8-8 Solving Radical Equations and Inequalities

Example 1 Solving Equations Containing One Radical

Solve each equation.

A. $5 + \sqrt{x+1} = 16$

$\sqrt{x+1} = 16 - 5$ *Subtract 5.*

$\sqrt{x+1} = 11$ *Simplify.*

$(\sqrt{x+1})^2 = (11)^2$

  square both sides.

$x + 1 = 121$ *Simplify Solve for x.*

$x = 120$

B. $7\sqrt[3]{5x-7} = 84$

$\dfrac{7\sqrt[3]{5x-7}}{7} = \dfrac{84}{7}$ *Divide by 7.*

$\sqrt[3]{5x-7} = 12$ *Simplify.*

$(\sqrt[3]{5x-7}{}^3) = (12)^3$ *Cube both sides.*

$5x - 7 = 1728$ *Simplify.*

$5x = 1735$ *Solve for x.*

$x = 347$

Check $\quad 5 + \sqrt{x+1} = 16$

$5 + \sqrt{120 + 1}$	16
$5 + \sqrt{121}$	16
16	16 ✔

Check $\quad 7\sqrt[3]{5x-7} = 84$

$7\sqrt[3]{5(347)-7}$	84
$7\sqrt[3]{1728}$	84
84	84 ✔

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Example 2 Solving Equations Containing Two Radicals

Solve $\sqrt{7x + 2} = 3\sqrt{3x - 2}$.

$(\sqrt{7x + 2})^2 = (3\sqrt{3x - 2})^2$ *Square both sides.*

$\quad 7x + 2 = 9(3x - 2)$ *Simplify.*

$\quad 7x + 2 = 27x - 18$ *Distribute 9.*

$\quad 20 = 20x$ *Solve for x.*

$\quad 1 = x$

Check $\quad \sqrt{7x + 2} = 3\sqrt{3x - 2}$

$$\begin{array}{c|c} \sqrt{7(1) + 2} & 3\sqrt{3(1) - 2} \\ 3 & 3 \checkmark \end{array}$$

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Example 3 Solving Equations with Extraneous Solutions

Solve $\sqrt{-3x + 33} = 5 - x$.

Method 1 Use a graphing calculator.
Let **Y1** $= \sqrt{-3x + 33}$ and **Y2** $= 5 - x$.

The graphs intersect in only one point, so there is exactly one solution.

The solution is $x = -1$.

Method 2 Use algebra to solve the equation.

Step 1 Solve for x.

$$\sqrt{-3x + 33} = 5 - x$$

$$(\sqrt{-3x + 33})^2 = (5 - x)^2 \qquad \text{\textit{Square both sides.}}$$

$$-3x + 33 = 25 - 10x + x^2 \qquad \text{\textit{Simplify.}}$$

$$0 = x^2 - 7x - 8 \qquad \text{\textit{Write in standard form.}}$$

$$0 = (x - 8)(x + 1) \qquad \text{\textit{Factor.}}$$

$$x - 8 = 0 \text{ or } x + 1 = 0 \qquad \text{\textit{Solve for x.}}$$
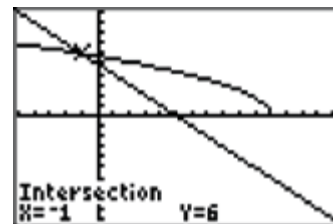
$$x = 8 \text{ or } x = -1$$

Step 2 Use substitution to check for extraneous solutions.

$\sqrt{-3x + 33} = 5 - x$	
$\sqrt{-3(8) + 33}$	$5 - 8$
3	-3 ✘

$\sqrt{-3x + 33} = 5 - x$	
$\sqrt{-3(-1) + 33}$	$5 - (-1)$
6	6 ✔

Because $x = 8$ is extraneous, the only solution is $x = -1$.

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Example 4 Solving Equations with Rational Exponents

Solve each equation.

A. $(5x + 7)^{\frac{1}{3}} = 3$

$\sqrt[3]{5x + 7} = 3$ *Write in radical form.*

$(\sqrt[3]{5x + 7})^3 = (3)^3$ *Cube both sides.*

$5x + 7 = 27$ *Simplify.*

$5x = 20$ *Solve for x.*

$x = 4$

B. $2x = (4x + 8)^{\frac{1}{2}}$

Step 1 Solve for x.

$(2x)^2 = \left[(4x + 8)^{\frac{1}{2}}\right]^2$ *Raise both sides to the reciprocal power.*

$4x^2 = 4x + 8$ *Simplify.*

$4x^2 - 4x - 8 = 0$ *Write in standard form.*

$4(x^2 - x - 2) = 0$ *Factor out the GCF, 4.*

$4(x - 2)(x + 1) = 0$ *Factor.*

$4 \neq 0, x - 2 = 0 \text{ or } x + 1 = 0$ *Solve for x.*

$x = 2 \text{ or } x = -1$

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8-8 Solving Radical Equations and Inequalities

Example 4 Solving Equations with Rational Exponents (continued)

Step 2 Use substitution to check for extraneous solutions.

$$2x = (4x + 8)^{\frac{1}{2}}$$

$2(2)$	$(4(2) + 8)^{\frac{1}{2}}$
4	$16^{\frac{1}{2}}$
4	4 ✔

$$2x = (4x + 8)^{\frac{1}{2}}$$

$2(-1)$	$(4(-1) + 8)^{\frac{1}{2}}$
-2	$4^{\frac{1}{2}}$
-2	2 ✖

The only solution is $x = 2$.

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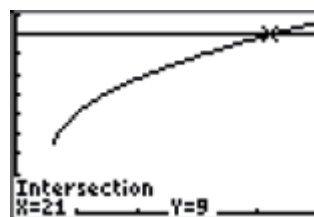
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Example 5 Solving Radical Inequalities

Solve $\sqrt{2x-6} + 3 \leq 9$.

Method 1 Use a graph and a table.

On a graphing calculator, let **Y1** $= \sqrt{2x-6} + 3$ and **Y2** $= 9$. The graph of **Y1** is at or below the graph of **Y2** for values of x between 3 and 21. Notice that **Y1** is undefined when $x < 3$.

The solution is $3 \leq x \leq 21$.

Method 2 Use algebra to solve the inequality.

Step 1 Solve for x.

$\sqrt{2x-6} + 3 \leq 9$	
$\sqrt{2x-6} \leq 6$	*Subtract 3.*
$(\sqrt{2x-6})^2 \leq (6)^2$	*Square both sides.*
$2x - 6 \leq 36$	*Simplify.*
$2x \leq 42$	*Solve for x.*
$x \leq 21$	

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8-8 # Solving Radical Equations and Inequalities

Example 5 Solving Radical Inequalities (continued)

Step 2 Consider the radicand.

$2x - 6 \geq 0$ *The radicand cannot be negative.*

$2x \geq 6$ *Solve for x.*

$x \geq 3$

The solution of $\sqrt{2x - 6} + 3 \leq 9$ is $x \geq 3$ and $x \leq 21$, or $3 \leq x \leq 21$.

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8-8 # Solving Radical Equations and Inequalities

Example 6 Automobile Application

The time *t* in seconds that it takes a car to travel a quarter mile when starting from a full stop can be estimated by using the formula $t = 5.825\sqrt[3]{\dfrac{w}{P}}$, where *w* is the weight of the car in pounds and *P* is the power delivered by the engine in horsepower. If the quarter-mile time from a 3590 lb car is 13.4 s, how much power does its engine deliver? Round to the nearest horsepower.

Use the formula to determine the amount of horsepower the 3590 lb car has if it finishes the quarter-mile in 13.4 s.

$$t = 5.825\sqrt[3]{\dfrac{w}{P}}$$

$$13.4 = 5.825\sqrt[3]{\dfrac{3590}{P}} \qquad \textit{Substitute 13.4 for t and 3590 for w.}$$

$$(13.4)^3 = \left(5.825\sqrt[3]{\dfrac{3590}{P}}\right)^3 \qquad \textit{Cube both sides.}$$

$$2406.104 \approx 197.646\left(\dfrac{3590}{P}\right) \qquad \textit{Simplify.}$$

$$2406.104P \approx 709{,}548.747 \qquad \textit{Solve for P.}$$

$$P \approx 295$$

The engine delivers a power of about 295 hp.

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