Name ______________________ Date ____________ Class ____________

LESSON 8-8

Practice A

Solving Radical Equations and Inequalities

Rewrite each equation to isolate the radical.

1. $\sqrt{x} - 6 = 0$ **2.** $8 + \sqrt{3x} - x = 0$ **3.** $\sqrt{2x + 1} - 17 = 3x$

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Identify to what power each equation must be raised in order to solve. Then solve.

4. $\sqrt{x} = 4$ **5.** $\sqrt[4]{3x} = 12$ **6.** $\sqrt[3]{x + 1} = 4$

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Solve the equation. Then identify any extraneous solutions.

7. $2\sqrt{x + 2} = 4$ **8.** $\sqrt{x + 3} = x - 3$

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Solve each equation or inequality.

9. $\sqrt{x + 2} = 5$ **10.** $(4x)^{\frac{1}{2}} = 6$

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11. $(x + 1)^{\frac{1}{3}} = 3$ **12.** $2\sqrt{x - 3} = 10$

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13. $\sqrt{2x} - 6 < 0$ **14.** $\sqrt{3x + 1} \geq 8$

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Solve.

15. Ainsley and Ben each solve the inequality $\sqrt{x + 3} + 5 \leq 10$. Ainsley's solution is $x \leq 22$. Ben's solution is $-3 \leq x \leq 22$. Why are their solutions different? Which is correct?

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LESSON 8-8

Practice A

Solving Radical Equations and Inequalities

Rewrite each equation to isolate the radical.

1. $\sqrt{x} - 6 = 0$ **$\sqrt{x} = 6$**
2. $8 + \sqrt{3x} - x = 0$ **$\sqrt{3x} = x - 8$**
3. $\sqrt{2x + 1} - 17 = 3x$ **$\sqrt{2x+1} = 3x + 17$**

Identify to what power each equation must be raised in order to solve. Then solve.

4. $\sqrt{x} = 4$ **2; $x = 16$**
5. $\sqrt[4]{3x} = 12$ **4; $x = \frac{69}{2}$**
6. $\sqrt[3]{x + 1} = 4$ **3; $x = 63$**

Solve the equation. Then identify any extraneous solutions.

7. $2\sqrt{x + 2} = 4$ **$x = 2$; no extraneous solutions**
8. $\sqrt{x + 3} = x - 3$ **$x = 1$, $x = 6$; $x = 1$ is an extraneous solution.**

Solve each equation or inequality.

9. $\sqrt{x + 2} = 5$ **$x = 23$**
10. $(4x)^{\frac{1}{2}} = 6$ **$x = 9$**
11. $(x + 1)^{\frac{1}{3}} = 3$ **$x = 26$**
12. $2\sqrt{x - 3} = 10$ **$x = 28$**
13. $\sqrt{2x} - 6 < 0$ **$0 \le x < 18$**
14. $\sqrt{3x + 1} \ge 8$ **$x \ge 21$**

Solve.

15. Ainsley and Ben each solve the inequality $\sqrt{x + 3} + 5 \le 10$. Ainsley's solution is $x \le 22$. Ben's solution is $-3 \le x \le 22$. Why are their solutions different? Which is correct?

Ben's solution is correct. Ainsley forgot that the radicand cannot be negative.

LESSON 8-8

Practice B

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt{x + 6} = 7$ **$x = 43$**
2. $\sqrt{5x} = 10$ **$x = 20$**
3. $\sqrt{2x + 5} = \sqrt{3x - 1}$ **$x = 6$**
4. $\sqrt{x + 4} = 3\sqrt{x}$ **$x = \frac{1}{2}$**
5. $\sqrt[3]{x - 6} = \sqrt[3]{3x + 24}$ **$x = -15$**
6. $3\sqrt[3]{x} = \sqrt[3]{7x + 5}$ **$x = \frac{1}{4}$**
7. $\sqrt{-14x + 2} = x - 3$ **No solutions, since both -1 and -7 are extraneous**
8. $(x + 4)^{\frac{1}{2}} = 6$ **$x = 32$**
9. $4(x - 3)^{\frac{1}{2}} = 8$ **$x = 7$**
10. $4(x - 12)^{\frac{1}{3}} = -16$ **$x = -52$**

Solve each inequality.

11. $\sqrt{3x + 6} \le 3$ **$-2 \le x \le 1$**
12. $\sqrt{x - 4} + 3 > 9$ **$x > 40$**
13. $\sqrt{x + 7} \ge \sqrt{2x - 1}$ **$\frac{1}{2} \le x \le 8$**
14. $\sqrt{2x - 7} > 9$ **$x > 44$**

Solve.

15. A biologist is studying two species of animals in a habitat. The population, p_1, of one of the species is growing according to $p_1 = 500t^{\frac{3}{2}}$ and the population, p_2, of the other species is growing according to $p_2 = 100t^2$ where time, t, is measured in years. After how many years will the populations of the two species be equal?

25 years

LESSON 8-8

Practice C

Solving Radical Equations and Inequalities

Solve each equation.

1. $\sqrt[3]{4x + 1} - 5 = 0$ **$x = 31$**
2. $3\sqrt{x - 11} = 18$ **$x = 47$**
3. $\sqrt[4]{10x + 11} = 3$ **$x = 7$**
4. $\sqrt[3]{3x} = \sqrt[3]{2x + 9}$ **$x = 9$**
5. $x + 2 = \sqrt{3x + 6}$ **$x = -2$ and $x = 1$**
6. $(10x - 25)^{\frac{1}{2}} = x$ **$x = 5$**
7. $5(6x + 1)^{\frac{1}{4}} = 10$ **$x = \frac{5}{2}$**
8. $4(7x + 18)^{\frac{1}{2}} = 4x$ **$x = 9$; $x = -2$ is an extraneous solution.**

Solve each inequality.

9. $\sqrt{4x + 5} \le 3$ **$-\frac{5}{4} \le x \le 1$**
10. $\sqrt[3]{x + 3} \ge 2$ **$x \ge 5$**
11. $\sqrt{x - 7} + 9 < 12$ **$7 \le x < 16$**
12. $\sqrt[3]{x - 6} + 7 > 4$ **$x > -21$**
13. $\sqrt{3x - 1} > \sqrt{x + 7}$ **$x > 4$**
14. $\sqrt[3]{x + 2} - 1 \le 4$ **$-2 \le x \le 123$**

Solve.

15. Einstein's theory of relativity states that the mass of an object increases as the object's velocity increases. The mass, $m(v)$, of an object traveling with velocity, v, is given by $m(v) = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$, where c is the speed of light and m_0 is the mass of the object at rest. In terms of c, solve for the velocity at which the effective mass, $m(v)$, of the particle has increased to twice its mass at rest, m_0.

$v = \frac{\sqrt{3}}{2}c$

LESSON 8-8

Reteach

Solving Radical Equations and Inequalities

Solve radical equations by raising both sides of the equation to the power of the index of the radical. For example, the index of $\sqrt[n]{a}$ is n. Therefore,

$\sqrt{x} = 3$

$(\sqrt{x})^2 = 3^2$ — The index of $\sqrt{x}$ is 2. Raise both sides to the power of 2.

$x = 9$

Solve: $3\sqrt{x - 2} = 18$

Step 1 Isolate the radical.
Divide both sides of the equation by 3 and simplify.

$\frac{3\sqrt{x - 2}}{3} = \frac{18}{3}$

$\sqrt{x - 2} = 6$

Step 2 Square both sides of the equation and simplify.

$(\sqrt{x - 2})^2 = 6^2$ — Remember: $(\sqrt[n]{a})^n = a$.

$x - 2 = 36$

Step 4 Solve.

$x = 38$

Step 5 Check.

$3\sqrt{x - 2} = 18$ — Always check for extraneous solutions when solving radical equations.

$3\sqrt{38 - 2} = 3\sqrt{36} = 3(6) = 18$

Solve each equation. Check your answer.

1. $4\sqrt[3]{2x + 11} = 12$

   $\frac{4\sqrt[3]{2x + 11}}{4} = \frac{12}{4}$

   $\sqrt[3]{2x + 11} = 3$

   $(\sqrt[3]{2x + 11})^3 = 3^3$

   $2x + 11 = 27$

   $2x = 16$; $x = 8$

   $4\sqrt[3]{2(8) + 11} = 12$

   $4\sqrt[3]{36} = 12$ ✓

2. $5 + \sqrt{x - 3} = 9$

   $5 - 5 + \sqrt{x - 3} = 9 - 5$

   $\sqrt{x - 3} = 4$

   $x - 3 = 16$

   $x = 19$

   $5 + \sqrt{19 - 3} = 5$

   $+\sqrt{16} = 5 + 4$

   $= 9$ ✓

3. $2\sqrt{x + 4} = 10$

   $\sqrt{x + 4} = 5$

   $x + 4 = 25$

   $x = 21$

   $2\sqrt{21 + 4} =$

   $2\sqrt{25} = 2 \cdot 5$

   $= 10$ ✓