Name ______________________ Date __________ Class __________

LESSON 8-8

Practice C
Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt[3]{4x + 1} - 5 = 0$

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2. $3\sqrt{x - 11} = 18$

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3. $\sqrt[4]{10x + 11} = 3$

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4. $\sqrt[3]{3x} = \sqrt[3]{2x + 9}$

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5. $x + 2 = \sqrt{3x + 6}$

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6. $(10x - 25)^{\frac{1}{2}} = x$

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7. $5(6x + 1)^{\frac{1}{4}} = 10$

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8. $4(7x + 18)^{\frac{1}{2}} = 4x$

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Solve each inequality.

9. $\sqrt{4x + 5} \leq 3$

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10. $\sqrt[3]{x + 3} \geq 2$

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11. $\sqrt{x - 7} + 9 < 12$

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12. $\sqrt[3]{x - 6} + 7 > 4$

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13. $\sqrt{3x - 1} > \sqrt{x + 7}$

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14. $\sqrt[3]{x + 2} - 1 \leq 4$

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Solve.

15. Einstein's theory of relativity states that the mass of an object increases as the object's velocity increases. The mass, $m(v)$, of an object traveling with velocity, v, is given by $m(v) = \dfrac{m_0}{\sqrt{1 - \dfrac{v^2}{c^2}}}$, where c is the speed of light and m_0 is the mass of the object at rest. In terms of c, solve for the velocity at which the effective mass, $m(v)$, of the particle has increased to twice its mass at rest, m_0.

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LESSON 8-8

Practice A

Solving Radical Equations and Inequalities

Rewrite each equation to isolate the radical.

1. $\sqrt{x} - 6 = 0$ — $\sqrt{x} = 6$
2. $8 + \sqrt{3x} - x = 0$ — $\sqrt{3x} = x - 8$
3. $\sqrt{2x+1} - 17 = 3x$ — $\sqrt{2x+1} = 3x + 17$

Identify to what power each equation must be raised in order to solve. Then solve.

4. $\sqrt{x} = 4$ — 2; $x = 16$
5. $\sqrt[4]{3x} = 12$ — 4; $x = \frac{69}{2}$
6. $\sqrt[3]{x+1} = 4$ — 3; $x = 63$

Solve the equation. Then identify any extraneous solutions.

7. $2\sqrt{x+2} = 4$ — $x = 2$; no extraneous solutions
8. $\sqrt{x+3} = x - 3$ — $x = 1$, $x = 6$; $x = 1$ is an extraneous solution.

Solve each equation or inequality.

9. $\sqrt{x+2} = 5$ — $x = 23$
10. $(4x)^{\frac{1}{2}} = 6$ — $x = 9$
11. $(x+1)^{\frac{1}{3}} = 3$ — $x = 26$
12. $2\sqrt{x-3} = 10$ — $x = 28$
13. $\sqrt{2x} - 6 < 0$ — $0 \le x < 18$
14. $\sqrt{3x+1} \ge 8$ — $x \ge 21$

Solve.

15. Ainsley and Ben each solve the inequality $\sqrt{x+3} + 5 \le 10$. Ainsley's solution is $x \le 22$. Ben's solution is $-3 \le x \le 22$. Why are their solutions different? Which is correct?

Ben's solution is correct. Ainsley forgot that the radicand cannot be negative.

LESSON 8-8

Practice B

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x+6} = 7$ — $x = 43$
2. $\sqrt{5x} = 10$ — $x = 20$
3. $\sqrt{2x+5} = \sqrt{3x-1}$ — $x = 6$
4. $\sqrt{x+4} = 3\sqrt{x}$ — $x = \frac{1}{2}$
5. $\sqrt[3]{x-6} = \sqrt[3]{3x+24}$ — $x = -15$
6. $3\sqrt[3]{x} = \sqrt[3]{7x+5}$ — $x = \frac{1}{4}$
7. $\sqrt{-14x+2} = x - 3$ — No solutions, since both -1 and -7 are extraneous
8. $(x+4)^{\frac{1}{2}} = 6$ — $x = 32$
9. $4(x-3)^{\frac{1}{2}} = 8$ — $x = 7$
10. $4(x-12)^{\frac{1}{3}} = -16$ — $x = -52$

Solve each inequality.

11. $\sqrt{3x+6} \le 3$ — $-2 \le x \le 1$
12. $\sqrt{x-4} + 3 > 9$ — $x > 40$
13. $\sqrt{x+7} \ge \sqrt{2x-1}$ — $\frac{1}{2} \le x \le 8$
14. $\sqrt{2x-7} > 9$ — $x > 44$

Solve.

15. A biologist is studying two species of animals in a habitat. The population, p_1, of one of the species is growing according to $p_1 = 500t^{\frac{3}{2}}$ and the population, p_2, of the other species is growing according to $p_2 = 100t^2$ where time, t, is measured in years. After how many years will the populations of the two species be equal?

25 years

LESSON 8-8

Practice C

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt[3]{4x+1} - 5 = 0$ — $x = 31$
2. $3\sqrt{x-11} = 18$ — $x = 47$
3. $\sqrt[4]{10x+11} = 3$ — $x = 7$
4. $\sqrt[3]{3x} = \sqrt[3]{2x+9}$ — $x = 9$
5. $x + 2 = \sqrt{3x+6}$ — $x = -2$ and $x = 1$
6. $(10x - 25)^{\frac{1}{2}} = x$ — $x = 5$
7. $5(6x+1)^{\frac{1}{4}} = 10$ — $x = \frac{5}{2}$
8. $4(7x+18)^{\frac{1}{2}} = 4x$ — $x = 9$; $x = -2$ is an extraneous solution.

Solve each inequality.

9. $\sqrt{4x+5} \le 3$ — $-\frac{5}{4} \le x \le 1$
10. $\sqrt[3]{x+3} \ge 2$ — $x \ge 5$
11. $\sqrt{x-7} + 9 < 12$ — $7 \le x < 16$
12. $\sqrt[3]{x-6} + 7 > 4$ — $x > -21$
13. $\sqrt{3x-1} > \sqrt{x+7}$ — $x > 4$
14. $\sqrt[3]{x+2} - 1 \le 4$ — $-2 \le x \le 123$

Solve.

15. Einstein's theory of relativity states that the mass of an object increases as the object's velocity increases. The mass, $m(v)$, of an object traveling with velocity, v, is given by $m(v) = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is the speed of light and m_0 is the mass of the object at rest. In terms of c, solve for the velocity at which the effective mass, $m(v)$, of the particle has increased to twice its mass at rest, m_0.

$v = \frac{\sqrt{3}}{2}c$

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities

Solve radical equations by raising both sides of the equation to the power of the index of the radical. For example, the index of $\sqrt[n]{a}$ is n. Therefore,

$\sqrt{x} = 3$

$(\sqrt{x})^2 = 3^2$

$x = 9$

The index of $\sqrt{x}$ is 2. Raise both sides to the power of 2.

Solve: $3\sqrt{x-2} = 18$

Step 1 Isolate the radical.
Divide both sides of the equation by 3 and simplify.

$\frac{3\sqrt{x-2}}{3} = \frac{18}{3}$

$\sqrt{x-2} = 6$

Step 2 Square both sides of the equation and simplify.

$(\sqrt{x-2})^2 = 6^2$

$x - 2 = 36$

Remember: $(\sqrt[n]{a})^n = a$.

Step 4 Solve.

$x = 38$

Step 5 Check.

$3\sqrt{x-2} = 18$

$3\sqrt{38-2} = 3\sqrt{36} = 3(6) = 18$

Always check for extraneous solutions when solving radical equations.

Solve each equation. Check your answer.

1. $4\sqrt[3]{2x+11} = 12$

   $\frac{4\sqrt[3]{2x+11}}{4} = \frac{12}{4}$

   $\sqrt[3]{2x+11} = 3$

   $(\sqrt[3]{2x+11})^3 = 3^3$

   $2x + 11 = 27$

   $2x = 16$; $x = 8$

   $4\sqrt[3]{2(8)+11} = 12$

   $4\sqrt[3]{36} = 12$ ✓

2. $5 + \sqrt{x-3} = 9$

   $5 - 5 + \sqrt{x-3} = 9 - 5$

   $\sqrt{x-3} = 4$

   $x - 3 = 16$

   $x = 19$

   $5 + \sqrt{19-3} = 5$

   $+\sqrt{16} = 5 + 4$

   $= 9$ ✓

3. $2\sqrt{x+4} = 10$

   $\sqrt{x+4} = 5$

   $x + 4 = 25$

   $x = 21$

   $2\sqrt{21+4} =$

   $2\sqrt{25} = 2 \cdot 5$

   $= 10$ ✓