terms inside parentheses may not be similar to other terms in the polynomial before they are multiplied. However, after the Distributive Property has been used, there may be more similar terms in the polynomial. Point out that this is the reason for using the Distributive Property first.

Direct students' attention to the example. Ask: **Before you use the Distributive Property, can you tell what the like terms are?** (No) **Once the Distributive Property has been used, what are the like terms?** ($2x^2$ and x^2; $12x$ and $-10x$) Remind students to be careful of negatives.

Work through the last step of the example and then have students complete the practice exercises.

PRACTICE ON YOUR OWN

In exercises 1–10, students simplify polynomial expressions.

CHECK

Determine that students know how to simplify polynomial expressions.

Students who successfully complete the **Practice on Your Own** and **Check** are ready to move on to the next skill.

COMMON ERRORS

Students may not recognize like terms, particularly when there are multiple variables with different exponents.

Students who made more than 3 errors in the **Practice on Your Own,** or who were not successful in the **Check** section, may benefit from the **Alternative Teaching Strategy.**

Alternative Teaching Strategy

Objective Simplify polynomial expressions.

Some students may benefit from physically matching like terms using circles, squares, triangles, etc.

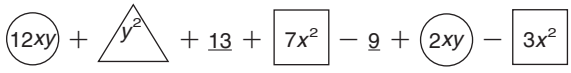
Remind students that like terms must have identical variable factors, regardless of how many different variables are part of the term.

Write the following problem on the board:

$$12xy + y^2 + 13 + 7x^2 - 9 + 2xy - 3x^2$$

Ask a volunteer to identify the four different types of terms in this expression. (xy, y^2, x^2, and constants) Ask: **How many terms have *xy* in them?** (2) Instruct students to place a circle around those two terms. Ask: **How many terms have a y^2 in them?** (1) Instruct students to place a triangle around that term. Ask: **How many terms have an x^2 in them?** (2) Instruct students to place a square around those terms. Finally, ask: **How many terms are constants?** (2) Instruct students to underline the constants.

Students' expressions should now look like the following:



Instruct students to simplify the expression by combining like terms. Point out that they should be careful with negatives; if a negative (or a subtraction sign) precedes a term, it goes with that term.

$(14xy + y^2 + 4x^2 + 4)$

Have students work the following problems using this technique:

$9t^2 + 3t - 2 + 8t - 4 + 2t^2$
(Answer: $11t^2 + 11t - 6$)

$5x^2y + 14xy - 2xy^2 + 8xy^2 - 9xy + x^2y$
(Answer: $6x^2y + 5xy + 6xy^2$)

Explain to students that if an expression contains any parentheses, they must first distribute before they can match up like terms. Work a few examples.

Name _____ Date _____ Class _____

SKILL 63

Are You Ready?

Simplify Polynomial Expressions

To simplify a polynomials expression:

- Step 1: Remove all parentheses by using the Distributive Property (if needed).
- Step 2: Identify and combine like terms.

Example: Simplify $2x(x + 6) + x^2 - 10x$.

$= 2x(x + 6) + x^2 - 10x$

$= 2x(x) + 2x(6) + x^2 - 10x$ Use the Distributive Property.

$= 2x^2 + 12x + x^2 - 10x$ Multiply using properties of exponents.

$= 3x^2 + 2x$ Combine like terms.

Practice on Your Own

Simplify each expression.

1. $12m + 7n - 9n$

2. $4x - y - 6x + 9y$

3. $7p - 3q + 4(2p + q)$

4. $5(d - 2e) + 3(5d + 4e)$

5. $24t^2 + 5t - 15t + t$

6. $2r^2s - rs + 5rs^2 - r^2s + 7rs$

7. $5(7f^2 - 3f + 1) - 20f^2 + 10f$

8. $p^2(3p - 5) + p(7p + 8)$

9. $18g^2(1 - 2g) - 5g^2 + 10g^3$

10. $jk(3j^2 + 5k) + j^2(2jk + 4)$

Check

Simplify each expression.

11. $2x - 7x + 5y$

12. $9a + 14b + b - 7a$

13. $20g - 14h + 6(g + 2h)$

14. $9u^3 - 4u^2 + 5u - u^3 - 10u - 3u^2$

15. $10(2p^2 - p - 1) + 7p^2 - 3$

16. $2c(9 - 5c) + 6(c^2 - c)$

SKILL 56 ANSWERS:

Practice on Your Own

1. $5x + 30$

2. $5z - 35$

3. $2n - 4$

4. $12 + 4k$

5. $48 - 8y$

6. $6m + 18$

7. $10p + 10$

8. $60 - 3c$

9. $7q - 7$

10. $55 + 11t$

11. $14 + 2b$

12. $36 - 9w$

Check

13. $12c + 24$

14. $15 - 5a$

15. $25 + 25d$

16. $50 - 10j$

17. $4x + 12$

18. $30 + 15y$

19. $3g - 75$

20. $9m - 9$

SKILL 57 ANSWERS:

Practice on Your Own

1. $12x$

2. $4m$

3. $7a^2$

4. $-7t$

5. $-3b$

6. $8d^2$

7. $-x$

8. 0

9. $11h$

10. $-9y - 9$

11. $10 + 10x$

12. $5 - 5u$

13. $13y + 6x$

14. 4

Check

15. $10x$

16. $-3c$

17. $-3a^2$

18. $8.4z$

19. $10m + 11$

20. $8q - 5r$

SKILL 58 ANSWERS:

Practice on Your Own

1. $5 + n$

2. a number decreased by 15; 15 less than a number; the difference between a number and 15; etc.

3. $C = 3(9.95) + 2(14.98)$

4. $P = 7 + 10 + s$

5. $V = 12{,}000 + 500y$

6. $n = 56 - 3w$

Check

7. $n - 6$

8. $C = 6(6.99) + 2(22.98)$

9. $A = 400 + 150m$

SKILL 59 ANSWERS:

Practice on Your Own

1. $10x^2$

2. $-21a^4$

3. $-16mn$

7. 49; $(x + 7)^2$

8. 144; $(x - 12)^2$

9. $\frac{81}{4}$; $\left(x + \frac{9}{2}\right)^2$

Check

10. 9; $(x + 3)^2$

11. 64; $(x - 8)^2$

12. $\frac{25}{4}$; $\left(x + \frac{5}{2}\right)^2$

13. 100; $(x - 10)^2$

14. 1; $(x + 1)^2$

15. $\frac{49}{4}$; $\left(x - \frac{7}{2}\right)^2$

SKILL 83 ANSWERS:

Practice on Your Own

1. 28%

2. 30%

3. 23%

4. 19%

5. 58%

6. 81%

Check

7. 10%

8. 5%

9. 42%

SKILL 84 ANSWERS:

Practice on Your Own

1. mean = 8; median = 7; mode = 7

2. mean = $\frac{22}{7} \approx 3.1$; median = 3; mode = 4

3. mean = $\frac{39}{6} = 6.5$; median = 6.5; mode = none

4. mean = 5.25; median = 5.5; mode = 4 and 6

5. mean = 5; median = 4; mode = none

6. mean = 20; median = 20; mode = none

7. mean = 7; median = 6; mode = 5

8. mean = 8; median = 8; mode = 7 and 9

9. mean = 27; median = 25; mode = none

Check

10. mean = 8; median = 7; mode = 7

11. mean = 15; median = 15; mode = 15

12. mean = $\frac{51}{6} = 8.5$; median = 8.5; mode = 8 and 9

13. mean = 6; median = 5.5; mode = none

14. mean = $\frac{41}{7} \approx 5.9$; median = 5; mode = 5

15. mean = 20; median = 18; mode = none

SKILL 85 ANSWERS:

Practice on Your Own

1. 168

2. 432

3. 84%

4. 504

Check

5. 70

6. 25

7. 40%

8. 100

SKILL 86 ANSWERS:

Practice on Your Own

1. 7

2. 1

3. 19

4. 10

5. 2

6. 12

Check

7. 16

8. 8

9. 11

10. 5

SKILL 87 ANSWERS:

Practice on Your Own

1. Figure *ABCD* has 4 sides.

2. Angle *ABC* and angle *CBD* share a common vertex.

3. $\angle A$ and $\angle B$ are supplementary.

4. The third side of $\triangle DEF$ has length 8 or 13.

5. Point *P* lies in the third quadrant.

Check

6. The other two angles of triangle *MNP* have measures of 60 degrees.

7. The measure of $\angle HJK$ is greater than 90 degrees.

8. Lines *p* and *q* are perpendicular.

SKILL 88 ANSWERS:

Practice on Your Own

1. Hypothesis: a triangle is a right triangle; Conclusion: the sum of its acute angles is 90 degrees.

2. Hypothesis: two lines are parallel to a third line; Conclusion: the lines are parallel to each other

3. True
Converse: If the sum of the measures of a polygon's angles is 180 degrees, then the polygon is a triangle.
True

4. True
Converse: If two angles are congruent, then they are right angles.
False

Check

5. Hypothesis: two planes intersect; Conclusion: they intersect in a line.

6. True
Converse: If two angles are congruent, then they are vertical.
False

7. True
Converse: If a parallelogram is a rhombus, then the diagonals of the parallelogram are perpendicular.
True

SKILL 89 ANSWERS:

Practice on Your Own

1. any real number other than 0, 1, or 2 are all counterexamples

2. any negative number

3. $n = 3$

4. any negative number or 0

5. $n = -1$

6. any fraction such that $-1 < n < 1$,
e.g. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots \frac{2}{3}, \frac{3}{4}, \frac{5}{6}, \ldots$

Check

7. $n = 1$

8. any negative number of any positive fraction such that $0 < n < 1$

9. any negative number or 0

10. $n = 0$

SKILL 62 ANSWERS:

Practice on Your Own

1. $16d + 40$
2. $m^2 - m$
3. $9b^2 + 9b$
4. $48 - 60q$
5. $5p^2 + 30p$
6. $50w - 30w^2$
7. $-10r - 2r^4$
8. $n^4 + n^2$
9. $9g^2 + 3g$
10. $2e^4 - 2e^3$
11. $2h^3 - 10h^2 + 30h$
12. $x^4y + x^2y^6$

Check

13. $60x - 30$
14. $4y^2 - 36y$
15. $55 - 22k$
16. $49t - 49t^2$
17. $w^3 - w^4$
18. $3g^3 - 15g^2$
19. $p^3 - 2p^2 + 5p$
20. $3u^3v^3 + u^2v^4$

SKILL 63 ANSWERS:

Practice on Your Own

1. $12m - 2n$
2. $-2x + 8y$
3. $15p + q$
4. $20d + 2e$
5. $24t^2 - 9t$
6. $r^2s + 6rs + 5rs^2$
7. $15f^2 - 5f + 5$
8. $3p^3 + 2p^2 + 8p$
9. $13g^2 - 26g^3$
10. $5j^3k + 5jk^2 + 4j^2$

Check

11. $-5x + 5y$
12. $2a + 15b$
13. $26g - 2h$
14. $8u^3 - 7u^2 - 5u$
15. $27p^2 - 10p - 13$
16. $12c - 4c^2$

SKILL 64 ANSWERS:

Practice on Your Own

1. $n^2 + 9n + 18$
2. $c^2 + 7c - 60$
3. $10q^2 + 43q + 12$
4. $3k^2 + 20k - 7$
5. $u^2 - 1$
6. $r^2 + 12r + 36$
7. $25a^2 - 16$
8. $24g^2 + 44g + 12$
9. $20z^2 + 22z - 16$
10. $8p^2 - 22p + 9$

Check

11. $x^2 + 11x + 28$
12. $2w^2 - 12w - 54$
13. $p^2 + 10p + 25$
14. $4t^2 - 49$
15. $7y^2 - 10y + 3$
16. $27m^2 + 42m + 8$