Name ______________________________ Date ___________ Class ___________

LESSON 8-7

Reading Strategy
Use a Visual Map

You can determine how a radical function has been reflected and translated on the coordinate plane by looking at its equation.

<table>
<tr><th>Parent Function</th><th colspan="2">Transformed Function</th></tr>
<tr><td rowspan="2">$f(x) = \sqrt{x}$</td><td colspan="2">$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$</td></tr>
<tr><td>Reflection:
$a < 0$ = across x-axis
$b < 0$ = across y-axis</td><td>Translation:
$+h$ = right h units
$-h$ = left h units
$+k$ = up k units
$-k$ = down k units</td></tr>
</table>

Identify the constants and describe the transformation from the parent function.

1. $g(x) = \sqrt{x} - 3$

a. a = ____________ **b.** b = ____________

c. h = ____________ **d.** k = ____________

e. Describe the transformation. ____________

2. $g(x) = -\sqrt{x - 1} + 2$

a. a = ____________ **b.** b = ____________

c. h = ____________ **d.** k = ____________

e. Describe the transformation. ____________

3. $g(x) = \sqrt{-x + 4} - 5$

a. a = ____________ **b.** b = ____________

c. h = ____________ **d.** k = ____________

e. Describe the transformation. ____________

4. $g(x) = -\sqrt{x + 3} + 2$

a. a = ____________ **b.** b = ____________

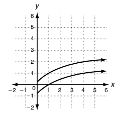
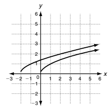
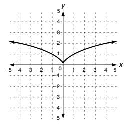
c. h = ____________ **d.** k = ____________

e. Describe the transformation. ____________

LESSON **8-7**

Reteach
Radical Functions (continued)

Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

Transformations	
Vertical Translations $y = \sqrt{x} + k$ Shifts $f(x)$ up k units for $k > 0$ Shifts $f(x)$ down k units for $k < 0$	
Horizontal Translations $y = \sqrt{x - h}$ Shifts $f(x)$ right h units for $h > 0$ Shifts $f(x)$ left h units for $h < 0$	
Reflections $y = -\sqrt{x}$ reflects $f(x)$ across x-axis $y = \sqrt{-x}$ reflects $f(x)$ across y-axis	

$k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down.

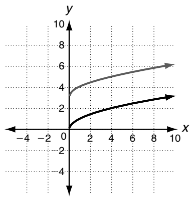
$h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left.

$r(x) = \sqrt{-x}$ reflects $f(x)$ across the y-axis.

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

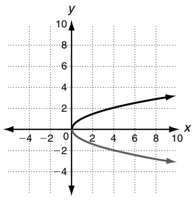
2. $s(x) = \sqrt{x} + 3$

$k = 3$: **Shifts $f(x)$ 3 units up**



3. $p(x) = -\sqrt{x}$

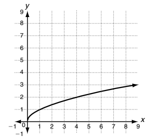
Reflects $f(x)$ across the x-axis



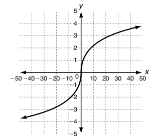
LESSON **8-7**

Challenge
Graphs of Radical Functions

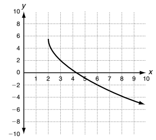
The first graph shows the function $y = \sqrt{x}$. Observe that both the domain and the range consist of the set of nonnegative numbers. The graph begins at the point $(0, 0)$ and includes points $(1, 1)$, $(4, 2)$, and $(9, 3)$. As x increases from 0 to 1, then from 1 to 4, and then from 4 to 9, the y value increases by 1 each time.



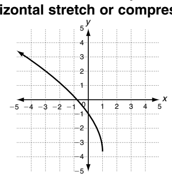
The second graph shows the function $y = \sqrt[3]{x}$. In this case the domain and range are both the set of real numbers. As x increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the y-value increases by 1 each time.



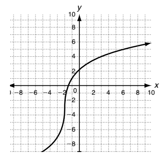
Look at the third graph. You can determine the equation from the graph. The square root function has been reflected over the x-axis. The starting point is at $(2, 6)$ so both a reflection and a translation are involved. As x increases from the starting point 1 unit to the right, the y-value decreases 4 units so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x - 2} + 6$.



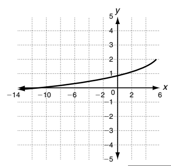
Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.

1. 

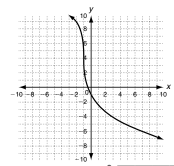
$y = 3\sqrt{-x + 1} - 4$

2. 

$y = 4\sqrt[3]{x + 2} - 3$

3. 

$y = -0.5\sqrt{-x + 5} + 2$

4. 

$y = 5\sqrt[3]{-x - 1} + 4$

LESSON **8-7**

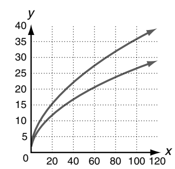
Problem Solving
Radical Functions

On Earth the distance, *d*, in kilometers that one can see to the horizon is a function of altitude, *a*, in meters, and can be found using the function $d(a) = 3.56\sqrt{a}$. To find the corresponding distance to the horizon on Mars, the function must be stretched horizontally by a factor of about $\frac{9}{5}$.

1. a. Write the function that corresponds to the given transformation.

$d(a) = 3.56\sqrt{\frac{5}{9}a}$

b. Use a graphing calculator to graph the function and the parent function. Sketch both curves on the coordinate plane.



c. Use your graph to determine the approximate distance to the horizon from an altitude of 100 meters:

on Earth **36 km**

on Mars **27 km**

Choose the letter for the best answer.

2. Which equation represents the radius of a sphere as a function of the volume of the sphere?

A $r = \sqrt[3]{\frac{3\pi}{4V}}$ C $r = \sqrt[3]{\frac{4V}{3\pi}}$

(B) $r = \sqrt[3]{\frac{3V}{4\pi}}$ D $r = \sqrt[3]{\frac{4\pi}{3V}}$

3. Alice graphed a function that is found only in the first quadrant. Which function could she have used?

A $f(x) = \sqrt{x + 2}$ (C) $f(x) = \sqrt{x} + 2$

B $f(x) = -\sqrt{x}$ D $f(x) = \sqrt{x - 2}$

4. Harry made a symmetrical design by graphing four functions, one in each quadrant. The graph of which function is in the third quadrant?

A $f(x) = 4\sqrt{x}$ C $f(x) = -4\sqrt{x}$

B $f(x) = 4\sqrt{-x}$ (D) $f(x) = -4\sqrt{-x}$

5. The side length of a cube can be represented by $s = \sqrt{\frac{T}{6}}$, where T is the surface area of the cube. What transformation is shown by $s = \sqrt{\frac{T}{3}}$?

(A) Horizontal compression by a factor of 0.5

B Horizontal stretch by a factor of 2

C Vertical compression by a factor of 0.5

D Vertical stretch by a factor of 2

6. The hypotenuse of a right isosceles triangle can be written $H = \sqrt{2x^2}$, where x is the length of one of the legs. Which function models the hypotenuse when the legs are lengthened by a factor of 2?

A $H = \sqrt{2x^2} + 2$ C $H = \sqrt{4x^2}$

B $H = \sqrt{2x^2} + 4$ (D) $H = \sqrt{8x^2}$

LESSON **8-7**

Reading Strategy
Use a Visual Map

You can determine how a radical function has been reflected and translated on the coordinate plane by looking at its equation.

Parent Function	Transformed Function	
$f(x) = \sqrt{x}$	$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$	
	Reflection: $a < 0$ = across x-axis $b < 0$ = across y-axis	**Translation:** $+h$ = right h units $-h$ = left h units $+k$ = up k units $-k$ = down k units

Identify the constants and describe the transformation from the parent function.

1. $g(x) = \sqrt{x} - 3$

a. $a =$ **1** b. $b =$ **1**

c. $h =$ **0** d. $k =$ **−3**

e. Describe the transformation. **Translated 3 units down**

2. $g(x) = -\sqrt{x - 1} + 2$

a. $a =$ **−1** b. $b =$ **1**

c. $h =$ **1** d. $k =$ **2**

e. Describe the transformation. **Translated 2 units up, 1 unit right, and reflected across the x-axis**

3. $g(x) = \sqrt{-x + 4} - 5$

a. $a =$ **1** b. $b =$ **−1**

c. $h =$ **−4** d. $k =$ **−5**

e. Describe the transformation. **Translated 5 units down, 4 units left, and reflected across the y-axis**

4. $g(x) = -\sqrt{x + 3} + 2$

a. $a =$ **−1** b. $b =$ **1**

c. $h =$ **−3** d. $k =$ **2**

e. Describe the transformation. **Translated 2 units up, 3 units left, and reflected across the x-axis**