

8-5 Solving Rational Equations and Inequalities

Example 1 Solving Rational Equations

Solve the equation $x - \frac{18}{x} = 3$.

$$x(x) - \frac{18}{x}(x) = 3(x)$$

$$x^2 - 18 = 3x$$

$$x^2 - 3x - 18 = 0$$

$$(x - 6)(x + 3) = 0$$

$$x - 6 = 0 \text{ or } x + 3 = 0$$

$$x = 6 \text{ or } x = -3$$

Multiply each term by the LCD, x .

Simplify. Note that $x \neq 0$.

Write in standard form.

Factor.

Apply the Zero Product Property.

Solve for x .

Check $x - \frac{18}{x} = 3$

$6 - \frac{18}{6}$	3
$6 - 3$	3
3	$3 \checkmark$

$$x - \frac{18}{x} = 3$$

$(-3) - \frac{18}{(-3)}$	3
$-3 + 6$	3
3	$3 \checkmark$

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Example 2 Extraneous Solutions

Solve each equation.

A. $\frac{5x}{x-2} = \frac{3x+4}{x-2}$

$$\frac{5x}{x-2}(x-2) = \frac{3x+4}{x-2}(x-2)$$

Multiply each term by the LCD, $x - 2$.

$$\frac{5x}{\cancel{x-2}}(\cancel{x-2}) = \frac{3x+4}{\cancel{x-2}}(\cancel{x-2})$$

Divide out common factors.

$$5x = 3x + 4$$

Simplify. Note that $x \neq 2$.

$$x = 2$$

Solve for x .

The solution $x = 2$ is extraneous because it makes the denominators of the original equation equal to 0. Therefore, the equation has no solution.

Check Substitute 2 for x in the original equation.

$$\frac{5x}{x-2} = \frac{3x+4}{x-2}$$

$\frac{5(2)}{2-2}$	$\frac{3(2)+4}{2-2}$
$\frac{10}{0}$	$\frac{10}{0} \times$

Division by 0 is undefined.

B. $\frac{2x-5}{x-8} + \frac{x}{2} = \frac{11}{x-8}$

$$\frac{2x-5}{x-8} \cdot 2(x-8) + \frac{x}{2} \cdot 2(x-8) = \frac{11}{x-8} \cdot 2(x-8)$$

Multiply each term by the LCD, $2(x - 8)$.

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Example 2 Extraneous Solutions (continued)

$$\frac{2x-5}{x-8} \cdot 2(x-8) + \frac{x}{2} \cdot 2(x-8) = \frac{11}{x-8} \cdot 2(x-8) \quad \text{Divide out common factors.}$$

$$2(2x-5) + x(x-8) = 11(2) \quad \text{Simplify. Note that } x \neq 8.$$

$$4x - 10 + x^2 - 8x = 22 \quad \text{Use the Distributive Property.}$$

$$x^2 - 4x - 32 = 0 \quad \text{Write in standard form.}$$

$$(x-8)(x+4) = 0 \quad \text{Factor.}$$

$$x-8 = 0 \text{ or } x+4 = 0 \quad \text{Use the Zero Product Property.}$$

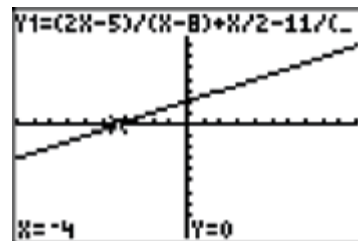
$$x = 8 \text{ or } x = -4 \quad \text{Solve for } x.$$

The solution $x = 8$ is extraneous because it makes the denominators of the original equation equal to 0. The only solution is $x = -4$.

Check Write $\frac{2x-5}{x-8} + \frac{x}{2} = \frac{11}{x-8}$ as $\frac{2x-5}{x-8} + \frac{x}{2} - \frac{11}{x-8} = 0$.

Graph the left side of the equation as **Y1**. Identify the values of x for which **Y1** = 0.

The graph intersects the x -axis only when $x = -4$. Therefore, $x = -4$ is the only solution.



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Example 3 Problem-Solving Application

A jet travels 3950 mi from Chicago, Illinois, to London, England, and 3950 mi on the return trip. The total flying time is 16.5 h. The return trip takes longer due to winds that generally blow from west to east. If the jet’s average speed with no wind is 485 mi/h, what is the average speed of the wind during the round-trip flight? Round to the nearest mile per hour.

1. Understand the Problem

The **answer** will be the average speed of the wind.

List the **important information**:

- The jet spent 16.5 h on the round-trip.
- It went 3950 mi east and 3950 mi west.
- Its average speed with no wind is 485 mi/h.

2. Make a Plan

Let w represent the speed of the wind. When the jet is going east, its speed is equal to its speed with no wind plus w . When the jet is going west, its speed is equal to its speed with no wind minus w .

	Distance (mi)	Average Speed (mi/h)	Time (h)
East	3950	$485 + w$	$\frac{3950}{485 + w}$
West	3950	$485 - w$	$\frac{3950}{485 - w}$

$$\begin{array}{rclclcl} \text{total time} & = & \text{time east} & + & \text{time west} \\ 16.5 & = & \frac{3950}{485 + w} & + & \frac{3950}{485 - w} \end{array}$$

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Example 3 Problem-Solving Application (continued)

3. Solve

$$16.5(485 + w)(485 - w) = \frac{3950}{485 + w}(485 + w)(485 - w) + \frac{3950}{485 - w}(485 + w)(485 - w)$$

The LCD is $(485 + w)(485 - w)$.

$$16.5(485 + w)(485 - w) = 3950(485 - w) + 3950(485 + w)$$

Simplify. Note that $w \neq \pm 485$.

$$3881212.5 - 16.5w^2 = 1915750 - 3950w + 1915750 + 3950w$$

Use the Distributive Property.

$$3881212.5 - 16.5w^2 = 3831500$$

Combine like terms.

$$-16.5w^2 = 49712.5$$

Solve for w .

$$w \approx \pm 55$$

The speed of the wind cannot be negative. Therefore, the average speed of the wind is 55 mi/h.

4. Look Back

If the speed of the wind is 55 mi/h, the jet's speed when going east is $485 + 55 = 540$ mi/h. It will take the jet approximately 7.3 h to travel 3950 mi east. The jet's speed when going west is $485 - 55 = 430$ mi/h. It will take the jet approximately 9.2 h to travel 3950 mi west. The total trip will take 16.5 h, which is the given time.

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Example 4 Work Application

Natalie can finish a 500-piece puzzle in about 8 hours. When Natalie and Renzo work together, they can finish a 500-piece puzzle in about 4.5 hours. About how long will it take Renzo to finish a 500-piece puzzle if he works by himself?

Natalie’s rate: $\frac{1}{8}$ of the puzzle per hour

Renzo’s rate: $\frac{1}{h}$ of the puzzle per hour, where h is the number of hours needed to finish the puzzle by himself

Natalie’s rate × hours worked	+	Renzo’s rate × hours worked	=	complete puzzle
$\frac{1}{8}(4.5)$	+	$\frac{1}{h}(4.5)$	=	1

$$\frac{1}{8}(4.5)(8h) + \frac{1}{h}(4.5)(8h) = 1(8h) \quad \text{Multiply by the LCD, } 8h.$$

$$4.5h + 36 = 8h \quad \text{Simplify.}$$

$$36 \approx 3.5h \quad \text{Solve for } h.$$

$$10.3 \approx h$$

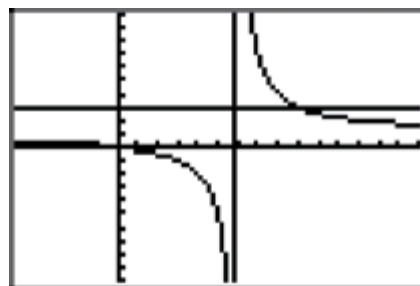
It will take Renzo about 10.3 hours, or 10 hours 17 minutes, to complete a 500-piece puzzle working by himself.

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Example 5 Solving Rational Inequalities by Using a Graph and a Table

Solve $\frac{x}{x-6} \leq 3$ by using a graph and a table.

Use a graph. On a graphing calculator, $Y1 = \frac{x}{x-6}$ and $Y2 = 3$.



The graph of **Y1** is at or below the graph of **Y2** when $x < 6$ or when $x \geq 9$.

Use a table. The table shows that **Y1** is undefined when $x = 6$ and that $Y1 \leq Y2$ when $x \geq 9$.

X	Y1	Y2
5	-2	3
6	ERROR	3
7	1	3
8	2	3
9	3	3
10	2.5	3

The solution of the inequality is $x < 6$ or $x \geq 9$.

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Example 6 Solving Rational Inequalities Algebraically

Solve the inequality $\frac{6}{x-8} \leq 3$ algebraically.

Case 1 LCD is positive.

Step 1 Solve for x .

$$\frac{6}{x-8}(x-8) \leq 3(x-8)$$

Multiply by the LCD.

$$6 \leq 3x - 24 \quad \text{Simplify. Note that } x \neq 8.$$

$$30 \leq 3x \quad \text{Solve for } x.$$

$$10 \leq x$$

$$x \geq 10 \quad \text{Rewrite with the variable on the left.}$$

Case 2 LCD is negative.

Step 1 Solve for x .

$$\frac{6}{x-8}(x-8) \geq 3(x-8)$$

*Multiply by the LCD.
Reverse the inequality.*

$$6 \geq 3x - 24 \quad \text{Simplify. Note that } x \neq 8.$$

$$30 \geq 3x \quad \text{Solve for } x.$$

$$10 \geq x$$

$$x \leq 10 \quad \text{Rewrite with the variable on the left.}$$

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Example 6 Solving Rational Inequalities Algebraically (continued)

Step 2 Consider the sign of the LCD.

$$x - 8 > 0 \quad \text{LCD is positive.}$$

$$x > 8 \quad \text{Solve for } x.$$

For Case 1, the solution must satisfy $x \geq 10$ and $x > 8$, which simplifies to $x \geq 10$.

Step 2 Consider the sign of the LCD.

$$x - 8 < 0 \quad \text{LCD is negative.}$$

$$x < 8 \quad \text{Solve for } x.$$

For Case 2, the solution must satisfy $x \leq 10$ and $x < 8$, which simplifies to $x < 8$.

The solution set of the original inequality is the union of the solutions to both Case 1 and Case 2. The solution to the

inequality $\frac{6}{x-8} \leq 3$ is $x < 8$ or $x \geq 10$, or $\{x \mid x < 8 \cup x \geq 10\}$.