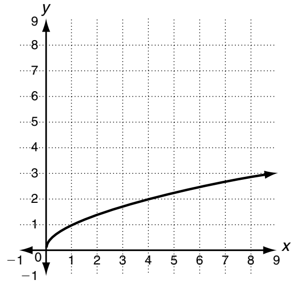
Name ________________________________________ Date ___________ Class ____________

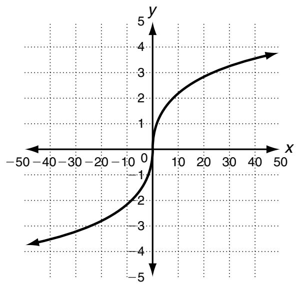
LESSON 8-7

# Challenge

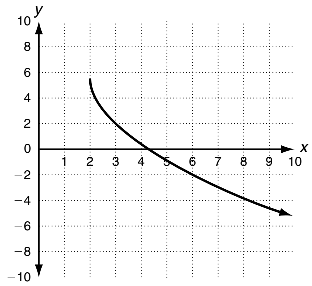
## *Graphs of Radical Functions*



The first graph shows the function $y = \sqrt{x}$. Observe that both the domain and the range consist of the set of nonnegative numbers. The graph begins at the point $(0, 0)$ and includes points $(1, 1)$, $(4, 2)$, and $(9, 3)$. As $x$ increases from 0 to 1, then from 1 to 4, and then from 4 to 9, the $y$ value increases by 1 each time.



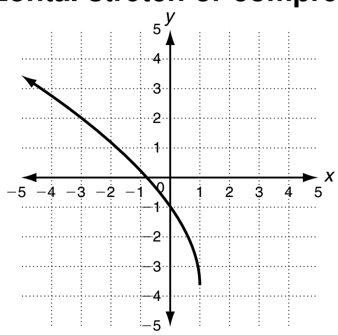
The second graph shows the function $y = \sqrt[3]{x}$. In this case the domain and range are both the set of real numbers. As $x$ increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the $y$-value increases by 1 each time.



Look at the third graph. You can determine the equation from the graph. The square root function has been reflected over the $x$-axis. The starting point is at $(2, 6)$ so both a reflection and a translation are involved. As $x$ increases from the starting point 1 unit to the right, the $y$-value decreases 4 units so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x - 2} + 6$.

**Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.**

1.



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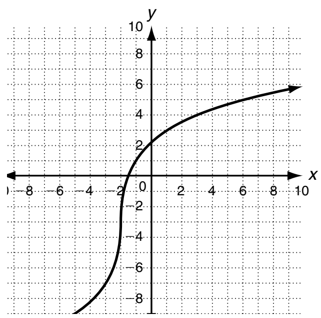
2.



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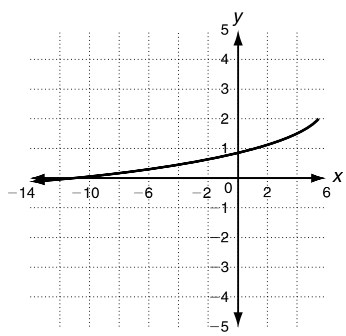
3.



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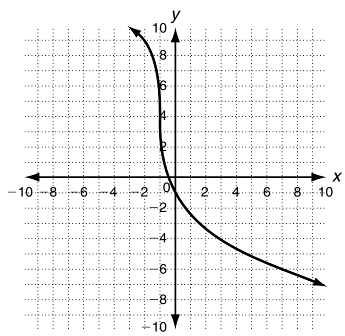
4.



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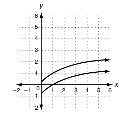
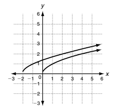
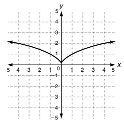
LESSON 8-7

## Reteach

### *Radical Functions (continued)*

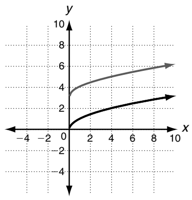
Transformations of the square root function, $f(x) = \sqrt{x}$, are similar to transformations of other functions.

| Transformations | | |
|---|---|---|
| **Vertical Translations** <br> $y = \sqrt{x} + k$ <br> Shifts $f(x)$ up $k$ units for $k > 0$ <br> Shifts $f(x)$ down $k$ units for $k < 0$ |  | $k = -1$, so $g(x) = \sqrt{x} - 1$ shifts $f(x)$ 1 unit down. |
| **Horizontal Translations** <br> $y = \sqrt{x - h}$ <br> Shifts $f(x)$ right $h$ units for $h > 0$ <br> Shifts $f(x)$ left $h$ units for $h < 0$ |  | $h = -2$, so $m(x) = \sqrt{x + 2}$ shifts $f(x)$ 2 units left. |
| **Reflections** <br> $y = -\sqrt{x}$ reflects $f(x)$ across $x$-axis <br> $y = \sqrt{-x}$ reflects $f(x)$ across $y$-axis |  | $r(x) = \sqrt{-x}$ reflects $f(x)$ across the $y$-axis. |

**Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.**

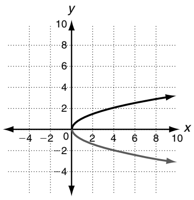
**2.** $s(x) = \sqrt{x} + 3$

$k = 3$: **Shifts $f(x)$ 3 units up**



**3.** $p(x) = -\sqrt{x}$

**Reflects $f(x)$ across the $x$-axis**

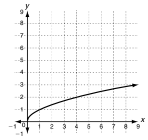





LESSON 8-7

## Challenge

### *Graphs of Radical Functions*



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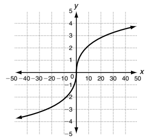
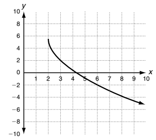


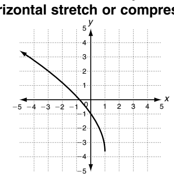
The second graph shows the function $y = \sqrt[3]{x}$. In this case the domain and range are both the set of real numbers. As $x$ increases from 0 to 1, then from 1 to 8, and then from 8 to 27, the $y$-value increases by 1 each time.



Look at the third graph. You can determine the equation from the graph. The square root function has been reflected over the $x$-axis. The starting point is at $(2, 6)$ so both a reflection and a translation are involved. As $x$ increases from the starting point 1 unit to the right, the $y$-value decreases 4 units so a vertical stretch is also indicated. Putting all these transformations together gives $y = -4\sqrt{x - 2} + 6$.

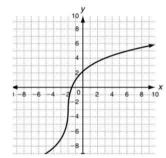
**Use these observations to write an equation for each graph. Note that the equations may not be unique since many times a vertical stretch or compression can also be written using a horizontal stretch or compression, respectively.**

**1.**



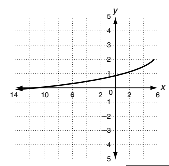
$y = 3\sqrt{-x + 1} - 4$

**2.**



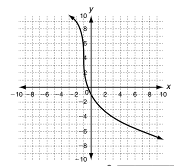
$y = 4\sqrt[3]{x + 2} - 3$

**3.**



$y = -0.5\sqrt{-x + 5} + 2$

**4.**



$y = 5\sqrt[3]{-x - 1} + 4$




LESSON 8-7

## Problem Solving

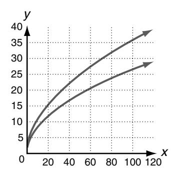
### *Radical Functions*

**On Earth the distance, *d*, in kilometers that one can see to the horizon is a function of altitude, *a*, in meters, and can be found using the function $d(a) = 3.56\sqrt{a}$. To find the corresponding distance to the horizon on Mars, the function must be stretched horizontally by a factor of about $\frac{9}{5}$.**

**1. a.** Write the function that corresponds to the given transformation.

$d(a) = 3.56\sqrt{\frac{5}{9}a}$

**b.** Use a graphing calculator to graph the function and the parent function. Sketch both curves on the coordinate plane.



**c.** Use your graph to determine the approximate distance to the horizon from an altitude of 100 meters:

on Earth **36 km**

on Mars **27 km**

**Choose the letter for the best answer.**

**2.** Which equation represents the radius of a sphere as a function of the volume of the sphere?

A $r = \sqrt[3]{\frac{3\pi}{4V}}$  C $r = \sqrt[3]{\frac{4V}{3\pi}}$

(B) $r = \sqrt[3]{\frac{3V}{4\pi}}$  D $r = \sqrt[3]{\frac{4\pi}{3V}}$

**4.** Harry made a symmetrical design by graphing four functions, one in each quadrant. The graph of which function is in the third quadrant?

A $f(x) = 4\sqrt{x}$  C $f(x) = -4\sqrt{x}$

B $f(x) = 4\sqrt{-x}$  (D) $f(x) = -4\sqrt{-x}$

**6.** The hypotenuse of a right isosceles triangle can be written $H = \sqrt{2x^2}$, where $x$ is the length of one of the legs. Which function models the hypotenuse when the legs are lengthened by a factor of 2?

A $H = \sqrt{2x^2} + 2$  C $H = \sqrt{4x^2}$

B $H = \sqrt{2x^2} + 4$  (D) $H = \sqrt{8x^2}$

**3.** Alice graphed a function that is found only in the first quadrant. Which function could she have used?

A $f(x) = \sqrt{x + 2}$  (C) $f(x) = \sqrt{x} + 2$

B $f(x) = -\sqrt{x}$  D $f(x) = \sqrt{x - 2}$

**5.** The side length of a cube can be represented by $s = \sqrt{\frac{T}{6}}$, where $T$ is the surface area of the cube. What transformation is shown by $s = \sqrt{\frac{T}{3}}$?

(A) Horizontal compression by a factor of 0.5

B Horizontal stretch by a factor of 2

C Vertical compression by a factor of 0.5

D Vertical stretch by a factor of 2




LESSON 8-7

## Reading Strategy

### *Use a Visual Map*

You can determine how a radical function has been reflected and translated on the coordinate plane by looking at its equation.

| Parent Function | Transformed Function | |
|---|---|---|
| $f(x) = \sqrt{x}$ | $f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$ | |
| | **Reflection:** <br> $a < 0$ = across $x$-axis <br> $b < 0$ = across $y$-axis | **Translation:** <br> $+h$ = right $h$ units <br> $-h$ = left $h$ units <br> $+k$ = up $k$ units <br> $-k$ = down $k$ units |

**Identify the constants and describe the transformation from the parent function.**

**1.** $g(x) = \sqrt{x} - 3$

**a.** $a =$ 1  **b.** $b =$ 1

**c.** $h =$ 0  **d.** $k =$ −3

**e.** Describe the transformation. **Translated 3 units down**

**2.** $g(x) = -\sqrt{x - 1} + 2$

**a.** $a =$ −1  **b.** $b =$ 1

**c.** $h =$ 1  **d.** $k =$ 2

**e.** Describe the transformation. **Translated 2 units up, 1 unit right, and reflected across the $x$-axis**

**3.** $g(x) = \sqrt{-x + 4} - 5$

**a.** $a =$ 1  **b.** $b =$ −1

**c.** $h =$ −4  **d.** $k =$ −5

**e.** Describe the transformation. **Translated 5 units down, 4 units left, and reflected across the $y$-axis**

**4.** $g(x) = -\sqrt{x + 3} + 2$

**a.** $a =$ −1  **b.** $b =$ 1

**c.** $h =$ −3  **d.** $k =$ 2

**e.** Describe the transformation. **Translated 2 units up, 3 units left, and reflected across the $x$-axis**