LESSON 8-6

Rational Expressions and Rational Exponents

Know it! Note

Lesson Objectives (p. 610):

Vocabulary

1. Index (p. 610):

2. Rational exponent (p. 611):

Key Concepts

3. Properties of *n*th Roots (p. 611):

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots		
Quotient Property of Roots		

4. Rational Exponents (p. 611):

For any natural number *n* and integer *m*,

WORDS	NUMBERS	ALGEBRA

LESSON 8-6 Rational Expressions and Rational Exponents

Know it! Note

Lesson Objectives (p. 610):

rewrite radical expressions by using rational exponents; simplify and evaluate radical expressions and expressions containing rational expressions.

Vocabulary

1. Index (p. 610): in the radical expression $\sqrt[n]{a}$, n is the index of the radical.

2. Rational exponent (p. 611): an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$.

Key Concepts

3. Properties of nth Roots (p. 611):

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots **The *n*th root of a product is equal to the product of the *n*th roots.**	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots **The *n*th root of a quotient is equal to the quotient of the *n*th roots.**	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

4. Rational Exponents (p. 611):

For any natural number n and integer m,

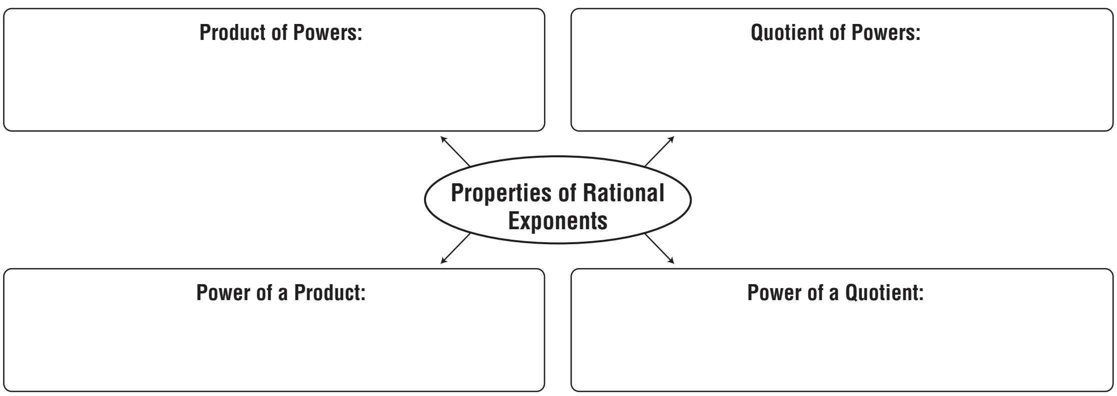
WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the *n*th root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the *n*th root raised to the *m*th power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

5. Properties of Rational Exponents (p. 612):

For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property		
Quotient of Powers Property		
Power of a Power Property		
Power of a Product Property		
Power of a Quotient Property		

6. Get Organized In each box, give a numeric and algebraic example of the given property of rational exponents. (p. 614).



5. Properties of Rational Exponents (p. 612):

For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property **To multiple powers with the same base, add the exponents.**	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property **To divide powers with the same base, subtract the exponents.**	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$a^{\frac{m}{n}} = a^{m-n}$
Power of a Power Property **To raise one power to another, multiple the exponents.**	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$\left(a^m\right)^n = a^{m \cdot n}$
Power of a Product Property **To find the power of a product, distribute the exponents.**	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property **To find the power of a quotient, distribute the exponent.**	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

6. Get Organized In each box, give a numeric and algebraic example of the given property of rational exponents. (p. 614).

Properties of Rational Exponents

Product of Powers:

$x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} = x^{\frac{3}{4}}$; $2^{\frac{1}{5}} \cdot 2^{\frac{2}{5}} = 2^{\frac{3}{5}}$

Quotient of Powers:

$\frac{x^{\frac{1}{2}}}{x^{\frac{1}{4}}} = x^{\frac{1}{4}}$; $\frac{2^{\frac{2}{5}}}{2^{\frac{1}{5}}} = 2^{\frac{1}{5}}$

Power of a Product:

$(xy)^{\frac{1}{2}} = \left(x^{\frac{1}{2}}\right)\left(y^{\frac{1}{2}}\right)$; $(4 \cdot 8)^{\frac{1}{3}} = \left(4^{\frac{1}{3}}\right)\left(8^{\frac{1}{3}}\right)$

Power of a Quotient:

$\left(\frac{x}{y}\right)^{\frac{1}{2}} = \frac{x^{\frac{1}{2}}}{y^{\frac{1}{2}}}$; $\left(\frac{4}{8}\right)^{\frac{1}{3}} = \frac{4^{\frac{1}{3}}}{8^{\frac{1}{3}}}$