Name ______________________________ Date __________ Class __________

LESSON 8-5

Reteach

Solving Rational Equations and Inequalities

To solve a rational equation, clear any denominators by multiplying each term on both sides of the equation by the least common denominator, LCD.

Solve: $x + \frac{12}{x} = 7$.

Step 1 The LCD is *x*. Multiply each term by *x*.

$x(x) + \frac{12}{x}(x) = 7(x)$

Step 2 Simplify.

$x^2 + 12 = 7x$

This makes the equation a quadratic equation.

Step 3 Write in standard form.

$x^2 - 7x + 12 = 0$

Set one side equal to 0 to solve a quadratic equation.

Step 4 Factor the quadratic equation.

$(x - 3)(x - 4) = 0$

Step 5 Set each factor equal to 0.

$x - 3 = 0 \qquad x - 4 = 0$

Step 6 Solve each equation.

$x = 3 \qquad x = 4$

Check $x + \frac{12}{x} = 7$

Always check the solutions to rational equations.

$x = 3$	$x = 4$
$3 + \frac{12}{3} = 3 + 4 = 7\checkmark$	$4 + \frac{12}{4} = 4 + 3 = 7\checkmark$

Solve each equation.

1. $\frac{x}{2} + 1 = \frac{4}{x}$

$\frac{x}{2}(2x) + 1(2x) = \frac{4}{x}(2x)$

$x^2 + 2x = 8$

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2. $x - \frac{6}{x} = 1$

$x(x) - \frac{6}{x}(x) = 1(x)$

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3. $x = 4 + \frac{5}{x}$

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Name ______________________________ Date __________ Class __________

LESSON 8-5

Reteach

Solving Rational Equations and Inequalities (continued)

Check all solutions to rational equations. If the solution to a rational equation makes the denominator equal to zero, then that solution is NOT a solution. It is called an **extraneous** solution.

Solve: $\frac{x+4}{x-6} + \frac{x}{2} = \frac{10}{x-6}$.

Step 1 The LCD is $2(x-6)$. Multiply each term by $2(x-6)$.

$$\frac{x+4}{x-6} \cdot 2(x-6) + \frac{x}{2} \cdot 2(x-6) = \frac{10}{x-6} \cdot 2(x-6)$$

Remember to multiply EVERY term by the LCD.

Step 2 Simplify.

$2(x+4) + x(x-6) = 10(2)$

$2x + 8 + x^2 - 6x = 20$

Step 3 Write in standard form.

$x^2 - 4x - 12 = 0$

Step 4 Factor the quadratic equation.

$(x+2)(x-6) = 0$

Step 5 Set each factor equal to 0 and solve.

$x + 2 = 0$ $\quad$ $x - 6 = 0$

$x = -2$ $\quad$ $x = 6$

Step 6 Check: $\frac{x+4}{x-6} + \frac{x}{2} = \frac{10}{x-6}$

$x = -2$

$\frac{-2+4}{-2-6} + \frac{-2}{2} = \frac{10}{-2-6}$?

$\frac{2}{-8} + (-1) = \frac{10}{-8}$ ✓

The only solution is $x = -2$.

$x = 6$ is extraneous.

This value makes the denominators of the original equation equal to 0.

Solve each equation.

4. $\frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x}{5}$

$\frac{1}{x+2} \cdot 5(x+2) +$ ____________________

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5. $\frac{x}{3} + \frac{x+3}{x-1} = \frac{4}{x-1}$

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LESSON 8-5

Practice A

Solving Rational Equations and Inequalities

Find the least common denominator (LCD) for each pair.

1. x and $\frac{3}{x}$ — x
2. $\frac{3}{x-6}$ and $\frac{x}{4}$ — $4(x-6)$
3. x^2 and x^3 — x^3

Solve each equation.

4. $2 + \frac{1}{x} = 4$ — $x = \frac{1}{2}$
5. $\frac{12}{x} + 4 = 3$ — $x = -12$
6. $x + 2 = \frac{3}{x}$ — $x = -3,\ x = 1$
7. $\frac{5}{6} + \frac{4}{x} = 3$ — $x = \frac{24}{13}$

Solve each inequality.

8. $\frac{8}{x+2} < 2$ — $x < -2$ or $x > 2$
9. $\frac{10}{x-5} \geq 2$ — $5 < x \leq 10$
10. $\frac{3}{x-1} < 3$ — $x < 1$ or $x > 2$
11. $\frac{6}{x+4} > 2$ — $-4 < x \leq -1$

Solve.

12. List all of the extraneous solutions for the equation $\frac{2x}{x+4} = \frac{x}{x-1}$.

    $x = -4$ and 1 because they make the denominators of the original equation equal to 0

13. Virat and Ari are washing the family car. When Virat washes the car by himself it takes him 3 hours, but with Ari helping it takes only 2 hours.

    a. In the equation $\frac{1}{3}(2) + \frac{1}{m}(2) = 1$, what does m represent?

    The length of time it would take Ari to wash the car himself

    b. Find the value of m.

    $m = 6$

LESSON 8-5

Practice B

Solving Rational Equations and Inequalities

Solve each equation.

1. $x - \frac{6}{x} = 5$ — $x = -1$ or $x = 6$
2. $\frac{15}{4} = \frac{6}{x} + 3$ — $x = 8$
3. $x = \frac{3}{x} + 2$ — $x = 3$ or $x = -1$
4. $\frac{4}{x^2-4} = \frac{1}{x-2}$ — no solution.

Solve each inequality by using a graphing calculator and a table.

5. $\frac{6}{x+1} < -3$ — $-3 < x < -1$
6. $\frac{x}{x-2} \geq 0$ — $x \leq 0$ or $x > 2$
7. $\frac{2x}{x+5} \leq 0$ — $-5 < x \leq 0$
8. $\frac{-x}{x-3} \geq 0$ — $0 \leq x < 3$

Solve each inequality algebraically.

9. $\frac{12}{x+4} \leq 4$ — $x < -4$ or $x \geq -1$
10. $\frac{7}{x+3} < -5$ — $-\frac{22}{5} < x < -3$
11. $\frac{x}{x-2} > 9$ — $2 < x < \frac{9}{4}$
12. $\frac{2x}{x-5} \geq 3$ — $5 < x \leq 15$

Solve.

13. The time required to deliver and install a computer at a customer's location is $t = 4 + \frac{d}{r}$, where t is time in hours, d is the distance, in miles, from the warehouse to the customer's location, and r is the average speed of the delivery truck. If it takes 6.2 hours for the employee to deliver and install a computer for a customer located 100 miles from the warehouse, what is the average speed of the delivery truck?

    About 45.5 miles per hour

LESSON 8-5

Practice C

Solving Rational Equations and Inequalities

Solve each equation.

1. $\frac{12r}{r+2} = \frac{4}{r+2} - 6$ — $r = -\frac{4}{9}$
2. $\frac{4x}{x-4} = \frac{2x+8}{x-4}$ — no solution.
3. $-\frac{6}{x} + 1 = \frac{7}{x^2}$ — $x = 7$ and $x = -1$
4. $\frac{2}{d+2} + \frac{8}{d-2} = \frac{14}{d^2-4}$ — $d = \frac{1}{5}$

Solve each inequality by using a graphing calculator and a table.

5. $\frac{x-1}{x} < 2$ — $x < -1$ or $x > 0$
6. $\frac{3x}{x+5} \leq -4$ — $-5 < x \leq -3$
7. $\frac{2-x}{x+3} \geq 4$ — $-3 < x \leq -2$
8. $\frac{x}{4-x} < 3$ — $x < 3$ OR $x > 4$

Solve each inequality algebraically.

9. $\frac{14}{m} \leq \frac{7}{2}$ — $m < 0$ or $m \geq 4$
10. $\frac{12}{s-5} > 3$ — $5 < s < 9$
11. $\frac{7z}{z-4} \geq 6$ — $z \leq -24$ or $z > 4$
12. $\frac{-9x}{x+12} < -5$ — $x < -12$ or $x > 15$

Solve.

13. An artist is designing a picture frame whose length, l, and width, w, satisfy the Golden Ratio, which is $\frac{w}{l} = \frac{l}{l+w}$. If the length of the frame is 24 inches, what is the width of the frame?

    About 14.83 in.

14. Team A can wash all the windows in the school in x hours. It takes Team B 3 hours longer to do the same job. If the teams work together, they can complete the job in 8.5 hours. How long does it take Team B to do the job alone?

    About 18.6 h

LESSON 8-5

Reteach

Solving Rational Equations and Inequalities

To solve a rational equation, clear any denominators by multiplying each term on both sides of the equation by the least common denominator, LCD.

Solve: $x + \frac{12}{x} = 7$.

Step 1 The LCD is x. Multiply each term by x.

$x(x) + \frac{12}{x}(x) = 7(x)$

Step 2 Simplify.

$x^2 + 12 = 7x$

This makes the equation a quadratic equation.

Step 3 Write in standard form.

$x^2 - 7x + 12 = 0$

Set one side equal to 0 to solve a quadratic equation.

Step 4 Factor the quadratic equation.

$(x-3)(x-4) = 0$

Step 5 Set each factor equal to 0.

$x - 3 = 0 \qquad x - 4 = 0$

Step 6 Solve each equation.

$x = 3 \qquad x = 4$

Check $x + \frac{12}{x} = 7$

Always check the solutions to rational equations.

$x = 3$: $3 + \frac{12}{3} = 3 + 4 = 7\checkmark$

$x = 4$: $4 + \frac{12}{4} = 4 + 3 = 7\checkmark$

Solve each equation.

1. $\frac{x}{2} + 1 = \frac{4}{x}$

   $\frac{x}{2}(2x) + 1(2x) = \frac{4}{x}(2x)$

   $x^2 + 2x = 8$

   $x^2 + 2x - 8 = 0$

   $(x+4)(x-2) = 0$

   $x = -4,\ x = 2$

2. $x - \frac{6}{x} = 1$

   $x(x) - \frac{6}{x}(x) = 1(x)$

   $x^2 - 6 = x$

   $x^2 - x - 6 = 0$

   $(x-3)(x+2) = 0$

   $x = 3,\ x = -2$

3. $x = 4 + \frac{5}{x}$

   $x(x) = 4(x) + \frac{5}{x}$

   $x^2 = 4x + 5$

   $x^2 - 4x - 5 = 0$

   $(x-5)(x+1) = 0$

   $x = 5,\ x = -1$

LESSON 8-5

Reteach

Solving Rational Equations and Inequalities (continued)

Check all solutions to rational equations. If the solution to a rational equation makes the denominator equal to zero, then that solution is NOT a solution. It is called an **extraneous** solution.

Solve: $\frac{x+4}{x-6} + \frac{x}{2} = \frac{10}{x-6}$.

Step 1 The LCD is $2(x - 6)$. Multiply each term by $2(x - 6)$.

$$\frac{x+4}{x-6} \cdot 2(x-6) + \frac{x}{2} \cdot 2(x-6) = \frac{10}{x-6} \cdot 2(x-6)$$

Remember to multiply EVERY term by the LCD.

Step 2 Simplify.

$2(x + 4) + x(x - 6) = 10(2)$

$2x + 8 + x^2 - 6x = 20$

Step 3 Write in standard form.

$x^2 - 4x - 12 = 0$

Step 4 Factor the quadratic equation.

$(x + 2)(x - 6) = 0$

Step 5 Set each factor equal to 0 and solve.

$x + 2 = 0 \qquad x - 6 = 0$

$x = -2 \qquad x = 6$

Step 6 Check: $\frac{x+4}{x-6} + \frac{x}{2} = \frac{10}{x-6}$

$x = -2 \qquad\qquad x = 6$ is extraneous.

$\frac{-2+4}{-2-6} + \frac{-2}{2} = \frac{10}{-2-6}$?

$\frac{2}{-8} + (-1) = \frac{10}{-8}$ ✓

The only solution is $x = -2$.

This value makes the denominators of the original equation equal to 0.

Solve each equation.

4. $\frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x}{5}$

$\frac{1}{x+2} \cdot 5(x+2) + \frac{x+1}{x+2} \cdot 5(x+2)$

$= \frac{x}{5} \cdot 5(x+2)$

$5 + 5(x + 1) = x(x + 2)$

$x^2 - 3x - 10 = 0;\ x = 5$

5. $\frac{x}{3} + \frac{x+3}{x-1} = \frac{4}{x-1}$

$\frac{x}{3} \cdot 3(x-1) + \frac{x+3}{x-1} \cdot 3(x-1)$

$= \frac{4}{x-1} \cdot 3(x-1)$

$x(x - 1) + 3(x + 3) = 12$

$x^2 + 2x - 3 = 0;\ x = -3$

LESSON 8-5

Challenge

A Rational Method

Rational inequalities can be tricky to solve algebraically since it is often necessary to examine various possibilities.

Solve the inequality $\frac{8}{x+5} \le 4$ algebraically.

$$\frac{8}{x+5} - 4 \le 0$$

$$\frac{8}{x+5} - \frac{4(x+5)}{x+5} \le 0$$

$$\frac{-4x-12}{x+5} \le 0$$

$$\frac{-4(x+3)}{x+5} \le 0$$

This last inequality is true if the expression on the left of the inequality sign is equal to either 0 or a negative number. For the expression to be equal to 0, the numerator must be 0 and this occurs only at $x = -3$. For it to be negative, the numerator and denominator must have opposite signs. The table below lists the signs for the numerator and denominator in the intervals where neither numerator nor denominator is zero.

Interval	$x < -5$	$-5 < x < -3$	$x > -3$
Numerator	+	+	−
Denominator	−	+	+
Rational Expression	−	+	−

For the inequality to be true, $x < -5$ or $x \ge -3$.

Solve each inequality algebraically.

1. $\frac{x+1}{x+3} < 2$ — $x < -5$ or $x > -3$
2. $\frac{x+4}{2x-1} \le 3$ — $x < \frac{1}{2}$ or $x \ge \frac{7}{5}$
3. $\frac{3}{x+3} > \frac{3}{x-2}$ — $-3 < x < 2$
4. $\frac{1}{x+1} > \frac{2}{x-1}$ — $x < -3$ or $-1 < x < 1$
5. $\frac{x^2-3x+2}{x^2-2x-3} \ge 0$ — $x < -1$ or $1 \le x \le 2$ or $x > 3$
6. $\frac{x^2-4}{x^2-x-2} < 1$ — $x < -1$
7. $\frac{5-x}{x^2-25} \ge 1$ — $-6 \le x < -5$
8. $\frac{3x}{x-5} < \frac{2}{8x-1}$ — $\frac{1}{8} < x < 5$
9. $\frac{5x}{x-2} < 7$ — $x < 2$ OR $x > 7$

LESSON 8-5

Problem Solving

Solving Rational Equations and Inequalities

Norton and Jessie have a lawn service business. Sometimes they work by themselves, and sometimes they work together. They want to know if it is worthwhile to work together on some jobs.

1. Norton can mow a large lawn in about 4.0 hours. When Norton and Jesse work together, they can mow the same lawn in about 2.5 hours. Jesse wants to know how long it would take her to mow the lawn if she worked by herself.

   a. Write an expression for Jessie's rate, using j for the number of hours she would take to mow the lawn by herself. $\frac{1}{j}$

   b. Write an equation to show the amount of work completed when they work together. $\left[\frac{1}{4}(2.5)\right] + \left[\frac{1}{j}(2.5)\right] = 1$

   c. How long would it take Jessie to mow the lawn by herself? $6\frac{2}{3}$ h

2. Jessie can weed a garden in about 30 minutes. When Norton helps her, they can weed the same garden in about 20 minutes. Norton wants to know how long it would take him to weed the garden if he worked by himself.

   a. Write an expression for Norton's rate, using n for the number of hours he would take to weed the garden by himself. $\frac{1}{n}$

   b. Write an equation to show the amount of work completed when they work together. $\left[\frac{1}{n}\left(\frac{1}{3}\right)\right] + \left[\frac{1}{\frac{1}{2}}\left(\frac{1}{3}\right)\right] = 1$

   c. How long would it take Norton to weed the garden by himself? 1 h

Choose the letter for the best answer.

3. Norton can edge a large lawn in about 3.0 hours. Jessie can edge a similar lawn in about 2.5 hours. Which equation could be used to find the time it would take them to edge that lawn if they worked together?

   A $\frac{1}{3} - \frac{1}{2.5} = \frac{1}{t}$

   B $\frac{1}{3} - \frac{1}{2.5} = t$

   (C) $\frac{1}{3} + \frac{1}{2.5} = \frac{1}{t}$

   D $\frac{1}{3} + \frac{1}{2.5} = t$

4. When Jessie helps Norton trim trees, they cut Norton's time to trim trees in half. What can be said about the time it would take Jessie to do the job alone?

   (A) Jessie would take the same amount of time as Norton.

   B Jessie would take half the time that Norton takes.

   C Jessie would take twice the time that Norton takes.

   D There is not enough information.

LESSON 8-5

Reading Strategy

Analyze Information

The solutions to a rational function are those values that result from solving the equation. Some solutions, however, are extraneous. **Extraneous solutions** are solutions of a derived equation that are not solutions of the original equation.

Write the equation.

$$\frac{2x}{x-4} = \frac{x+4}{x-4}$$

Find the solution.

$$\frac{2x}{x-4}(x-4) = \frac{x+4}{x-4}(x-4)$$

$2x = x + 4$

$x = 4$

Substitute the solution into the original equation.

$$\frac{2(4)}{4-4} = \frac{4+4}{4-4}$$

Analyze.

The solution gives a denominator of 0, so the solution is extraneous. This equation has no solution.

Identify values of x that would be extraneous solutions for each equation.

1. $\frac{x}{x-2} + \frac{1}{2} = \frac{x+6}{x-2}$ — $x = 2$
2. $\frac{1}{x} + \frac{1}{3} = \frac{8}{3}x$ — $x = 0$
3. $\frac{-6}{x-3} = 1$ — $x = 3$
4. $\frac{x+1}{x-1} + \frac{2}{x} = \frac{x}{x+1}$ — $x = -1, 0, 1$
5. $\frac{1}{5} - \frac{1}{x+5} = \frac{4}{3x^2}$ — $x = -5, 0$
6. $\frac{7x}{3x+2} = 2$ — $x = -\frac{2}{3}$
7. $\frac{4}{x^2-9} + \frac{1}{x-3} = \frac{2x}{x+3}$ — $x = -3, 3$
8. $\frac{1}{x} + \frac{4}{4x-1} = \frac{1}{7}$ — $x = 0, \frac{1}{4}$
9. $\frac{-2}{x^2+x-2} = \frac{1}{8}$ — $x = -2, 1$

Solve.

10. Ralph solved the inequality $\frac{x}{2x-1} \le 1$. He found the solutions to be 1 and $\frac{1}{2}$. He knows the solution has to be expressed as an inequality. He thinks the solution should be written $x \ge 1$ or $x \le \frac{1}{2}$. Is he correct? How do you know?

    No; possible answer: the solution should be $x \ge 1$ or $x < \frac{1}{2}$. $x = \frac{1}{2}$ is an extraneous solution.