Name ______________________ Date __________ Class ___________

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities

Solve radical equations by raising both sides of the equation to the power of the index of the radical. For example, the index of $\sqrt[n]{a}$ is n. Therefore,

$$\sqrt{x} = 3$$

$$(\sqrt{x})^2 = 3^2$$

$$x = 9$$

The index of $\sqrt{x}$ is 2. Raise both sides to the power of 2.

Solve: $3\sqrt{x-2} = 18$

Step 1 Isolate the radical.
Divide both sides of the equation by 3 and simplify.

$$\frac{3\sqrt{x-2}}{3} = \frac{18}{3}$$

$$\sqrt{x-2} = 6$$

Step 2 Square both sides of the equation and simplify.

$$(\sqrt{x-2})^2 = 6^2$$

$$x - 2 = 36$$

Remember: $(\sqrt[n]{a})^n = a$.

Step 4 Solve.
$x = 38$

Step 5 Check.

$$3\sqrt{x-2} = 18$$

$$3\sqrt{38-2} = 3\sqrt{36} = 3(6) = 18$$

Always check for extraneous solutions when solving radical equations.

Solve each equation. Check your answer.

1. $4\sqrt[3]{2x+11} = 12$

$$\frac{4\sqrt[3]{2x+11}}{4} = \frac{12}{4}$$

$$\sqrt[3]{2x+11} = 3$$

$$(\sqrt[3]{2x+11})^3 = 3^3$$

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2. $5 + \sqrt{x-3} = 9$

$$5 - 5 + \sqrt{x-3} = 9 - 5$$

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3. $2\sqrt{x+4} = 10$

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Name ________________________ Date ____________ Class ____________

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities (continued)

Solving equations with rational exponents is similar to solving radical equations.

Solve: $x = (x + 20)^{\frac{1}{2}}$.

Step 1 Raise both sides to the reciprocal power.

$x^2 = \left[(x + 20)^{\frac{1}{2}}\right]^2$

> Think: $\left(a^{\frac{1}{n}}\right)^n = a$

> The reciprocal of $\frac{1}{2}$ is 2.

Step 2 Square both sides.

$x^2 = x + 20$

Step 3 Write the quadratic equation in standard form.

$x^2 - x - 20 = 0$

> Set one side of the equation equal to zero.

Step 4 Factor.

$(x + 4)(x - 5) = 0$

Step 5 Solve.

$(x + 4) = 0$ or $(x - 5) = 0$

$x = -4$ $\quad$ $x = 5$

Step 6 Check for extraneous solutions.

$x = (x + 20)^{\frac{1}{2}}$

$x = -4$ $\quad$ $x = 5$

> This is the only solution.

$-4 \ ? \ (-4 + 20)^{\frac{1}{2}}$ $\quad$ $5 \ ? \ (5 + 20)^{\frac{1}{2}}$

$-4 \neq (16)^{\frac{1}{2}}$ ✗ $\quad$ $5 = (25)^{\frac{1}{2}}$ ✓

Solve each equation.

4. $(5x + 6)^{\frac{1}{4}} = 3$

$\left[(5x + 6)^{\frac{1}{4}}\right]^4 = 3^4$

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5. $(6x - 8)^{\frac{1}{3}} = 4$

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6. $x = (x + 6)^{\frac{1}{2}}$

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LESSON 8-8

Practice A

Solving Radical Equations and Inequalities

Rewrite each equation to isolate the radical.

1. $\sqrt{x} - 6 = 0$ **$\sqrt{x} = 6$**
2. $8 + \sqrt{3x} - x = 0$ **$\sqrt{3x} = x - 8$**
3. $\sqrt{2x + 1} - 17 = 3x$ **$\sqrt{2x+1} = 3x + 17$**

Identify to what power each equation must be raised in order to solve. Then solve.

4. $\sqrt{x} = 4$ **2; $x = 16$**
5. $\sqrt[4]{3x} = 12$ **4; $x = \frac{69}{2}$**
6. $\sqrt[3]{x + 1} = 4$ **3; $x = 63$**

Solve the equation. Then identify any extraneous solutions.

7. $2\sqrt{x + 2} = 4$ **$x = 2$; no extraneous solutions**
8. $\sqrt{x + 3} = x - 3$ **$x = 1$, $x = 6$; $x = 1$ is an extraneous solution.**

Solve each equation or inequality.

9. $\sqrt{x + 2} = 5$ **$x = 23$**
10. $(4x)^{\frac{1}{2}} = 6$ **$x = 9$**
11. $(x + 1)^{\frac{1}{3}} = 3$ **$x = 26$**
12. $2\sqrt{x - 3} = 10$ **$x = 28$**
13. $\sqrt{2x} - 6 < 0$ **$0 \le x < 18$**
14. $\sqrt{3x + 1} \ge 8$ **$x \ge 21$**

Solve.

15. Ainsley and Ben each solve the inequality $\sqrt{x + 3} + 5 \le 10$. Ainsley's solution is $x \le 22$. Ben's solution is $-3 \le x \le 22$. Why are their solutions different? Which is correct?

Ben's solution is correct. Ainsley forgot that the radicand cannot be negative.

LESSON 8-8

Practice B

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x + 6} = 7$ **$x = 43$**
2. $\sqrt{5x} = 10$ **$x = 20$**
3. $\sqrt{2x + 5} = \sqrt{3x - 1}$ **$x = 6$**
4. $\sqrt{x + 4} = 3\sqrt{x}$ **$x = \frac{1}{2}$**
5. $\sqrt[3]{x - 6} = \sqrt[3]{3x + 24}$ **$x = -15$**
6. $3\sqrt[3]{x} = \sqrt[3]{7x + 5}$ **$x = \frac{1}{4}$**
7. $\sqrt{-14x + 2} = x - 3$ **No solutions, since both -1 and -7 are extraneous**
8. $(x + 4)^{\frac{1}{2}} = 6$ **$x = 32$**
9. $4(x - 3)^{\frac{1}{2}} = 8$ **$x = 7$**
10. $4(x - 12)^{\frac{1}{3}} = -16$ **$x = -52$**

Solve each inequality.

11. $\sqrt{3x + 6} \le 3$ **$-2 \le x \le 1$**
12. $\sqrt{x - 4} + 3 > 9$ **$x > 40$**
13. $\sqrt{x + 7} \ge \sqrt{2x - 1}$ **$\frac{1}{2} \le x \le 8$**
14. $\sqrt{2x - 7} > 9$ **$x > 44$**

Solve.

15. A biologist is studying two species of animals in a habitat. The population, p_1, of one of the species is growing according to $p_1 = 500t^{\frac{3}{2}}$ and the population, p_2, of the other species is growing according to $p_2 = 100t^2$ where time, t, is measured in years. After how many years will the populations of the two species be equal?

25 years

LESSON 8-8

Practice C

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt[3]{4x + 1} - 5 = 0$ **$x = 31$**
2. $3\sqrt{x - 11} = 18$ **$x = 47$**
3. $\sqrt[4]{10x + 11} = 3$ **$x = 7$**
4. $\sqrt[3]{3x} = \sqrt[3]{2x + 9}$ **$x = 9$**
5. $x + 2 = \sqrt{3x + 6}$ **$x = -2$ and $x = 1$**
6. $(10x - 25)^{\frac{1}{2}} = x$ **$x = 5$**
7. $5(6x + 1)^{\frac{1}{4}} = 10$ **$x = \frac{5}{2}$**
8. $4(7x + 18)^{\frac{1}{2}} = 4x$ **$x = 9$; $x = -2$ is an extraneous solution.**

Solve each inequality.

9. $\sqrt{4x + 5} \le 3$ **$-\frac{5}{4} \le x \le 1$**
10. $\sqrt[3]{x + 3} \ge 2$ **$x \ge 5$**
11. $\sqrt{x - 7} + 9 < 12$ **$7 \le x < 16$**
12. $\sqrt[3]{x - 6} + 7 > 4$ **$x > -21$**
13. $\sqrt{3x - 1} > \sqrt{x + 7}$ **$x > 4$**
14. $\sqrt[3]{x + 2} - 1 \le 4$ **$-2 \le x \le 123$**

Solve.

15. Einstein's theory of relativity states that the mass of an object increases as the object's velocity increases. The mass, $m(v)$, of an object traveling with velocity, v, is given by $m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is the speed of light and m_0 is the mass of the object at rest. In terms of c, solve for the velocity at which the effective mass, $m(v)$, of the particle has increased to twice its mass at rest, m_0.

$v = \frac{\sqrt{3}}{2}c$

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities

Solve radical equations by raising both sides of the equation to the power of the index of the radical. For example, the index of $\sqrt[n]{a}$ is n. Therefore,

$\sqrt{x} = 3$

$(\sqrt{x})^2 = 3^2$ — The index of $\sqrt{x}$ is 2. Raise both sides to the power of 2.

$x = 9$

Solve: $3\sqrt{x - 2} = 18$

Step 1 Isolate the radical.
Divide both sides of the equation by 3 and simplify.

$\frac{3\sqrt{x-2}}{3} = \frac{18}{3}$

$\sqrt{x - 2} = 6$

Step 2 Square both sides of the equation and simplify.

$(\sqrt{x - 2})^2 = 6^2$ — Remember: $(\sqrt[n]{a})^n = a$.

$x - 2 = 36$

Step 4 Solve.

$x = 38$

Step 5 Check.

$3\sqrt{x - 2} = 18$ — Always check for extraneous solutions when solving radical equations.

$3\sqrt{38 - 2} = 3\sqrt{36} = 3(6) = 18$

Solve each equation. Check your answer.

1. $4\sqrt[3]{2x + 11} = 12$

$\frac{4\sqrt[3]{2x+11}}{4} = \frac{12}{4}$

$\sqrt[3]{2x + 11} = 3$

$(\sqrt[3]{2x + 11})^3 = 3^3$

$2x + 11 = 27$

$2x = 16$; $x = 8$

$4\sqrt[3]{2(8) + 11} = 12$

$4\sqrt[3]{36} = 12$ ✓

2. $5 + \sqrt{x - 3} = 9$

$5 - 5 + \sqrt{x - 3} = 9 - 5$

$\sqrt{x - 3} = 4$

$x - 3 = 16$

$x = 19$

$5 + \sqrt{19 - 3} = 5$

$+\sqrt{16} = 5 + 4$

$= 9$ ✓

3. $2\sqrt{x + 4} = 10$

$\sqrt{x + 4} = 5$

$x + 4 = 25$

$x = 21$

$2\sqrt{21 + 4} =$

$2\sqrt{25} = 2 \cdot 5$

$= 10$ ✓

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities (continued)

Solving equations with rational exponents is similar to solving radical equations.

Solve: $x = (x + 20)^{\frac{1}{2}}$.

Step 1 Raise both sides to the reciprocal power.
$x^2 = \left[(x + 20)^{\frac{1}{2}}\right]^2$

Think: $\left(a^{\frac{1}{n}}\right)^n = a$

The reciprocal of $\frac{1}{2}$ is 2.

Step 2 Square both sides.
$x^2 = x + 20$

Step 3 Write the quadratic equation in standard form.
$x^2 - x - 20 = 0$

Set one side of the equation equal to zero.

Step 4 Factor.
$(x + 4)(x - 5) = 0$

Step 5 Solve.
$(x + 4) = 0$ or $(x - 5) = 0$
$x = -4$ $\quad$ $x = 5$

Step 6 Check for extraneous solutions.
$x = (x + 20)^{\frac{1}{2}}$
$x = -4$ $\quad$ $x = 5$

This is the only solution.

$-4 \,?\, (-4 + 20)^{\frac{1}{2}}$ $\quad$ $5 \,?\, (5 + 20)^{\frac{1}{2}}$
$-4 \neq (16)^{\frac{1}{2}}$ ✗ $\quad$ $5 = (25)^{\frac{1}{2}}$ ✓

Solve each equation.

4. $(5x + 6)^{\frac{1}{4}} = 3$
$\left[(5x + 6)^{\frac{1}{4}}\right]^4 = 3^4$
$5x + 6 = 81$
$5x = 75$
$x = 15$

5. $(6x - 8)^{\frac{1}{3}} = 4$
$\left[(6x - 8)^{\frac{1}{3}}\right]^3 = 4^3$
$6x - 8 = 64$
$6x = 72$
$x = 12$

6. $x = (x + 6)^{\frac{1}{2}}$
$x^2 = \left[(x + 6)^{\frac{1}{2}}\right]^2$
$x^2 = x + 6$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3$

LESSON 8-8

Challenge

Multiple Radicals

Equations may involve more than one radical. In that case, the solution process is repeated to eliminate multiple radicals. For example:

$$\sqrt{x} + \sqrt{x - 5} = 5$$

To solve, isolate one radical and square both sides as shown below.

$$\sqrt{x} = 5 - \sqrt{x - 5}$$
$$(\sqrt{x})^2 = (5 - \sqrt{x - 5})^2$$
$$x = 25 - 10\sqrt{x - 5} + (x - 5)$$

Notice that now there is only one radical in the equation. Repeat the process, isolate the radical, square, and solve.

$$x = 25 - 10\sqrt{x - 5} + (x - 5)$$
$$-20 = -10\sqrt{x - 5}$$
$$2 = \sqrt{x - 5}$$
$$2^2 = \left(\sqrt{x - 5}\right)^2$$
$$4 = x - 5$$
$$9 = x$$

Solve each equation. Check each answer to ensure that it does not include extraneous solutions.

1. $\sqrt{x - 3} = \sqrt{x + 15} - 2$ — 15.25
2. $\sqrt{x + 16} = x - \sqrt{x + 7}$ — 9
3. $\sqrt{x - 3} - \sqrt{x - 2} = 1$ — No solution
4. $\sqrt{\sqrt{x - 3}} = \sqrt{x - 15}$ — 19
5. $\sqrt{x - 3} = \dfrac{2}{\sqrt{x - 3}}$ — 5
6. $\sqrt{x^2 - 7x + 12} - x = x - 6$ — 3
7. $\sqrt{\sqrt{3x + 1}} = \sqrt{\sqrt{\sqrt{50x + 6}}}$ — 5 or $-\frac{1}{9}$
8. $\sqrt[3]{x - 7} = \sqrt[3]{x} - 1$ — 8 or -1
9. $\sqrt{x + 2} = 1 + \sqrt{x - 3}$ — 7
10. $\sqrt[3]{x + 2} = \sqrt[3]{\frac{x}{2} + 5}$ — 6

LESSON 8-8

Problem Solving

Solving Radical Equations and Inequalities

The formula $s = \sqrt{30fd}$ can be used to estimate the speed, *s*, in miles per hour that a car is traveling when it goes into a skid, where *f* is the coefficient of friction and *d* is the length of the skid marks in feet.

1. How does the speed vary as the length of the skid marks? Directly

2. Kody skids to a stop on a street with a speed limit of 35 mi/h. His skid marks measure 52 ft, and the coefficient of friction is 0.7. Kody says that he was driving only about 30 mi/h. Kody wants to prove that he was not speeding.
   - **a.** Solve the equation for *d* in terms of *s*. $d = \dfrac{s^2}{30f}$
   - **b.** How long would the skid marks be if he had been driving at a speed of 35 mi/h? About 58 ft
   - **c.** Was Kody speeding or not? Explain how you know. No; possible answer: his skid marks were only 52 ft, not 58 ft.
   - **d.** Find his actual speed. About 33 mi/h

3. Ashley skids to a stop on a street with a speed limit of 15 mi/h to avoid a dog who runs into the street about 20 ft ahead of her. Ashley claims to have been going less than 15 mi/h. The coefficient of friction is 0.7.
   - **a.** If Ashley were driving the speed limit, by what distance would she have missed the dog? About 9 ft
   - **b.** If Ashley were driving less than 10 mi/h, by what distance would she have missed the dog? By at least 15 ft

Choose the letter for the best answer.

4. Barney was driving at 25 mi/h. A car pulls out 30 ft ahead of him. Which statement is true?
   - **A** Barney hits the car.
   - **(B)** Barney stops less than a foot from the car.
   - **C** Barney misses the car by 3 ft.
   - **D** Barney's skid marks measure 23 ft.

5. On a busy highway with a speed limit of 70 mi/h, a truck ahead of Verna jack-knifes across the road. Verna skids to a stop 10 ft short of the truck. Her skid marks measure 260 ft. Was Verna speeding?
   - **(A)** Yes; her speed was 73.9 mi/h.
   - **B** Yes; her speed was 75.3 mi/h.
   - **C** No; her speed was 70 mi/h.
   - **D** No; her speed was only 63 mi/h.

LESSON 8-8

Reading Strategy

Use Vocabulary

You can solve radical equations in a three-step process. For example, to solve the equation $\sqrt{x} - 4 = 0$, follow the steps below.

STEP 1: Isolate the radical
$\sqrt{x} = 4$

→ **STEP 2:** Raise both sides to the appropriate power.
$(\sqrt{x})^2 = (4)^2$

→ **STEP 3:** Simplify and solve.
$x = 16$

To **isolate** the radical means to have only the radical on one side of the equation and all other variables and constants on the other side of the equation.

To understand what it means to **raise** a number to a power, think about how the exponent is located slightly higher than the base.

Isolate the radical in each equation.

1. $\sqrt{x} + 3 = 0$ — $\sqrt{x} = -3$
2. $\sqrt{x + 2} - 6 = 0$ — $\sqrt{x + 2} = 6$
3. $4\sqrt{x} + 8 = 0$ — $\sqrt{x} = -2$
4. $\frac{1}{2}\sqrt{x} - 9 = 0$ — $\sqrt{x} = 18$
5. $-2\sqrt{x + 6} = -4$ — $\sqrt{x + 6} = 2$

To what power should both sides of each equation be raised?

6. $\sqrt[3]{x} = 2$ — Third power
7. $\sqrt{x + 2} = 1$ — Second Power
8. $\sqrt[4]{x - 3} = 3$ — Fourth power
9. $\sqrt[3]{x} = -4$ — Third power

Solve the following equations.

10. $\sqrt{x} - 7 = 1$ — $x = 64$
11. $\sqrt{x + 2} = 3$ — $x = 7$