Name ______________________________________ Date ___________ Class ____________

LESSON **8-6**

Challenge

Using Rational Exponents

Rational exponents make it possible to simplify radical expressions. Consider the simple operation of multiplication of radicals. Is it possible to multiply the radical expression $\sqrt{2}\cdot\sqrt[3]{2}$? When written in radical form the answer is no, or at least it is very difficult. When written in exponent form it may be done as follows:

$$\sqrt{2}\cdot\sqrt[3]{2} = 2^{\frac{1}{2}}\cdot 2^{\frac{1}{3}} = 2^{\frac{1}{2}+\frac{1}{3}} = 2^{\frac{3}{6}+\frac{2}{6}} = 2^{\frac{5}{6}} = \sqrt[6]{2^5} = \sqrt[6]{32}.$$

When you look at the radicals, it may not be obvious that the first and last expressions are equal. The operation with rational exponents verifies the result. Other fraction operations are possible when using rational exponents instead of radicals.

$$\sqrt[3]{2}\cdot\sqrt[5]{3^2} = 2^{\frac{1}{3}}\cdot 3^{\frac{2}{5}} = 2^{\frac{5}{15}}\cdot 3^{\frac{6}{15}} = (2^5\cdot 3^6)^{\frac{1}{15}} = \sqrt[15]{2^5\cdot 3^6} = \sqrt[15]{23,328}$$

Once again, the radical operations are quite difficult, but the rational exponents greatly simplify the multiplication.

Simplify. Assume all variables have positive values.

1. $\sqrt{18}\cdot\sqrt[4]{18}$

____

2. $\sqrt{30}\cdot\sqrt[3]{900}\cdot\sqrt[5]{630}$

____

3. $\sqrt{35xy}\cdot\sqrt[4]{35x^2y^3}$

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4. $\sqrt{3xy}\cdot\sqrt[4]{75x^2y^3}\cdot\sqrt[3]{25x^2y}$

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5. $\sqrt{20}\div\sqrt[4]{20}$

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6. $\sqrt{210}\div\sqrt[4]{8820}\div\sqrt[5]{1296}$

____

7. $\sqrt{3xy}\div\sqrt[3]{3x^2y}$

____

8. $\sqrt{3xy}\div\sqrt[3]{75x^2y^2}\div\sqrt[6]{25x^3y^4}$

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9. $\dfrac{\sqrt{175}\cdot\sqrt[3]{175}}{\sqrt[5]{175}}$

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10. $\dfrac{(\sqrt{75}\cdot 3\sqrt{108})\sqrt[3]{9}}{9\sqrt[4]{15}}$

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LESSON 8-6

Reteach

Radical Expressions and Rational Exponents (continued)

The *n*th root of a number can be represented using a rational, or fractional, exponent: $\sqrt[n]{a} = a^{\frac{1}{n}}$.

This means to take the nth root of *a*.

Examples: $121^{\frac{1}{2}} = \sqrt{121} = 11$

$216^{\frac{1}{3}} = \sqrt[3]{216} = 6$

$256^{\frac{1}{4}} = \sqrt[4]{256} = 4$

Powers and roots can be expressed using rational exponents $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$.

The denominator is the root and the numerator is the power.

Examples:

$64^{\frac{2}{3}} = (\sqrt[3]{64})^2 = 4^2 = 16$

$(-32)^{\frac{4}{5}} = (\sqrt[5]{-32})^4 = (-2)^4 = 16$

To write expressions using rational exponents, use the definitions.

$\sqrt[n]{a} = a^{\frac{1}{n}}$ and $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Note that $(\sqrt[n]{a})^m = \sqrt[n]{a^m}$.

Examples: $\sqrt{5} = 5^{\frac{1}{2}}$ $\sqrt[4]{6^3} = 6^{\frac{3}{4}}$

Think: The root is $n = 4$.
The power is $m = 3$.

Write each expression in radical form and simplify.

7. $27^{\frac{4}{3}} = (\sqrt[3]{27})^4$ **81**

8. $49^{\frac{3}{2}}$ **$(\sqrt{49})^3 = 343$**

9. $16^{\frac{3}{4}}$ **$(\sqrt[4]{16})^3 = 8$**

Write each expression by using rational exponents.

10. $\sqrt[5]{4^2}$ Think: $m = 2$, $n = 5$ **$4^{\frac{2}{5}}$**

11. $\sqrt{19}$ **$19^{\frac{1}{2}}$**

12. $\sqrt[4]{6^5}$ **$6^{\frac{5}{4}}$**

Simplify each expression.

13. $\left(\frac{24}{3x^3}\right)^{\frac{1}{3}}$ **$\frac{2}{x}$**

14. $\sqrt{49} \cdot \sqrt[3]{8x^6}$ **$14x^2$**

15. $\sqrt{\frac{117}{13}}$ **3**

LESSON 8-6

Challenge

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When you look at the radicals, it may not be obvious that the first and last expressions are equal. The operation with rational exponents verifies the result. Other fraction operations are possible when using rational exponents instead of radicals.

$$\sqrt[3]{2} \cdot \sqrt[5]{3^2} = 2^{\frac{1}{3}} \cdot 3^{\frac{2}{5}} = 2^{\frac{5}{15}} \cdot 3^{\frac{6}{15}} = (2^5 \cdot 3^6)^{\frac{1}{15}} = \sqrt[15]{2^5 \cdot 3^6} = \sqrt[15]{23{,}328}$$

Once again, the radical operations are quite difficult, but the rational exponents greatly simplify the multiplication.

Simplify. Assume all variables have positive values.

1. $\sqrt{18} \cdot \sqrt[4]{18}$ **$3\sqrt[4]{72}$**

2. $\sqrt{30} \cdot \sqrt[3]{900} \cdot \sqrt[5]{630}$ **$30\sqrt[30]{3^{17} \cdot 7^6 \cdot 10^{11}} \approx 30\sqrt[30]{1.52 \times 10^{24}}$**

3. $\sqrt{35xy} \cdot \sqrt[4]{35x^2y^3}$ **$xy\sqrt[4]{35^3y} = xy\sqrt[4]{42{,}875y}$**

4. $\sqrt{3xy} \cdot \sqrt[4]{75x^2y^3} \cdot \sqrt[3]{25x^2y}$ **$5xy\sqrt[12]{3^9 \cdot 5^2x^8y^7} = 5xy\sqrt[12]{492{,}075x^8y^7}$**

5. $\sqrt{20} \div \sqrt[4]{20}$ **$\sqrt[4]{20}$**

6. $\sqrt{210} \div \sqrt[4]{8820} \div \sqrt[5]{1296}$ **$\frac{\sqrt[20]{5^5 \cdot 6^4}}{6} = \frac{\sqrt[20]{4{,}050{,}000}}{6}$**

7. $\sqrt{3xy} \div \sqrt[3]{3x^2y}$ **$\frac{\sqrt[6]{3y}}{\sqrt[6]{x}} = \frac{\sqrt[6]{3x^5y}}{x}$**

8. $\sqrt{3xy} \div \sqrt[3]{75x^2y^2} \div \sqrt[6]{25x^3y^4}$ **$\frac{\sqrt[6]{3}}{5\sqrt[6]{x^4y^5}} = \frac{\sqrt[6]{3x^2y}}{5xy}$**

9. $\frac{\sqrt{175} \cdot \sqrt[3]{175}}{\sqrt[5]{175}}$ **$175^{\frac{19}{30}} = 5\sqrt[30]{5^8 \cdot 7^{19}}$**

10. $\frac{(\sqrt{75} \cdot 3\sqrt{108})\sqrt[3]{9}}{9\sqrt[4]{15}}$ **$6\sqrt[12]{5^9 \cdot 3^5} = 6\sqrt[12]{474{,}609{,}375}$**

LESSON 8-6

Problem Solving

Radical Expressions and Rational Exponents

Louise is building a guitar-like instrument. It has small metal bars, called frets, positioned across its neck so that it can produce notes of a specific scale on each string. The distance a fret should be placed from the bridge is related to a string's root note length by the function $d(n) = r\left(2^{-\frac{n}{12}}\right)$, where *r* is the length of the root note string and *n* is the number of notes higher than that string's root note. Louise wants to know where to place frets to produce different notes on a 50-cm string.

1. Find the distance from the bridge for a fret that produces a note exactly one octave (12 notes) higher than the root note.
   a. Substitute values for *r* and *n* in the given function. **$d(12) = 50\left(2^{-\frac{12}{12}}\right)$**
   b. How far from the bridge should the fret be placed? **25 cm**
   c. What fraction of the string length is the distance of this fret from the bridge? **$\frac{1}{2}$**
2. Complete the table to find the distance from the bridge, for frets that produce every other note of an entire scale on this string.

Notes Higher than the Root Note	2	4	6	8	10	12
Distance of Fret from Bridge (cm)	44.5	39.7	35.4	31.5	28.1	25

Choose the letter for the best answer.

3. Rafael made a ceramic cube in art class. The cube has a volume of 336 cm³. What is the side length of the cube to the nearest centimeter?
   [✓ A] 7
   B 12
   C 18
   D 56

4. Yolanda has an exercise ball with a volume of 7234 in.³. Find the radius of the exercise ball to the nearest inch.
   A 24
   B 21
   C 19
   [✓ D] 12

5. Which formula could you use to find the area of one side of a cube if the volume were given?
   A $A = V^{\frac{3}{2}}$
   [✓ C] $A = V^{\frac{2}{3}}$
   B $A = V^{-\frac{3}{2}}$
   D $A = V^{-\frac{2}{3}}$

6. A party tent in the shape of a hemisphere has a volume of 14,130 m³. What is the area of the ground that the tent covers in square meters?
   A 653.1
   [✓ C] 1121.5
   B 706.5
   D 1256.0

LESSON 8-6

Reading Strategy

Use a Concept Map

$$\sqrt[n]{a} = a^{\frac{1}{n}}$$

Vocabulary

n is called the **index** of the expression

The number or expression inside the radical sign *a* is called the **radicand**.

Relationship

$\sqrt[n]{a^m} = a^{\frac{m}{n}}$

Example:

$\sqrt[3]{5^6} = 5^{\frac{6}{3}} = 5^2 = 25$

Product Property

$\sqrt[n]{ab} = \sqrt[n]{a} \bullet \sqrt[n]{b}$

Example:

$\sqrt[3]{216} = \sqrt[3]{27} \bullet \sqrt[3]{8} = 3 \bullet 2 = 6$

Quotient Property

$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

Example: $\sqrt{\frac{4}{9}} = \frac{\sqrt{4}}{\sqrt{9}} = \frac{2}{3}$

Solve.

1. Explain how to simplify $8^{\frac{2}{3}}$.
   Write as a radical, $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$
2. Identify the radicand in the expression $\sqrt[7]{7x - 4}$.
   $7x - 4$
3. Identify the property you would use to solve $\sqrt{\frac{49}{x^4}}$. Then solve.
   Quotient Property; $\frac{7}{x^2}$

Write each expression as a radical and simplify.

4. $9^{\frac{1}{2}}$ **3**

5. $8^{\frac{1}{3}}$ **2**

6. $(x^8)^{\frac{1}{4}}$ **x^2**

7. $25^{\frac{3}{2}}$ **125**

8. $(8x^6)^{\frac{1}{3}}$ **$2x^2$**

9. $\left(\frac{4}{9}\right)^{\frac{3}{2}}$ **$\frac{8}{27}$**