Name ______________________________ Date __________ Class __________

LESSON **8-7**

Reteach

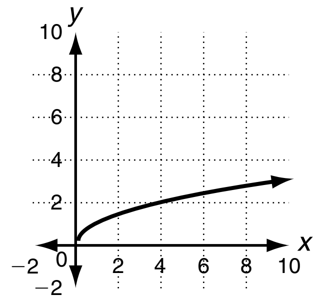
Radical Functions

The square root function, $f(x) = \sqrt{x}$, is a radical function.

The domain of $f(x) = \sqrt{x}$ is $\{x \mid x \geq 0\}$.

The range is $\{y \mid y \geq 0\}$.

Note that x and y have only nonnegative values.



You can make a table of values to graph a radical function.

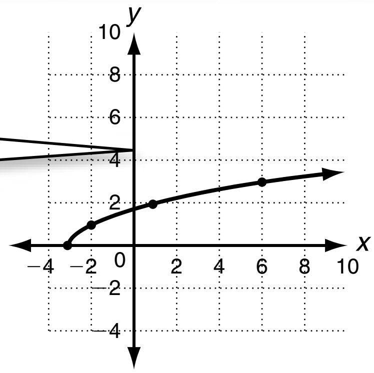
Graph: $f(x) = \sqrt{x + 3}$

x	$f(x) = \sqrt{x + 3}$	$(x, f(x))$
-3	$f(-3) = \sqrt{-3 + 3} = \sqrt{0} = 0$	$(-3, 0)$
-2	$f(-2) = \sqrt{-2 + 3} = \sqrt{1} = 1$	$(-2, 1)$
1	$f(1) = \sqrt{1 + 3} = \sqrt{4} = 2$	$(1, 2)$
6	$f(6) = \sqrt{6 + 3} = \sqrt{9} = 3$	$(6, 3)$

First choose the value of x that makes $f(x) = 0$.

First choose the value of x that make perfect squares.

The domain is $\{x \mid x \geq -3\}$.
The range is $\{y \mid y \geq 0\}$.



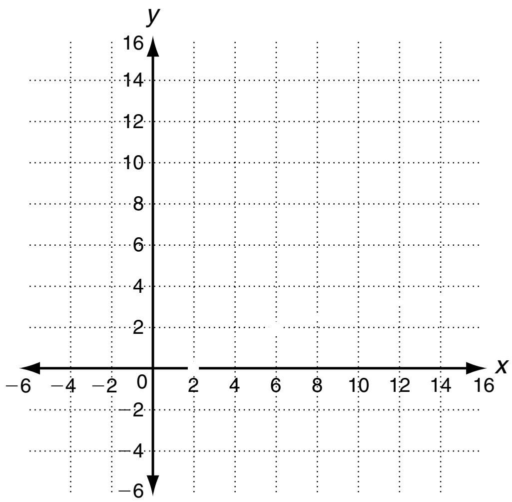
Graph the function. Identify its domain and range.

1. $f(x) = \sqrt{x - 2}$

x	$f(x) = \sqrt{x - 2}$	$(x, f(x))$
2		
3		

Domain: ____________________

Range: ____________________



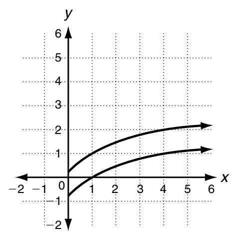
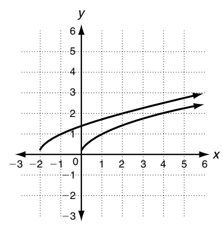
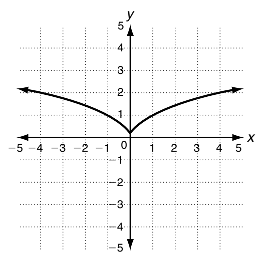
Name ______________________ Date ____________ Class ____________

LESSON 8-7

Reteach

Radical Functions (continued)

Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

Transformations	
Vertical Translations $y = \sqrt{x} + k$ Shifts $f(x)$ up k units for $k > 0$ Shifts $f(x)$ down k units for $k < 0$	
Horizontal Translations $y = \sqrt{x - h}$ Shifts $f(x)$ right h units for $h > 0$ Shifts $f(x)$ left h units for $h < 0$	
Reflections $y = -\sqrt{x}$ reflects $f(x)$ across x-axis $y = \sqrt{-x}$ reflects $f(x)$ across y-axis	

$k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down.

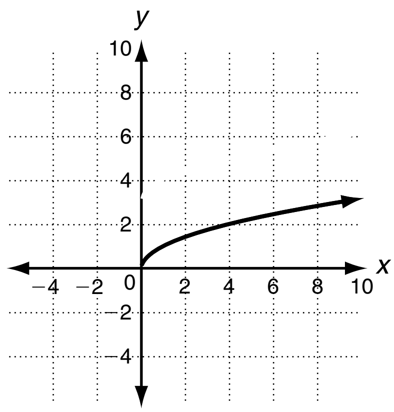
$h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left.

$r(x) = \sqrt{-x}$ reflects $f(x)$ across the y-axis.

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

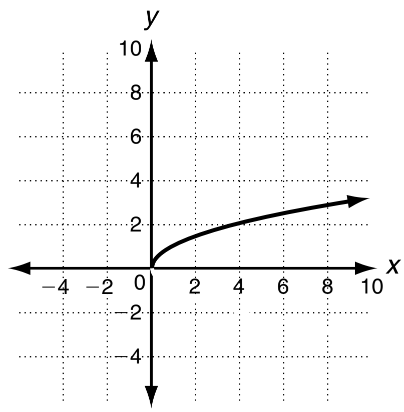
2. $s(x) = \sqrt{x} + 3$

$k = 3$: ______________________________



3. $p(x) = -\sqrt{x}$

____



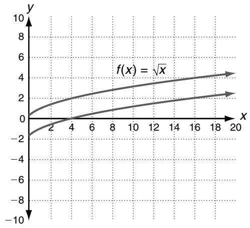
LESSON **8-7**

Practice A
Radical Functions

Graph each function.

1. $g(x) = \sqrt{x} - 2$

x	$g(x)$	$(x, g(x))$
0	$\sqrt{0} - 2 = -2$	$(0, -2)$
1	$\sqrt{1} - 2 = -1$	$(1, -1)$
4	$\sqrt{4} - 2 = 0$	$(4, 0)$
9	$\sqrt{9} - 2 = 1$	$(9, 1)$
16	$\sqrt{16} - 2 = 2$	$(16, 2)$



a. Describe the transformation from the parent function.

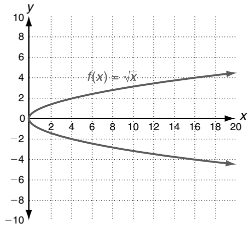
Translation 2 units down

b. Identify the domain and range. **Domain: $\{x \mid x \geq 0\}$; range: $\{y \mid y \geq -2\}$**

2. $g(x) = -\sqrt{x}$

a. Complete the table of values, then graph.

x	$g(x)$	$(x, g(x))$
0	$-\sqrt{0} = 0$	$(0, 0)$
1	$-\sqrt{1} = -1$	$(1, -1)$
4	$-\sqrt{4} = -2$	$(4, -2)$
9	$-\sqrt{9} = -3$	$(9, -3)$
16	$-\sqrt{16} = -4$	$(16, -4)$



b. Describe the transformation from the parent function.

Reflection across the x-axis

c. Identify the domain and range. **Domain: $\{x \mid x \geq 0\}$; range: $\{y \mid y \leq 0\}$**

Solve.

3. Dale wants to horizontally stretch the function $f(x) = \sqrt{x + 5}$ by a factor of 3. He writes the function $f(x) = \sqrt{3(x + 5)}$. Is he correct? If not, what is the correct function?

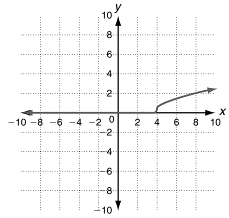
No; $g(x) = \sqrt{\frac{1}{3}(x + 5)}$

LESSON **8-7**

Practice B
Radical Functions

Graph each function, and identify its domain and range.

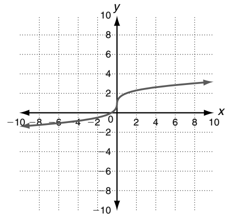
1. $f(x) = \sqrt{x - 4}$



Domain: $\{x \mid x \geq -4\}$

Range: $\{y \mid y \geq 0\}$

2. $f(x) = \sqrt[3]{x} + 1$



Domain: **all real numbers**

Range: **all real numbers**

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation.

3. $g(x) = 4\sqrt{x + 8}$ **Vertical stretch by a factor of 4 and translate 8 units left**

4. $g(x) = -\sqrt{3x} + 2$ **Reflection across the x-axis, horizontal compression by a factor of $\frac{1}{3}$, and translate 2 units up**

Use the description to write the square root function g.

5. The parent function $f(x) = \sqrt{x}$ is reflected across the y-axis, vertically stretched by a factor of 7, and translated 3 units down. $g(x) = 7\sqrt{-x} - 3$

6. The parent function $f(x) = \sqrt{x}$ is translated 2 units right, compressed horizontally by a factor of $\frac{1}{2}$, and reflected across the x-axis. $g(x) = -\sqrt{2(x - 2)}$

Solve.

7. For a gas with density, n, measured in atoms per cubic centimeter, the average distance, d, between atoms is given by $d = \left(\frac{3}{4\pi n}\right)^{\frac{1}{3}}$. The gas in a certain region of space has a density of just 10 atoms per cubic centimeter. Find the average distance between the atoms in that region of space.

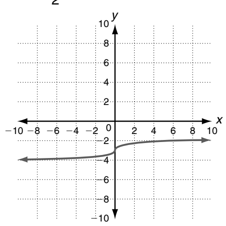
0.29 cm

LESSON **8-7**

Practice C
Radical Functions

Graph each function or inequality.

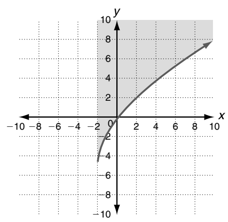
1. $g(x) = \frac{1}{2}\sqrt[3]{x} - 3$



a. Identify the domain and range.

Domain: all real numbers; range: all real numbers

2. $y \geq 4\sqrt{x + 2} - 6$



a. Describe the solution region.

The region above the curve including the line where $x \geq -2$

Use the description to write the square root function g.

3. The parent function $f(x) = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{4}$, reflected across the x-axis, and translated 6 units up. $g(x) = -\frac{1}{4}\sqrt{x} + 6$

4. The parent function $f(x) = \sqrt{x}$ is translated 8 units left, reflected across the y-axis, and stretched horizontally by a factor of 3. $g(x) = \sqrt{-\frac{1}{3}(x + 8)}$

Solve.

5. The frequency, f, in Hz, at which a simple pendulum rocks back and forth is given by $f = \frac{1}{2\pi}\sqrt{\frac{g}{l}}$, where g is the strength of the gravitational field at the location of the pendulum, and l is the length of the pendulum.

a. Find the frequency of a pendulum whose length is 1 foot and where the gravitational field is approximately 32 ft/s^2. **0.90 Hz**

b. The strength of the gravitational field on the moon is about $\frac{1}{6}$ as strong as on Earth. Find the frequency of the same pendulum on the moon. **0.37 Hz**

LESSON **8-7**

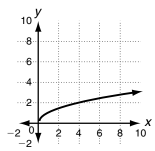
Reteach
Radical Functions

The square root function, $f(x) = \sqrt{x}$, is a radical function.

The domain of $f(x) = \sqrt{x}$ is $\{x \mid x \geq 0\}$.

The range is $\{y \mid y \geq 0\}$.

Note that x and y have only nonnegative values.



You can make a table of values to graph a radical function.

Graph: $f(x) = \sqrt{x + 3}$

x	$f(x) = \sqrt{x + 3}$	$(x, f(x))$
-3	$f(-3) = \sqrt{-3 + 3} = \sqrt{0} = 0$	$(-3, 0)$
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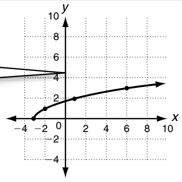
1	$f(1) = \sqrt{1 + 3} = \sqrt{4} = 2$	$(1, 2)$
6	$f(6) = \sqrt{6 + 3} = \sqrt{9} = 3$	$(6, 3)$

First choose the value of x that makes $f(x) = 0$.

First choose the value of x that make perfect squares.

The domain is $\{x \mid x \geq -3\}$. The range is $\{y \mid y \geq 0\}$.



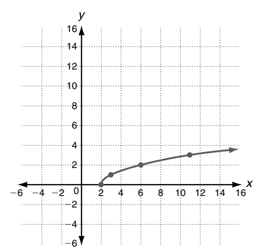
Graph the function. Identify its domain and range.

1. $f(x) = \sqrt{x - 2}$

x	$f(x) = \sqrt{x - 2}$	$(x, f(x))$
2	0	$(2, 0)$
3	1	$(3, 1)$
6	2	$(6, 2)$
11	3	$(11, 3)$

Domain: $\{x \mid x \geq 2\}$

Range: $\{y \mid y \geq 0\}$

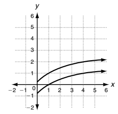
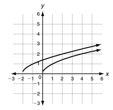
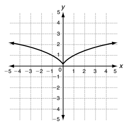


LESSON **8-7**

Reteach

Radical Functions (continued)

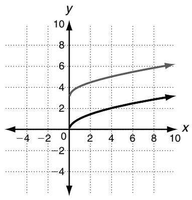
Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

Transformations		
Vertical Translations $y = \sqrt{x} + k$ Shifts $f(x)$ up k units for $k > 0$ Shifts $f(x)$ down k units for $k < 0$		$k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down.
Horizontal Translations $y = \sqrt{x - h}$ Shifts $f(x)$ right h units for $h > 0$ Shifts $f(x)$ left h units for $h < 0$		$h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left.
Reflections $y = -\sqrt{x}$ reflects $f(x)$ across x-axis $y = \sqrt{-x}$ reflects $f(x)$ across y-axis		$r(x) = \sqrt{-x}$ reflects $f(x)$ across the y-axis.

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

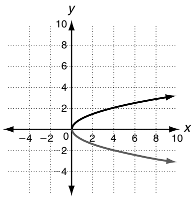
2. $s(x) = \sqrt{x} + 3$

$k = 3$: **Shifts $f(x)$ 3 units up**



3. $p(x) = -\sqrt{x}$

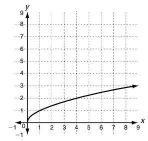
Reflects $f(x)$ across the x-axis



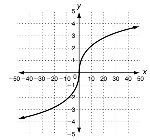
LESSON **8-7**

Challenge

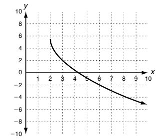
Graphs of Radical Functions



The first graph shows the function $y = \sqrt{x}$. Observe that both the domain and the range consist of the set of nonnegative numbers. The graph begins at the point $(0, 0)$ and includes points $(1, 1)$, $(4, 2)$, and $(9, 3)$. As x increases from 0 to 1, then from 1 to 4, and then from 4 to 9, the y value increases by 1 each time.



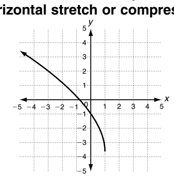
The second graph shows the function $y = \sqrt[3]{x}$. In this case the domain and range are both the set of real numbers. As x increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the y-value increases by 1 each time.



Look at the third graph. You can determine the equation from the graph. The square root function has been reflected over the x-axis. The starting point is at $(2, 6)$ so both a reflection and a translation are involved. As x increases from the starting point 1 unit to the right, the y-value decreases 4 units so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x - 2} + 6$.

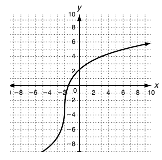
Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.

1.



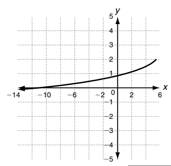
$y = 3\sqrt{-x + 1} - 4$

2.



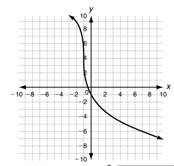
$y = 4\sqrt[3]{x + 2} - 3$

3.



$y = -0.5\sqrt{-x + 5} + 2$

4.



$y = 5\sqrt[3]{-x - 1} + 4$

LESSON **8-7**

Problem Solving

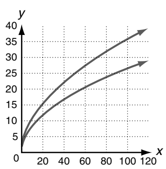
Radical Functions

On Earth the distance, *d*, in kilometers that one can see to the horizon is a function of altitude, *a*, in meters, and can be found using the function $d(a) = 3.56\sqrt{a}$. To find the corresponding distance to the horizon on Mars, the function must be stretched horizontally by a factor of about $\frac{9}{5}$.

1. a. Write the function that corresponds to the given transformation.

$d(a) = 3.56\sqrt{\frac{5}{9}a}$

b. Use a graphing calculator to graph the function and the parent function. Sketch both curves on the coordinate plane.



c. Use your graph to determine the approximate distance to the horizon from an altitude of 100 meters:

on Earth **36 km**

on Mars **27 km**

Choose the letter for the best answer.

2. Which equation represents the radius of a sphere as a function of the volume of the sphere?

A $r = \sqrt[3]{\frac{3\pi}{4V}}$ C $r = \sqrt[3]{\frac{4V}{3\pi}}$

(B) $r = \sqrt[3]{\frac{3V}{4\pi}}$ D $r = \sqrt[3]{\frac{4\pi}{3V}}$

3. Alice graphed a function that is found only in the first quadrant. Which function could she have used?

A $f(x) = \sqrt{x + 2}$ (C) $f(x) = \sqrt{x} + 2$

B $f(x) = -\sqrt{x}$ D $f(x) = \sqrt{x - 2}$

4. Harry made a symmetrical design by graphing four functions, one in each quadrant. The graph of which function is in the third quadrant?

A $f(x) = 4\sqrt{x}$ C $f(x) = -4\sqrt{x}$

B $f(x) = 4\sqrt{-x}$ (D) $f(x) = -4\sqrt{-x}$

5. The side length of a cube can be represented by $s = \sqrt{\frac{T}{6}}$, where T is the surface area of the cube. What transformation is shown by $s = \sqrt{\frac{T}{3}}$?

(A) Horizontal compression by a factor of 0.5

B Horizontal stretch by a factor of 2

C Vertical compression by a factor of 0.5

D Vertical stretch by a factor of 2

6. The hypotenuse of a right isosceles triangle can be written $H = \sqrt{2x^2}$, where x is the length of one of the legs. Which function models the hypotenuse when the legs are lengthened by a factor of 2?

A $H = \sqrt{2x^2} + 2$ C $H = \sqrt{4x^2}$

B $H = \sqrt{2x^2} + 4$ (D) $H = \sqrt{8x^2}$

LESSON **8-7**

Reading Strategy

Use a Visual Map

You can determine how a radical function has been reflected and translated on the coordinate plane by looking at its equation.

Parent Function	Transformed Function	
$f(x) = \sqrt{x}$	$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$	
	Reflection: $a < 0$ = across x-axis $b < 0$ = across y-axis	**Translation:** $+h$ = right h units $-h$ = left h units $+k$ = up k units $-k$ = down k units

Identify the constants and describe the transformation from the parent function.

1. $g(x) = \sqrt{x} - 3$

a. a = **1** **b.** b = **1**

c. h = **0** **d.** k = **−3**

e. Describe the transformation. **Translated 3 units down**

2. $g(x) = -\sqrt{x - 1} + 2$

a. a = **−1** **b.** b = **1**

c. h = **1** **d.** k = **2**

e. Describe the transformation. **Translated 2 units up, 1 unit right, and reflected across the x-axis**

3. $g(x) = \sqrt{-x + 4} - 5$

a. a = **1** **b.** b = **−1**

c. h = **−4** **d.** k = **−5**

e. Describe the transformation. **Translated 5 units down, 4 units left, and reflected across the y-axis**

4. $g(x) = -\sqrt{x + 3} + 2$

a. a = **−1** **b.** b = **1**

c. h = **−3** **d.** k = **2**

e. Describe the transformation. **Translated 2 units up, 3 units left, and reflected across the x-axis**