8-6 Radical Expressions and Rational Exponents

Numbers and Types of Real Roots		
Case	**Roots**	**Example**
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ± 2.
Even index; negative radicand	0 real roots	-16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

Properties of *n*th Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots The *n*th root of a product is equal to the product of the *n*th roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots The *n*th root of a quotient is equal to the quotient of the *n*th roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Rational Exponents

For any natural number n and integer m,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the *n*th root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the *n*th root raised to the *m*th power.	$8^{\frac{2}{3}} = \left(\sqrt[3]{8}\right)^2 = 2^2 = 4$	$a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$

TEACHING TRANSPARENCY

8-6 Radical Expressions and Rational Exponents (continued)

Properties of Rational Exponents

For all nonzero real numbers *a* and *b* and rational numbers *m* and *n*,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2} + \frac{3}{2}} = 12^{2} = 144$	$a^{m} \cdot a^{n} = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3} - \frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^{m}}{a^{n}} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^{3} = 8^{\frac{2}{3} \cdot 3} = 8^{2} = 64$	$\left(a^{m}\right)^{n} = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^{m} = a^{m}b^{m}$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}$