Name _____ Date _____ Class _____

LESSON 8-6

Practice C

Radical Expressions and Rational Exponents

Simplify each expression. Assume all variables are positive.

1. $\sqrt[4]{(2x)^8} \cdot \sqrt[3]{(2x)^6}$

2. $\sqrt{\frac{x^{16}y^8}{81}}$

3. $\sqrt{\frac{x^7}{27x^3}}$

Write each expression in radical form, and simplify.

4. $216^{\frac{2}{3}}$

5. $1000^{-\frac{2}{3}}$

6. $(16x^3)^{\frac{3}{2}}$

Write each expression by using rational exponents.

7. $\sqrt[5]{(3x)^4}$

8. $(\sqrt[5]{-6})^3$

9. $\sqrt[4]{30x^3}$

Simplify each expression.

10. $25^{\frac{1}{4}} \cdot 25^{-\frac{7}{4}}$

11. $(-64)^{\frac{1}{3}}$

12. $\left(\frac{x^8}{y^4}\right)^{\frac{3}{4}}$

13. $\left(\frac{x^3}{125}\right)^{\frac{1}{3}}$

14. $(-8x^{18})^{\frac{2}{3}}\left(\sqrt[3]{y^6}\right)$

15. $(a^4b^8)^{-\frac{1}{4}}$

16. $(\sqrt[3]{-8x^9})^2$

17. $(3x)^{\frac{2}{3}}(3x)^{\frac{7}{3}}$

18. $\left(\frac{m^8}{n^{12}}\right)^{-\frac{1}{4}}$

Solve.

19. Each key on a piano produces a frequency that is $2^{\frac{1}{12}}$ times higher than the frequency of the key immediately to its left. Moving n keys to the right of any key increases the frequency of the starting note by a factor $2^{\frac{n}{12}}$. The key corresponding to Concert A has a frequency of 440 Hz. What is the frequency of note D, which is 5 keys to the right of Concert A?

LESSON 8-6

Practice A

Radical Expressions and Rational Exponents

Answer each question.

1. List all of the square roots of 36. — 6 and -6
2. What is the inverse of the square of a number? — The square root
3. Express $n^{\frac{1}{2}}$ without a fractional exponent. — $\sqrt{n}$
4. Express $n^{\frac{7}{4}}$ without a fractional exponent. — $\sqrt[4]{n^7}$
5. Write the following root: the radicand is 10 and the index is 12. — $\sqrt[12]{10}$

Find all real roots.

6. 4th roots of 1: ± 1
7. cube roots of 27: 3
8. square roots of 81: ± 9

Write each expression in radical form, and simplify.

9. $6^{\frac{1}{2}}$: $\sqrt{6}$
10. $8^{\frac{2}{3}}$: $\sqrt[3]{8^2} = 4$
11. $5^{\frac{4}{3}}$: $\sqrt[3]{5^4}$

Write each expression by using rational exponents.

12. $\sqrt{7^2}$: 7^1 or 7
13. $\sqrt[4]{5^3}$: $5^{\frac{3}{4}}$
14. $\sqrt[3]{10^5}$: $10^{\frac{5}{3}}$

Simplify each expression. Assume all variables are positive.

15. $\sqrt[3]{8x^3}$: $2x$
16. $\sqrt{\frac{36}{16}}$: $\frac{3}{2}$
17. $2^2 \cdot 2^3$: 2^5 or 32
18. $\frac{(3x)^4}{(3x)^2}$: $9x^2$
19. $(5^2)^2$: 5^4 or 625
20. $\left(\frac{8x^3}{27}\right)^{\frac{1}{3}}$: $\frac{2x}{3}$
21. $\sqrt{\frac{8x^2}{2x^4}}$: $\frac{2}{x}$
22. $2 \cdot \left(\frac{1}{8}\right)^{\frac{1}{3}}$: 1
23. $(\sqrt{25x^2})^3$: $125x^3$

LESSON 8-6

Practice B

Radical Expressions and Rational Exponents

Simplify each expression. Assume all variables are positive.

1. $\sqrt[3]{125x^9}$: $5x^3$
2. $\sqrt[4]{\frac{x^8}{81}}$: $\frac{x^2}{3}$
3. $\sqrt[3]{\frac{64x^3}{8}}$: $2x$

Write each expression in radical form, and simplify.

4. $64^{\frac{2}{6}}$: 32
5. $27^{\frac{2}{3}}$: 9
6. $(-8)^{\frac{4}{3}}$: 16

Write each expression by using rational exponents.

7. $\sqrt[5]{51^4}$: $51^{\frac{4}{5}}$
8. $(\sqrt{169})^3$: $169^{\frac{3}{2}}$
9. $\sqrt[7]{36^{14}}$: 36^2

Simplify each expression.

10. $4^{\frac{3}{2}} \cdot 4^{\frac{5}{2}}$: 256
11. $\frac{27^{\frac{4}{3}}}{27^{\frac{2}{3}}}$: 9
12. $\left(125^{\frac{2}{3}}\right)^{\frac{1}{2}}$: 5
13. $(27 \cdot 64)^{\frac{2}{3}}$: 144
14. $\left(\frac{1}{243}\right)^{\frac{1}{5}}$: $\frac{1}{3}$
15. $64^{-\frac{1}{3}}$: $\frac{1}{4}$
16. $(-27x^6)^{\frac{1}{3}}$: $-3x^2$
17. $\frac{(25x)^{\frac{3}{2}}}{5 \cdot x^{\frac{1}{2}}}$: $25x$
18. $(4x)^{-\frac{1}{2}} \cdot (9x)^{\frac{1}{2}}$: $\frac{3}{2}$

Solve.

19. In every atom, electrons orbit the nucleus with a certain characteristic velocity known as the Fermi–Thomas velocity, equal to $\frac{Z^{\frac{2}{3}}}{137}c$, where Z is the number of protons in the nucleus and c is the speed of light. In terms of c, what is the characteristic Fermi-Thomas velocity of the electrons in Uranium, for which $Z = 92$?

About $0.15c$

LESSON 8-6

Practice C

Radical Expressions and Rational Exponents

Simplify each expression. Assume all variables are positive.

1. $\sqrt[4]{(2x)^8} \cdot \sqrt[3]{(2x)^6}$: $16x^4$
2. $\sqrt[4]{\frac{x^{16}y^8}{81}}$: $\frac{x^4y^2}{3}$
3. $\sqrt[3]{\frac{x^7}{27x^3}}$: $\frac{x\sqrt[3]{x}}{3}$

Write each expression in radical form, and simplify.

4. $216^{\frac{2}{3}}$: 36
5. $1000^{-\frac{2}{3}}$: $\frac{1}{100}$
6. $(16x^3)^{\frac{3}{2}}$: $64x^4\sqrt{x}$

Write each expression by using rational exponents.

7. $\sqrt[5]{(3x)^4}$: $(3x)^{\frac{4}{5}}$
8. $(\sqrt[5]{-6})^3$: $(-6)^{\frac{3}{5}}$
9. $\sqrt[4]{30x^3}$: $30^{\frac{1}{4}} \cdot x^{\frac{3}{4}}$

Simplify each expression.

10. $25^{\frac{1}{4}} \cdot 25^{-\frac{7}{4}}$: $\frac{1}{125}$
11. $(-64)^{\frac{1}{3}}$: -4
12. $\left(\frac{x^8}{y^4}\right)^{\frac{3}{4}}$: $\frac{x^6}{y^3}$
13. $\left(\frac{x^3}{125}\right)^{\frac{1}{3}}$: $\frac{x}{5}$
14. $(-8x^{18})^{\frac{2}{3}}\left(\sqrt[3]{y^6}\right)$: $4x^{12}y^2$
15. $(a^4b^8)^{-\frac{1}{4}}$: $\frac{1}{ab^2}$
16. $\left(\sqrt[3]{-8x^9}\right)^2$: $4x^6$
17. $(3x)^{\frac{2}{3}}(3x)^{\frac{7}{3}}$: $27x^3$
18. $\left(\frac{m^8}{n^{12}}\right)^{-\frac{1}{4}}$: $\frac{n^3}{m^2}$

Solve.

19. Each key on a piano produces a frequency that is $2^{\frac{1}{12}}$ times higher than the frequency of the key immediately to its left. Moving n keys to the right of any key increases the frequency of the starting note by a factor $2^{\frac{n}{12}}$. The key corresponding to Concert A has a frequency of 440 Hz. What is the frequency of note D, which is 5 keys to the right of Concert A?

About 587.3 Hz

LESSON 8-6

Reteach

Radical Expressions and Rational Exponents

Use **Properties of *n* th Roots** to simplify radical expressions.

Product Property: $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

Simplify: $\sqrt[4]{81x^8}$.

$\sqrt[4]{3^4} \cdot x^4 \cdot x^4$ — *Factor into perfect fourth roots.*

$\sqrt[4]{3^4}\sqrt[4]{x^4}\sqrt[4]{x^4}$ — *Use the Product Property.*

$3 \cdot x \cdot x$

$3x^2$

Think: $\sqrt[n]{a^n} = a$, so $\sqrt[4]{3^4} = 3$ and $\sqrt[4]{x^4} = x$.

Quotient Property: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Simplify: $\sqrt[3]{\frac{x^9}{2}}$.

Always **rationalize the denominator** when an expression contains a radical in the denominator.

$\frac{\sqrt[3]{x^9}}{\sqrt[3]{2}}$ — *Use the Quotient Property.*

$\frac{x^3}{\sqrt[3]{2}}$

Simplify the numerator. Think: $\sqrt[3]{x^9} = \sqrt[3]{x^3}\sqrt[3]{x^3}\sqrt[3]{x^3} = x \cdot x \cdot x$.

$\frac{x^3}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}}$ — *Rationalize the denominator.*

$\frac{x^3\sqrt[3]{2^2}}{\sqrt[3]{2^3}}$ — *Use the Product Property.*

$\frac{x^3\sqrt[3]{4}}{2}$ — *Simplify.*

Simplify each expression.

1. $\sqrt[3]{x^8} \cdot \sqrt[3]{x^4}$: $\sqrt[3]{x^8 \cdot x^4}$; x^4
2. $\sqrt[4]{\frac{x^8}{6}}$: $\frac{\sqrt[4]{x^8}}{\sqrt[4]{6}}$; $\frac{x^2\sqrt[4]{216}}{6}$
3. $\sqrt[3]{125x^6}$: $\sqrt[3]{125} \cdot \sqrt[3]{x^6}$; $= 5x^2$
4. $\sqrt[5]{\frac{64}{2x^{10}}}$: $\frac{2}{x^2}$
5. $\sqrt[3]{2x} \cdot \sqrt[3]{4x^2}$: $2x$
6. $\sqrt[4]{625x^8}$: $5x^2$