

Example 1 Graphing Radical Functions

Graph each function and identify its domain and range.

A. $f(x) = \sqrt{x-3}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Because the square root of a negative number is imaginary, choose only nonnegative values for x - 3.

| X | $f(\mathbf{x}) = \sqrt{\mathbf{x} - 3}$ | $(\mathbf{x}, \mathbf{f}(\mathbf{x}))$ |
|----|---|--|
| 3 | $f(3) = \sqrt{3-3} = \sqrt{0} = 0$ | (3, 0) |
| 4 | $f(4) = \sqrt{4-3} = \sqrt{1} = 1$ | (4, 1) |
| 7 | $f(7) = \sqrt{7-3} = \sqrt{4} = 1$ | (7, 2) |
| 12 | $f(12) = \sqrt{12 - 3} = \sqrt{9} = 3$ | (12, 3) |



The domain is $\{x \mid x \ge 3\}$, and the range is $\{y \mid y \ge 0\}$.



Example 1 Graphing Radical Functions (continued)

B. $f(x) = 2\sqrt[3]{x-2}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Choose both negative and positive values for x.

| X | $f(\mathbf{x}) = 2\sqrt[3]{\mathbf{x}-2}$ | $(\mathbf{x}, \mathbf{f}(\mathbf{x}))$ |
|----|--|--|
| -6 | $f(-6) = 2\sqrt[3]{-6-2} = 2\sqrt[3]{-8} = -4$ | (6,4) |
| 1 | $f(1) = 2\sqrt[3]{1-2} = 2\sqrt[3]{-1} = -2$ | (1, -2) |
| 2 | $f(2) = 2\sqrt[3]{2-2} = 2\sqrt[3]{0} = 0$ | (2, 0) |
| 3 | $f(3) = 2\sqrt[3]{3-2} = 2\sqrt[3]{1} = 2$ | (3, 2) |
| 10 | $f(10) = 2\sqrt[3]{10 - 2} = 2\sqrt[3]{8} = 4$ | (10, 4) |



The domain is the set of all real numbers. The range is also the set of all real numbers.

Check Graph the function on a graphing calculator.







Example 2 Transforming Square-Root Functions

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph the function $g(x) = \sqrt{x} + 5$.

g(x) = f(x) + 5

Translate f 5 units up.





Example 3 Applying Multiple Transformations

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph the function $g(x) = -\sqrt{x-4}$.

Reflect *f* across the *x*-axis, and translate it 4 units to the right.





Example 4 Writing Transformed Square-Root Functions

Use the description to write the square-root function *g*. The parent function $f(x) = \sqrt{x}$ is reflected across the *x*-axis, compressed vertically by a factor of $\frac{1}{5}$, and translated down 5 units.

Step 1 Identify how each transformation affects the function.

Reflection across the *x*-axis: *a* is negative)

Vertical compression by *a* factor of $\frac{1}{5}$ $\begin{cases} a = -\frac{1}{5} \\ a = -\frac{1}{5} \end{cases}$

Translation 5 units down: k = -5

Step 2 Write the transformed function.

$$g(x) = a\sqrt{x} + k$$

$$g(x) = \left(-\frac{1}{5}\right)\sqrt{x} + (-5)$$

$$g(x) = -\frac{1}{5}\sqrt{x} - 5$$

Substitute $-\frac{1}{5}$ for a and -5 for h.
Simplify.

Check Graph both functions on a graphing calculator. The graph of *g* indicates the given transformations of *f*.





Example 5 Business Application

A framing store uses the function $c(a) = 0.5\sqrt{a} + 0.2$ to determine the cost *c* in dollars of glass for a picture with an area *a* in square inches. The store charges an additional \$6.00 in labor to install the glass. Write the function *d* for the total cost of a piece of glass, including installation, and use it to estimate the total cost of glass for a picture with an area of 192 in².

Step 1 To increase c by 6.00, add 6 to *c*.

 $d(a) = c(a) + 6 = 0.5\sqrt{a} + 6.2$

Step 2 Find the value of *d* for a picture with an area of 192 in².

 $d(192) = 0.5\sqrt{192} + 6.2 \approx 13.13$

Substitute 192 for a and simplify.

The cost for the glass of a picture with an area of 192 in² is about \$13.13 including installation.



Example 6 Graphing Radical Inequalities

Graph the inequality $y > 2\sqrt{x} - 3$.

Step 1 Use the related equation $y = 2\sqrt{x} - 3$ to make a table of values.

| X | 0 | 1 | 4 | 9 |
|---|----|----|---|---|
| У | -3 | -1 | 1 | 3 |

Step 2 Use the table to graph the boundary curve. The inequality sign is >, so use a dashed curve and shade the area above it.



Because the value of x cannot be negative, do not shade left of the y-axis.

Check Choose a point in the solution region, such as (1, 0), and test it in the inequality.

- $y > 2\sqrt{x} 3$
- 0 > 2(1) 3
- 0 > −1 **✓**