CHAPTER
8

Vocabulary

The table contains important vocabulary terms from Chapter 8. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
combined variation			
constant of variation			
continuous function			
direct variation			
discontinuous function			
extraneous solution			
inverse variation			

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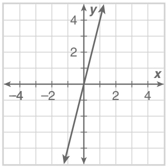
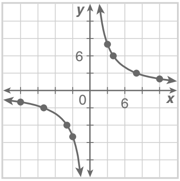
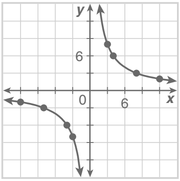
Vocabulary

The table contains important vocabulary terms from Chapter 8. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
combined variation	**572**	**A relationship containing both direct and inverse variation.**	$y = \frac{kx}{z}$, **where** k **is the constant of variation**
constant of variation	**569**	**The constant** k **in direct, inverse, joint, and combined variation equations.**	$y = \underset{\uparrow}{5}x$ **constant of variation**
continuous function	**593**	**A function whose graph is an unbroken line or curve with no gaps or breaks.**	
direct variation	**569**	**A linear relationship between two variables,** x **and** y**, that can be written in the form** $y = kx$**, where** k **is a nonzero constant.**	$c = 2\pi r$
discontinuous function	**593**	**A function whose graph has one or more jumps, breaks, or holes.**	
extraneous solution	**600**	**A solution of a derived equation that is not a solution of the original equation.**	**To solve** $\sqrt{x} = -2$**, square both sides;** $x = 4$**.** **Check** $\sqrt{4} = -2$ **is false; so 4 is an extraneous solution.**
inverse variation	**570**	**A relationship between two variables,** x **and** y**, that can be written in the form** $y = \frac{k}{x}$**, where** k **is a nonzero constant and** $x \neq 0$**.**	$y = \frac{24}{x}$ 

Term	Page	Definition	Clarifying Example
joint variation			
radical function			
radical inequality			
rational equation			
rational exponent			
rational expression			
rational function			
rational inequality			
square-root function			

Term	Page	Definition	Clarifying Example
joint variation	570	**A relationship among three variables that can be written in the form $y = kxz$, where k is a nonzero constant.**	$y = 3xz$
radical function	619	**A function whose rule contains a variable within a radical.**	$f(x) = \sqrt{x}$
radical inequality	630	**An inequality that contains a variable within a radical.**	$\sqrt{x + 3} \leq 7$
rational equation	600	**An equation that contains one or more rational expressions.**	$\frac{x + 2}{x^2 + 3x - 1} = 6$
rational exponent	611	**An exponent that can be expressed as $\frac{m}{n}$ such that if m and n are integers, then $b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$.**	$16^{\frac{1}{4}}$
rational expression	577	**An algebraic expression whose numerator and denominator are polynomials and whose denominator has a degree ≥ 1.**	$\frac{x + 2}{x^2 + 3x - 1}$
rational function	592	**A function whose rule can be written as a rational expression.**	$f(x) = \frac{x + 2}{x^2 + 3x - 1}$
rational inequality	603	**An inequality that contains one or more rational expressions.**	$\frac{x + 2}{x^2 + 3x - 1} > 6$
square-root function	619	**A function whose rule contains a variable under a square-root sign.**	$f(x) = \sqrt{x}$

CHAPTER 8

Chapter Review

Know it! Note

8-1 Variation Functions

1. The cost c to fill a sandbox varies directly as the depth of the sand s. If a sandbox filled with 6 inches of sand cost \$80 to fill, what is the cost to fill a sandbox to a depth of 9 inches?

2. The time t in hours needed to paint a house varies inversely with the number of painter's p. If 4 painters can paint a 3000 square foot house in 48 hours, how many hours will it take 12 painters to paint the house?

8-2 Multiplying and Dividing Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

3. $\dfrac{6x^3}{12x^2 + 6x}$

4. $\dfrac{x^2 + x - 12}{x^2 + 9x + 20}$

5. $\dfrac{-x + 1}{x^2 + 4x - 5}$

Multiply or divide. Assume that all expressions are defined.

6. $\dfrac{x^2 - 25}{x + 3} \cdot \dfrac{2x - 6}{x + 5}$

7. $\dfrac{16x^8y^2}{30x^3y^6} \div \dfrac{x}{6x^3}$

8. $\dfrac{4x^3 - 8x^2}{x^2 - 4x - 12} \div \dfrac{x^2 + 5x - 14}{x^2 - 36}$

8-3 Adding and Subtracting Rational Expressions

Add or subtract. Identify any *x*-values for which the expression is undefined.

9. $\dfrac{4x + 7}{x - 3} - \dfrac{x + 2}{x - 3}$

10. $\dfrac{x^2 - 7x}{x^2 - 36} + \dfrac{4}{x + 6}$

11. $\dfrac{x}{x - 4} - \dfrac{1}{x + 4}$

CHAPTER
8

Chapter Review

8-1 Variation Functions

1. The cost *c* to fill a sandbox varies directly as the depth of the sand *s*. If a sandbox filled with 6 inches of sand cost $80 to fill, what is the cost to fill a sandbox to a depth of 9 inches?

$120

2. The time *t* in hours needed to paint a house varies inversely with the number of painter's *p*. If 4 painters can paint a 3000 square foot house in 48 hours, how many hours will it take 12 painters to paint the house?

16 hours

8-2 Multiplying and Dividing Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

3. $\frac{6x^3}{12x^2 + 6x}$

$\frac{x^2}{2x+1}$; $x \neq -\frac{1}{2}$

4. $\frac{x^2 + x - 12}{x^2 + 9x + 20}$

$\frac{x-3}{x+5}$; $x \neq -5$

5. $\frac{-x + 1}{x^2 + 4x - 5}$

$\frac{-1}{x+5}$; $x \neq -5,1$

Multiply or divide. Assume that all expressions are defined.

6. $\frac{x^2 - 25}{x + 3} \cdot \frac{2x - 6}{x + 5}$

$\frac{(x-5)(2x-6)}{x+3}$

7. $\frac{16x^8y^2}{30x^3y^6} \div \frac{x}{6x^3}$

$\frac{16x^7}{5y^4}$

8. $\frac{4x^3 - 8x^2}{x^2 - 4x - 12} \div \frac{x^2 + 5x - 14}{x^2 - 36}$

$\frac{4x^2(x+6)}{(x+2)(x+7)}$

8-3 Adding and Subtracting Rational Expressions

Add or subtract. Identify any *x*-values for which the expression is undefined.

9. $\frac{4x + 7}{x - 3} - \frac{x + 2}{x - 3}$

$\frac{3x+5}{x-3}$; $x \neq 3$

10. $\frac{x^2 - 7x}{x^2 - 36} + \frac{4}{x + 6}$

$\frac{x^2 - 3x - 24}{(x-6)(x+6)}$; $x \neq \pm 6$

11. $\frac{x}{x - 4} - \frac{1}{x + 4}$

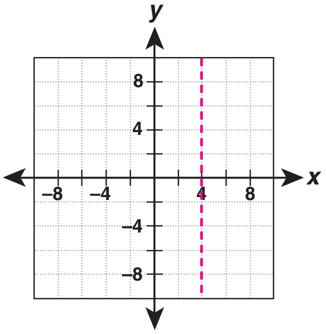
$\frac{x^2 + 3x + 4}{(x-4)(x+4)}$; $x \neq \pm 4$

12. A hot air balloon traveled from Austin, TX to a private island. The balloon averaged 10 mi/h. On the return trip the balloon averaged 12 mi/h. To the nearest mile per hour, what is the balloons average speed for the entire trip?

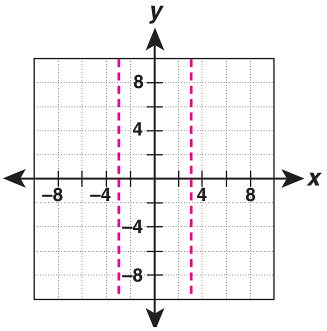
8-4 Rational Functions

Identify the zeros and asymptotes of each function. Then graph.

13. $f(x) = \dfrac{x^2 - 9}{x - 4}$



14. $f(x) = \dfrac{3x}{x^2 - 9}$



8-5 Solving Rational Equations and Inequalities

Solve each equation.

15. $x - \dfrac{24}{x} = -2$

16. $\dfrac{3x + 1}{x - 4} = \dfrac{6x + 5}{2x - 7}$

17. $\dfrac{3}{x - 2} + \dfrac{21}{x^2 - 4} = \dfrac{14}{x + 2}$

18. Marty and Carla Johnson work on refinishing tables. Working alone Carla can complete a table in 7 hours. If the two work together, the job takes 5 hours. How long will it take Mary to refinish the table working alone?

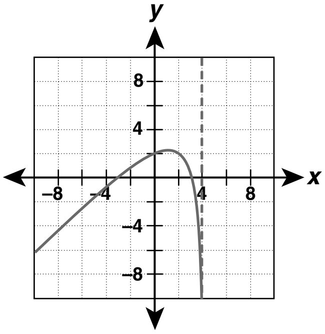
12. A hot air balloon traveled from Austin, TX to a private island. The balloon averaged 10 mi/h. On the return trip the balloon averaged 12 mi/h. To the nearest mile per hour, what is the balloons average speed for the entire trip?

10.9 mi/hr

8-4 Rational Functions

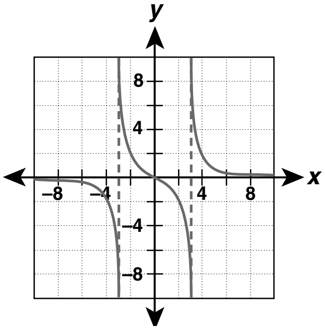
Identify the zeros and asymptotes of each function. Then graph.

13. $f(x) = \dfrac{x^2 - 9}{x - 4}$



zeros: 3, −3;
vertical asymptote: $x = 4$;
horizontal asymptote: none

14. $f(x) = \dfrac{3x}{x^2 - 9}$



zeros: 0;
vertical asymptote: $x = -3$, $x = 3$;
horizontal asymptote: $y = 0$

8-5 Solving Rational Equations and Inequalities

Solve each equation.

15. $x - \dfrac{24}{x} = -2$

−6, 4

16. $\dfrac{3x + 1}{x - 4} = \dfrac{6x + 5}{2x - 7}$

no solution

17. $\dfrac{3}{x - 2} + \dfrac{21}{x^2 - 4} = \dfrac{14}{x + 2}$

5

18. Marty and Carla Johnson work on refinishing tables. Working alone Carla can complete a table in 7 hours. If the two work together, the job takes 5 hours. How long will it take Mary to refinish the table working alone?

$17\frac{1}{2}$ hours

8-6 Radical Expressions and Rational Exponents

Simplify each expression. Assume that all variables are positive.

19. $\sqrt{75x^3}$

20. $\sqrt[3]{27y^{15}z^9}$

21. $\sqrt[4]{\frac{a^8}{8}}$

Write each expression in radical form, and simplify.

22. $36^{\frac{3}{2}}$

23. $27^{\frac{2}{3}}$

24. $(-125)^{\frac{2}{3}}$

Write each expression by using rational exponents.

25. $\sqrt[3]{7^4}$

26. $\left(\sqrt[5]{164}\right)^2$

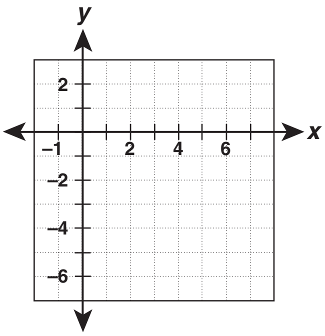
27. $\left(\sqrt[5]{-100}\right)^3$

28. In an experiment involving bacteria growth, the initial population is 250. The growth of the population can be modeled by the function $n(t) = 250 \cdot 2^{\frac{t}{50}}$, where n is the number of bacteria and t is the time in hours. Based on this model, what is the population of bacteria after 2 weeks?

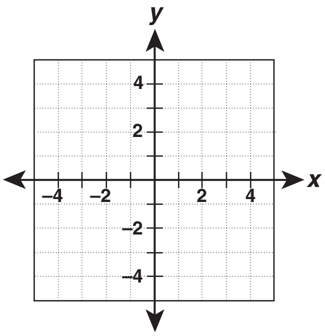
8-7 Radical Functions

Graph each function, and identify its domain and range.

29. $f(x) = -\sqrt{x} - 2$



30. $f(x) = \sqrt[3]{x + 2}$



8-6 Radical Expressions and Rational Exponents

Simplify each expression. Assume that all variables are positive.

19. $\sqrt{75x^3}$

$5x\sqrt{3x}$

20. $\sqrt[3]{27y^{15}z^9}$

$3y^5z^3$

21. $\sqrt[4]{\frac{a^8}{8}}$

$\frac{a^2\sqrt[4]{8^3}}{8}$

Write each expression in radical form, and simplify.

22. $36^{\frac{3}{2}}$

$\sqrt{36^3} = 216$

23. $27^{\frac{2}{3}}$

$\left(\sqrt[3]{27}\right)^2 = 9$

24. $(-125)^{\frac{2}{3}}$

$\left(\sqrt[3]{-125}\right)^2 = 25$

Write each expression by using rational exponents.

25. $\sqrt[3]{7^4}$

$7^{\frac{4}{3}}$

26. $\left(\sqrt[5]{164}\right)^2$

$164^{\frac{2}{5}}$

27. $\left(\sqrt[5]{-100}\right)^3$

$(-100)^{\frac{3}{5}}$

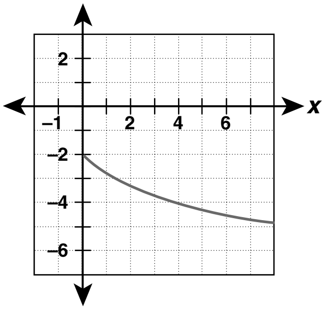
28. In an experiment involving bacteria growth, the initial population is 250. The growth of the population can be modeled by the function $n(t) = 250 \cdot 2^{\frac{t}{50}}$, where n is the number of bacteria and t is the time in hours. Based on this model, what is the population of bacteria after 2 weeks?

$\approx$ 26,355 bacteria

8-7 Radical Functions

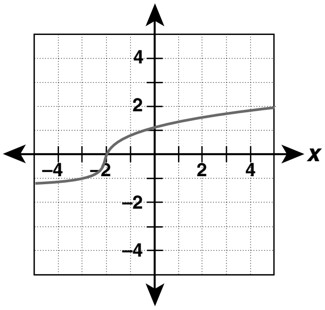
Graph each function, and identify its domain and range.

29. $f(x) = -\sqrt{x} - 2$



D: $\{x|x > 0\}$
R: $\{y|y \leq -2\}$

30. $f(x) = \sqrt[3]{x + 2}$



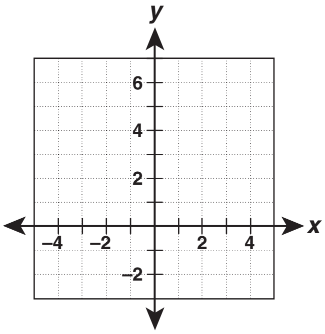
D: all real numbers
R: all real numbers

31. Oil is draining from a tank connected to two pipes. The speed *f* in feet per second at which oil drains through the first pipe can be modeled by $f(x) = \sqrt{36(x - 3)}$, where *x* is the depth of the oil in the tank in feet. The graph of the corresponding function for the second pipe is a translation of *f* 5 units right. Write a corresponding function *g*, and use it to estimate the speed at which oil drains through the second pipe when the depth of the water is 12 ft.

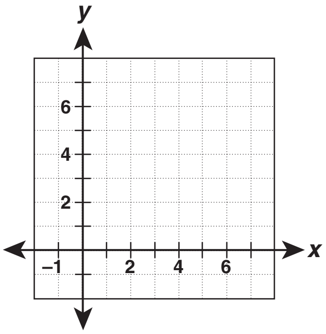
32. Use the description to write the square-root function *g*. The parent function $f(x) = \sqrt{x}$ is vertically stretched by a factor of 2 and then translated 3 units left and 2 units up.

Graph each function, and identify its domain and range.

33. $y > \sqrt{x} + 3$



34. $y \leq \sqrt{x - 4}$



8-8 Solving Radical Equations and Inequalities

Solve each equation.

35. $4\sqrt[3]{x + 1} = -4$

36. $\sqrt{9 - x} = x + 3$

37. $\sqrt[4]{a + 8} = \sqrt[4]{2a}$

31. Oil is draining from a tank connected to two pipes. The speed f in feet per second at which oil drains through the first pipe can be modeled by $f(x) = \sqrt{36(x - 3)}$, where x is the depth of the oil in the tank in feet. The graph of the corresponding function for the second pipe is a translation of f 5 units right. Write a corresponding function g, and use it to estimate the speed at which oil drains through the second pipe when the depth of the water is 12 ft.

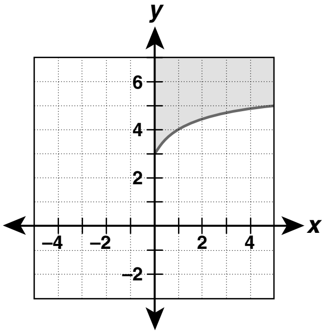
$g(x) = \sqrt{36(x - 8)}$; 12 ft/s

32. Use the description to write the square-root function g. The parent function $f(x) = \sqrt{x}$ is vertically stretched by a factor of 2 and then translated 3 units left and 2 units up.

$g(x) = 2\sqrt{x + 3} + 2$

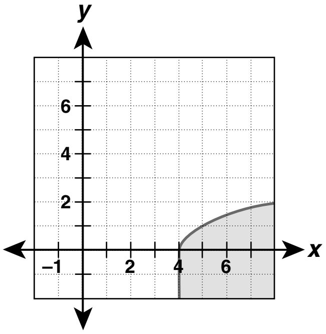
Graph each function, and identify its domain and range.

33. $y > \sqrt{x} + 3$



D: $\{x|x > 0\}$
R: $\{y|y > 3\}$

34. $y \leq \sqrt{x - 4}$



D: $\{x|x > 4\}$
R: $\{y|y \geq 0\}$

8-8 Solving Radical Equations and Inequalities

Solve each equation.

35. $4\sqrt[3]{x + 1} = -4$

-2

36. $\sqrt{9 - x} = x + 3$

0

37. $\sqrt[4]{a + 8} = \sqrt[4]{2a}$

8

38. The formula $d = \sqrt[3]{\frac{4w}{0.02847}}$ relates the average diameter *d* of a cultured pearl in millimeters to its weight *w* in carats. To the nearest tenth of a carat, what is the weight of a cultured pearl with an average diameter of 9 mm?

Solve each inequality.

39. $\sqrt{x + 7} < 5$

40. $\sqrt[3]{3x} \geq -6$

41. $\sqrt{x - 5} - 12 \leq 5$

38. The formula $d = \sqrt[3]{\frac{4w}{0.02847}}$ relates the average diameter d of a cultured pearl in millimeters to its weight w in carats. To the nearest tenth of a carat, what is the weight of a cultured pearl with an average diameter of 9 mm?

5.2 carats

Solve each inequality.

39. $\sqrt{x + 7} < 5$

$-7 \leq x < 18$

40. $\sqrt[3]{3x} \geq -6$

$x \geq -72$

41. $\sqrt{x - 5} - 12 \leq 5$

$5 \leq x \leq 294$

CHAPTER 8

Big Ideas

Answer these questions to summarize the important concepts from Chapter 8 in your own words.

1. Explain how to multiply and divide rational expressions.

2. Explain how to add and subtract rational expressions.

3. Explain how rational exponents and radicals are related.

4. Explain how to solve a radical equation.

For more review of Chapter 8:

- Complete the Chapter 8 Study Guide and Review on pages 638–641 of your textbook.
- Complete the Ready to Go On quizzes on pages 609 and 637 of your textbook.

CHAPTER
8

Big Ideas

Answer these questions to summarize the important concepts from Chapter 8 in your own words.

1. Explain how to multiply and divide rational expressions.

Answers will vary. Possible answer: First factor any expression that needs to be factored. Multiply rational expressions like you multiply fractions, multiplying the numerators and the denominators. You divide rational expressions by multiplying by the reciprocal of the second fraction.

2. Explain how to add and subtract rational expressions.

Answers will vary. Possible answer: Rewrite expressions with a common denominator. Then add and subtract the expressions in the same way as you add or subtract fractions: add or subtract the numerators and keep the denominator the same. Then simplify.

3. Explain how rational exponents and radicals are related.

Answers will vary. Possible answer: The denominator of a fractional exponent is the index of a radical. For example: $a^{\frac{1}{5}} = \sqrt[5]{a}$. If the exponent is a fraction such as $\frac{2}{3}$ it indicates that the denominator is raised to the numerator power. For example: $a^{\frac{2}{3}} = \left(\sqrt[3]{a}\right)^2$.

4. Explain how to solve a radical equation.

Answers will vary. Possible answer: Isolate the radical expression to one side of the equation and raise both sides of the equation to an exponent that will eliminate the radical. For example: if $\sqrt{x + 2} = 10$, then square both sides because the square will eliminate the square root.

For more review of Chapter 8:

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