8-6 Radical Expressions and Rational Exponents

Example 1 Finding Real Roots

Find all real roots.

A. sixth roots of 64

A positive number has two real sixth roots. Because $2^6 = 64$ and $(-2)^6 = 64$, the roots are 2 and -2.

B. cube roots of −216

A negative number has one real cube root. Because $(-6)^3 = -216$, the root is -6.

C. fourth roots of −1024

A negative number has no real fourth roots.

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Example 2 Simplifying Radical Expressions

Simplify each expression. Assume that all variables are positive.

A. $\sqrt[4]{81x^{12}}$

$\sqrt[4]{3^4 \cdot x^4 \cdot x^4 \cdot x^4}$ — *Factor into perfect fourths.*

$\sqrt[4]{3^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^4} \cdot \sqrt[4]{x^4}$ — *Product Property*

$3 \cdot x \cdot x \cdot x$ — *Simplify.*

$3x^3$

B. $\sqrt[4]{\dfrac{16x^8}{5}}$

$\dfrac{\sqrt[4]{16x^8}}{\sqrt[4]{5}}$ — *Quotient Property*

$\dfrac{2x^2}{\sqrt[4]{5}}$ — *Simplify the numerator.*

$\dfrac{2x^2}{\sqrt[4]{5}} \cdot \dfrac{\sqrt[4]{5}}{\sqrt[4]{5}} \cdot \dfrac{\sqrt[4]{5}}{\sqrt[4]{5}} \cdot \dfrac{\sqrt[4]{5}}{\sqrt[4]{5}}$ — *Rationalize the denominator.*

$\dfrac{2x^2\sqrt[4]{5^3}}{\sqrt[4]{5^4}}$ — *Product Property*

$\dfrac{2x^2\sqrt[4]{125}}{5}$ — *Simplify.*

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Example 3 Writing Expressions in Radical Form

Write the expression $(-32)^{\frac{3}{5}}$ in radical form and simplify.

Method 1 Evaluate the root first.		Method 2 Evaluate the power first.	
$(\sqrt[5]{-32})^3$	*Write with a radical.*	$\sqrt[5]{(-32)^3}$	*Write with a radical.*
$(-2)^3$	*Evaluate the root.*	$\sqrt[5]{-32{,}768}$	*Evaluate the power.*
-8	*Evaluate the power.*	-8	*Evaluate the root.*

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Example 4 Writing Expressions by Using Rational Exponents

Write each expression by using rational exponents.

A. $\sqrt[8]{13^4}$

$13^{\frac{4}{8}}$ — $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$13^{\frac{1}{2}}$ — *Simplify.*

B. $\sqrt[5]{3^{15}}$

$3^{\frac{15}{5}}$ — $\sqrt[n]{a^m} = a^{\frac{m}{n}}$

$3^3 = 27$ — *Simplify.*

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Example 5 Simplifying Expressions with Rational Exponents

Simplify each expression.

A. $7^{\frac{7}{9}} \cdot 7^{\frac{11}{9}}$

$7^{\frac{7}{9}+\frac{11}{9}}$ *Product of Powers*

7^2 *Simplify.*

49 *Evaluate the power.*

Check Enter the expression in a graphing calculator.

```
7^(7/9)*7^(11/9)
                49
```

B. $\dfrac{16^{\frac{3}{4}}}{16^{\frac{5}{4}}}$

$16^{\frac{3}{4}-\frac{5}{4}}$ *Quotient of Powers*

$16^{-\frac{1}{2}}$ *Simplify.*

$\dfrac{1}{16^{\frac{1}{2}}}$ *Negative Exponent Property*

$\dfrac{1}{4}$ *Evaluate the power.*

Check Enter the expression in a graphing calculator.

```
16^(3/4)/16^(5/4
)
               .25
```

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Example 6 Chemistry Application

Radium-226 is a form of radioactive element that decays over time. An initial sample of radium-226 has a mass of 500 mg. The mass of radium-226 remaining from the initial sample after t years is given by $500\left(2^{-\frac{t}{1600}}\right)$. To the nearest milligram, how much radium-226 would be left after 800 years?

$$500\left(2^{-\frac{t}{1600}}\right) = 500\left(2^{-\frac{800}{1600}}\right)$$ *Substitute 800 for t.*

$$= 500\left(2^{-\frac{1}{2}}\right)$$ *Simplify.*

$$= 500\left(\frac{1}{2^{\frac{1}{2}}}\right)$$ *Negative Exponent Property*

$$= \frac{500}{2^{\frac{1}{2}}}$$ *Simplify.*

$$\approx 354$$ *Use a calculator.*

The amount of radium-226 left after 800 years would be about 354 mg.

```
500/2^(1/2)
        353.5533906
```

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