Name ______________________________ Date __________ Class ___________

SECTION 8A

Ready To Go On? Skills Intervention

8-1 Variation Functions

Find these vocabulary words in Lesson 8-1 and the Multilingual Glossary.

Vocabulary		
direct variation	constant of variation	joint variation
inverse variation	combined variation	

Writing Variation Functions

A. Given: y varies directly as x, and $y = 36$ when $x = 4.5$. Write the variation function.

$y =$ _____ — y varies __________ as x.

_____ $= k$________ — Substitute 36 for y and 4.5 for x.

_____ $= k$ — Solve for the constant of variation k.

$y =$ _____ — Write the variation function by using the value of k.

Check: Substitute the original values of x and y into the equation.

Does $36 = 8(4.5)$? _____

B. Given: y varies inversely as x, and $y = 8.4$ when $x = 5$. Write the variation function.

$y =$ _____ — y varies __________ as x.

_____ $=$ _____ — Substitute 8.4 for y and 5 for x.

_____ $= k$ — Solve for the constant of variation k.

$y =$ _____ — Write the variation function by using the value of k.

Check: Substitute the original values of x and y into the equation.

Does $8.4 = \frac{42}{5}$? _____

C. Given: y varies jointly as x and z, and $y = 52$ when $x = 16$ and $z = 13$. Write the variation function.

$y =$ _____ — y varies __________ as x and _____.

_____ $= k$__________ — Substitute 52 for y, 16 for x, and 13 for z.

$\frac{\square}{\square} = k$ — Solve for the constant of variation k.

_____ $= k$

$y =$ ________ — Write the variation function by using the value of k.

Check: Does $52 = 0.25(16)(13)$? _____

Name ______________________________ Date __________ Class __________

SECTION **8A**

Ready To Go On? Problem Solving Intervention

8-1 Variation Functions

Inverse variation describes a situation in which one quantity increases and the other decreases.

The time t that it takes for a group of volunteers to clean up the park after an event varies inversely as the number of volunteers v. If 15 volunteers can clean up the park in 35 working hours, how many volunteers would be needed to clean up the park in 25 working hours?

Understand the Problem

1. What are you being asked to do?

____

2. What type of variation is presented in the problem? __________

3. What information are you given? ______________________________

Make a Plan

4. What is the general form for determining the variation constant? $t = \frac{k}{\square}$

5. What do you need to determine first? ______________________________

6. What is t? ____ What is v? ____

Solve

7. Substitute known values to determine k.

$t = \frac{k}{v}$

$35 = \frac{k}{\square}$

$\_\_\_\_ = k$

8. Use the value for k and let $t = 25$ to determine how many volunteers are needed. $t = \frac{k}{v}$

$25 = \frac{\square}{v} \Rightarrow 25v = \_\_\_\_ \Rightarrow v = \_\_\_\_$

9. The park could be cleaned up in 25 hours if there were ____ volunteers.

Look Back

10. Complete the table to check your answer.

Volunteers	15	16	17	18	19	20	21
Hours	35	______	______	______	______	______	______
Constant	525	525	525	525	525	525	525

11. How many volunteers are needed to clean the park in 25 hours? ____

Does your answer match Exercise 9? ____

Name ______________________________ Date __________ Class __________

SECTION **8A**

Ready To Go On? Skills Intervention

8-2 Multiplying and Dividing Rational Expressions

Find this vocabulary word in Lesson 8-2 and the Multilingual Glossary.

Vocabulary
rational expression

Simplifying Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

A. $\dfrac{8x^2}{4x^3 + 8x}$

$\dfrac{\square(2x)}{\square(x^2 + 2)}$ Factor out the greatest common factor, ____.

$\dfrac{\square}{(x^2 + 2)}$ Divide out the common factor. Simplify.

The expression is undefined at $x =$ ____ because this value makes the denominator equal zero.

B. $\dfrac{x^2 - 4}{x^2 + 3x - 10}$

$\dfrac{(x - 2)(\square)}{(\square)(x - 2)}$ Factor the numerator and denominator.

$\dfrac{(\square)}{(\square)}$ Divide out the common factor. Simplify.

The expression is undefined at $x =$ ____ and $x =$ ____.

Dividing Rational Expressions

Divide. Assume that all expressions are defined.

$\dfrac{x^2 - 9}{12x^3} \div \dfrac{x^2 + 6x + 9}{6x^3 + 24x^2}$

$\dfrac{x^2 - 9}{12x^3} \cdot \dfrac{\square}{\square}$ Rewrite as ______________ by the reciprocal.

$\dfrac{(\square)(\square)}{12x^3} \cdot \dfrac{6x^2(\square)}{(\square)(x + 3)}$ Factor the numerators and denominators.

$\dfrac{(\square)(x + 4)}{2x(\square)}$ Divide out the common factor. Simplify.

Name ______________________________ Date __________ Class ___________

SECTION 8A

Ready To Go On? Skills Intervention

8-3 Adding and Subtracting Rational Expressions

Find this vocabulary word in Lesson 8-3 and the Multilingual Glossary.

Vocabulary
complex fraction

Adding Rational Expressions with Like Denominators

Add. Identify any x-values for which the expression is undefined.

$$\frac{7x-3}{x^2+4}+\frac{-6x+11}{x^2+4}$$ Notice these rational expressions have *like* denominators.

$$\frac{7x-\Box-\Box+11}{x^2+4}$$ Add the numerators.

$$\frac{\Box+\Box}{x^2+4}$$ Combine like terms.

The expression is undefined at $x =$ ______________ because these values make the denominator equal zero. In other words, the expression is always _________.

Subtracting Rational Expressions

Subtract. Identify any x-values for which the expression is undefined.

$$\frac{2x^2-8}{x^2-16}-\frac{x+1}{x+4}$$

$$\frac{2x^2-8}{(\Box)(\Box)}-\frac{x+1}{x+4}$$ Factor the denominators. The LCD is ____________.

$$\frac{2x^2-8}{(\Box)(\Box)}-\frac{x+1}{x+4}\cdot\frac{(x-4)}{(\Box)}$$ Make the 2nd expression's denominator equal to the LCD by multiplying the rational expression by a form of 1.

$$\frac{\Box-8-(x+1)(\Box)}{(x-4)(x+4)}$$ Subtract the numerators.

$$\frac{2x^2-\Box-(x^2-\Box-4)}{(x-4)(x+4)}$$ Multiply the binomials in the numerator.

$$\frac{\Box-8-x^2+\Box+4}{(x-4)(x+4)}$$ Distribute the negative sign.

$$\frac{\Box+3x-\Box}{(x-4)(x+4)}$$ Simplify the numerator. Write it in standard form.

$$\frac{(\Box)(\Box)}{(x-4)(x+4)}$$ Factor the numerator.

$$\frac{(\Box)}{(\Box)}$$ Divide out common factors and simplify.

The expression is undefined at $x =$ ____ and $x =$ ____ because these values make the denominator equal zero.

Name ________________________________ Date __________ Class __________

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-3 Adding and Subtracting Rational Expressions

Rational expressions can be used to determine average speed.

A ferryboat full of passengers averages 40 mi/h traveling to its destination and 50 mi/h on the return trip with no passengers. What is the ferryboat's average speed for the entire trip? Round to the nearest tenth.

Understand the Problem

1. Describe the ferryboat's speed for the trip. ________________________________

Make a Plan

2. What do you need to determine? ________________________________

3. Use the formula distance = rate × time ($d = rt$) to determine the average speed for the entire trip. Let d represent the one-way distance.

Total distance: __________ — Both legs of the trip have a distance of d.

Time to the destination: $t_1 = \frac{d}{\square}$ — Use the formula $t = \frac{d}{r}$.

Time on the return trip: $t_2 = \frac{d}{\square}$ — Use the formula $t = \frac{d}{r}$.

Total time: $t = \frac{d}{\square} + \frac{d}{\square}$ — Add the time for both legs.

Average speed: $r = \dfrac{\square}{\frac{d}{\square} + \frac{d}{50}}$ — Use average speed $= \dfrac{\text{total distance}}{\text{total time}}$

Solve

4. Multiply the terms in the average speed formula by the LCD, ______, and simplify.

$$\frac{\square(200)}{\frac{d}{\square}(200) + \frac{d}{\square}(200)} = \frac{\square d}{\square d + \square d} = \frac{\square d}{\square d} \approx \text{\_\_\_\_\_\_ mi/h}$$

5. The ferryboat's average speed is ______ mi/h.

Look Back

6. To check your solution, substitute the average speed from Exercise 5 into the average speed equation and choose a distance for d, for example 100 miles.

$$\text{Average speed} = \frac{\square(100)}{\frac{100}{40} + \frac{100}{50}} \approx \text{\_\_\_\_\_\_ mi/h}$$

7. Does your solution make sense? ______

Name ________________________________ Date __________ Class ___________

SECTION **8A**

Ready To Go On? Skills Intervention

8-4 Rational Functions

Find these vocabulary words in Lesson 8-4 and the Multilingual Glossary.

Vocabulary		
rational function	discontinuous function	continuous function

Graphing Rational Functions with Vertical Asymptotes

Identify the zeros and asymptotes of each function. Then graph.

A. $f(x) = \dfrac{2x^3}{x^2 - 25}$

$f(x) = \dfrac{2x^3}{(\square)(\square)}$ Factor the denominator.

Zero(s): __________   The numerator is zero when $x =$ ____.

Vertical asymptote(s): $x =$ ____ and $x =$ ____   The denominator is zero when $x =$ ____.

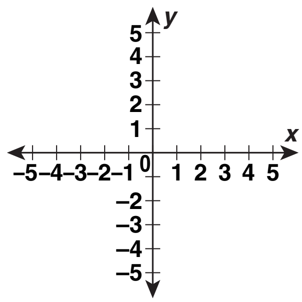
The degree of p (the numerator) is ____________ the degree of q (the denominator).

Horizontal asymptote(s): ________

Complete the table of values:

x	−5	−3	−1	0	1	3	5
y	______	______	______	______	______	______	______

Graph the function using the table of values.



B. $f(x) = \dfrac{3x^2 - 3}{x^2 - 9}$

$f(x) = \dfrac{3(x - 1)(\square)}{(\square)(x + 3)}$ Factor the numerator and denominator.

Zero(s): ___________   The numerator is zero when $x =$ ____.

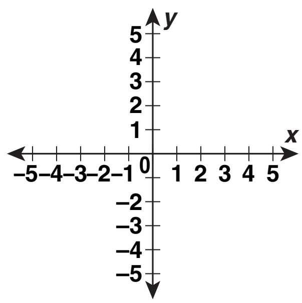
Vertical asymptote(s): $x =$ ____ and $x =$ ____   The denominator is zero when $x =$ ____.

Horizontal asymptote(s): ____   Degree of p ____ degree of q.

Complete the table of values:

x	−5	−3	−1	0	1	3	5
y	______	______	______	______	______	______	______

Graph the function using the table of values.



Name ______________________________ Date __________ Class __________

SECTION 8A

Ready To Go On? Skills Intervention

8-5 Solving Rational Equations and Inequalities

Find these vocabulary words in Lesson 8-5 and the Multilingual Glossary.

Vocabulary		
rational equation	extraneous solution	rational inequality

Solving Rational Equations

Solve the equation $\frac{18}{y} = 11 - y$.

The Least Common Denominator (LCD) is: ________.

$\frac{18}{y}(\_\_\_) = 11(\_\_\_) - y(\_\_\_)$ Multiply each term by the LCD.

$\_\_\_ = 11y - \_\_\_$ Simplify. Note that $y \neq$ ____.

$\_\_\_ - \_\_\_\_ + 18 = 0$ Write in standard form.

$(y - \_\_\_)(\_\_\_ - 2) = 0$ Factor.

$y - \_\_\_ = 0$ or $y - \_\_\_ = 0$ Apply the Zero Product Property.

$y = \_\_\_$ or $y = \_\_\_$ Solve for y.

Check your answer: Substitute the solutions for y into the original equation.

$\frac{18}{y} = 11 - y$

$\frac{18}{\square} = 11 - 9 \rightarrow 2 = 2$ ✓ $\frac{18}{\square} = 11 - 2 \rightarrow 9 = 9$ ✓

Extraneous Solutions

Solve the equation $\frac{4}{x^2 - 4} = \frac{1}{x + 2} + \frac{1}{x - 2}$.

The Least Common Denominator (LCD) is: ______________.

$\frac{4}{x^2 - 4}(x - 2)(x + 2) = \frac{1}{x + 2}(x - 2)(\_\_\_\_\_\_) + \frac{1}{x - 2}(\_\_\_\_\_\_)(\_\_\_\_\_\_)$ Multiply each term by the LCD.

$\_\_\_ = (\_\_\_\_\_\_) + (x + 2)$ Divide out common factors.

$4 = \_\_\_x$ Note that $x \neq$ ____. Simplify.

$\_\_\_ = x$ Solve for x.

The solution $x =$ ____ is ____________ because it makes the denominators of the original equation equal to ____. Therefore, the equation has ______________.

Name ______________________ Date __________ Class __________

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-5 Solving Rational Equations and Inequalities

Ruthann canoes 5 miles upstream and 5 miles downstream a river. Her entire trip takes 6 hours. Given, in still water, Ruthann paddles at an average speed of 2 mi/h, what is the average speed of the river's current? Round to the nearest tenth.

Understand the Problem

1. What is the total distance Ruthann canoed? __________ What was her average speed? __________

2. What do you need to determine? ______________________________

Make a Plan

3. Use the formula distance = rate × time ($d = rt$) to complete the table. Let c represent the speed of the current.

Direction	Distance (mi)	Average Speed (mi/h)	Time (h)
Upstream	5	$2 - c$	$\frac{\square}{2 - \square}$
Downstream	___	______	$\frac{5}{\square + c}$

4. Complete: Total time = time upstream + time downstream

$$6 = \frac{}{\_\_\_\_\_\_} + \frac{}{\_\_\_\_\_\_}$$

Solve

5. Multiply the terms in the equation from Exercise 4 by the LCD and simplify.

$$6(2 - c)(2 + c) = \left(\frac{\square}{2 - c}\right)(2 - \square)(2 + c) + \left(\frac{5}{\square + c}\right)(\square - c)(2 + \square)$$

$\square(2 - c)(2 + c) = \square(2 + c) + 5(\square - c)$ Simplify.

$6(4 - c^2) = (\square + 5c) + (10 - \square)$ Use the Distributive Property.

$\square - 6c^2 = \_\_\_ \rightarrow \_\_\_ = 6c^2 \rightarrow \frac{4}{\square} = c^2 \rightarrow \pm\_\_\_\_ = c$

The speed of the current is ____ mi/h. (Its speed cannot be negative).

Look Back

6. To check your solution, substitute the current's speed into the equation from Exercise 4. $6 = \frac{5}{\square} + \frac{5}{\square}$ Does your solution make sense? _____

Name ______________________ Date ___________ Class ___________

SECTION 8A

Ready To Go On? Quiz

8-1 Variation Functions

1. The price, P, paid for tomatoes varies directly as its weight, w, in pounds. If the price of 1.5 pounds of tomatoes is $2.97, what is the price of 4.99 pounds of tomatoes?

____

2. The simple interest, I, in dollars earned on a certain investment amount varies jointly as the interest rate, r, and the time, t. $I = \$124.80$ when $r = 4\%$ and $t = 2$ years. Find t when $I = \$300.30$ and $r = 5.5\%$.

____

8-2 Multiplying and Dividing Rational Expressions

Simplify. Identify any *x*-values for which the expression is undefined.

3. $\dfrac{12x}{4x^2 + 8x}$

____

4. $\dfrac{x^2 - 6x + 9}{x^2 - 9}$

____

Multiply or divide. Assume that all expressions are defined.

5. $\dfrac{7x - 7}{x^2 - x - 2} \cdot \dfrac{x + 1}{14x - 14}$

____

6. $\dfrac{16x^2 - 1}{x^3y^2} \div \dfrac{4x^2 + 3x - 1}{x^2y + xy}$

____

8-3 Adding and Subtracting Rational Expressions

Add or subtract. Identify any *x*-values for which the expression is undefined.

7. $\dfrac{x + 3}{x^2 + 7x + 6} + \dfrac{x + 1}{x + 6}$

____

8. $\dfrac{1}{x + 5} - \dfrac{x}{x - 5}$

____

9. Matt ran an average speed of 8.3 meters per second during the first lap of a race and an average speed of 7.45 meters per second during the second lap. What was Matt's average speed for the entire race? Round to the nearest hundredth.

____

Name ______________________ Date __________ Class __________

SECTION 8A

Ready To Go On? Quiz continued

8-4 Rational Functions

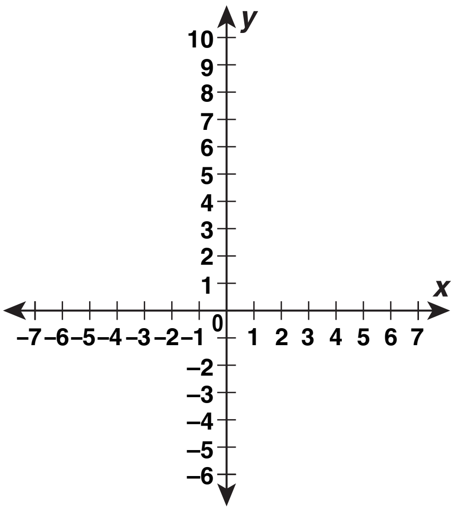
Identify the zeroes and asymptotes of each function. Then graph.

10. $f(x) = \dfrac{x^2 + x - 12}{x}$

zero(s): ______________

vertical asymptote(s): ______________

horizontal asymptote(s): ______________

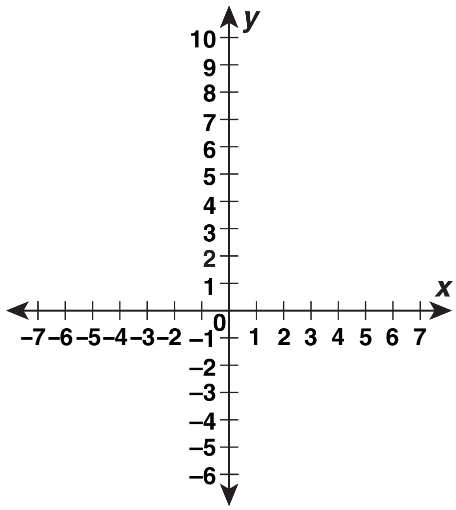


11. $f(x) = \dfrac{4x^2 + x}{x^2 - 4}$

zero(s): ______________

vertical asymptote(s): ______________

horizontal asymptote(s): ______________



8-5 Solving Rational Equations and Inequalities

Solve each equation.

12. $x - \dfrac{10}{x} = -3$

____

13. $\dfrac{x - 1}{x - 12} = \dfrac{2x - 13}{x - 12}$

____

14. Brian can lay a foundation for a house in 10 hours. Together, Brian and Robin can lay a foundation in 6.5 hours. How long will it take Robin to lay a foundation when working alone?

____

Name ____________________ Date __________ Class __________

SECTION 8A Ready To Go On? Enrichment

Continued Fractions

Lesson 8-3 introduced *complex fractions*. A complex fraction contains one or more fractions in its numerator, its denominator or both, as shown below.

$$\frac{\frac{4}{3}}{x+5} \qquad \frac{2+\frac{1}{x}}{x-7}$$

Continued fractions are more complicated. The two fractions below are continued fractions.

Finite: $$4+\cfrac{1}{5+\cfrac{1}{6+\cfrac{1}{2}}}$$

Infinite: $$1+\cfrac{1}{3+\cfrac{1}{3+\cfrac{1}{3+\dots}}}$$

Since each numerator is 1, both examples are *simple* continued fractions. A *finite* simple continued fraction represents a rational number, and an *infinite* simple continued fraction represents an irrational number.

Find a simple fraction for each continued fraction.

1. $7+\cfrac{1}{6+\cfrac{1}{5+\cfrac{1}{4}}}$

2. $1+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2}}}}$

3. $1+\cfrac{1}{1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{2}}}}$

__________ __________ __________

Exercise 1 above can also be written in the condensed form [7, 6, 5, 4]. Write the simple continued fractions in expanded form.

4. [2, 3, 5, 6]

5. $[2, \overline{2, 4}]$

Hint: This one is infinite.
Write the first several terms.

__________ __________

Name ______________________________ Date __________ Class ___________

SECTION 8B

Ready To Go On? Skills Intervention

8-6 Radical Expressions and Rational Exponents

Find these vocabulary words in Lesson 8-6 and the Multilingual Glossary.

Vocabulary	
rational exponent	index

Writing Expressions in Radical Form

Write each expression in radical form, and simplify.

> There are two methods to re-writing this expression:
> 1) evaluate the root first or
> 2) evaluate the power first

A. $(-64)^{\frac{2}{3}}$

$\left(\sqrt[\_]{\_\_\_\_}\right)^{\_}$ Write with a radical.

$\left(\sqrt[\_]{\_\_\_^3}\right)^{\_}$ Factor 64 into a perfect cube.

$(\_\_\_)^{\_}$ Evaluate the root.

$\_\_\_$ Evaluate the power.

B. $(9)^{\frac{5}{2}}$

$\left(\sqrt[\_]{9}\right)^{\_}$ Write with a radical.

$\left(\sqrt[\_]{\_\_^2}\right)^{\_}$ Factor 9 into a perfect square.

$(\_\_)^{\_}$ Evaluate the root.

$\_\_\_\_$ Evaluate the power.

Writing Expressions by using Rational Exponents

Write each expression by using rational exponents.

A. $(\sqrt[4]{100})^3$

$\left(100^{\_}\right)^3$ Rewrite the root as a rational exponent.

$100^{\frac{1}{4}(\_)}$ Apply the _______ of a Power Property of Rational Exponents.

$100^{\_}$ Simplify the exponent.

B. $(\sqrt[5]{-2})^4$

$\left((-2)^{\_}\right)^4$ Rewrite the root as a rational exponent.

$(-2)^{\frac{1}{5}(\_)}$ Apply the _______ of a Power Property of Rational Exponents.

$(-2)^{\_}$ Simplify the exponent.

Name ______________________________________ Date ___________ Class ____________

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-6 Radical Expressions and Rational Exponents

A rational exponent is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written using rational exponents.

The initial amount deposited in a savings account is \$2500. The amount a in dollars in the account after y years can be represented by the function $a(y) = 2500\left(2^{\frac{y}{24}}\right)$. To the nearest dollar, what will the amount in the account be after 8 years?

Understand the Problem

1. What variable are you being asked to solve for in the formula? ____
2. What does \$2500 represent? ____________________

Make a Plan

3. What does y represent in the formula? ________
4. How many years is the money in the account? ____

Solve

5. Substitute known values into the formula and solve.

$a(y) = 2500\left(2^{\frac{y}{24}}\right)$

$a(y) = 2500\left(2^{\frac{\square}{24}}\right)$ Substitute.

$a(y) = 2500\left(2^{\frac{1}{\square}}\right)$ Simplify the fraction.

$a(y) =$ _________ Use a calculator.

$a(y) =$ ______ Round.

6. The amount in the account, to the nearest whole dollar, after 8 years will be \$______.

Look Back

7. Using a graphing calculator and the formula $a(y) = 2500\left(2^{\frac{y}{24}}\right)$ to complete the table.

Year	0	2	4	6	8	10
Amount in Account	\$2500	\$2649	\$______	\$______	\$______	\$______

8. Does the amount in Exercise 6 match the amount in the table for year 8? _____

Name ______________________________ Date __________ Class __________

SECTION **8B**

Ready To Go On? Skills Intervention

8-7 Radical Functions

Find these vocabulary words in Lesson 8-7 and the Multilingual Glossary.

Vocabulary	
radical function	square-root function

Transformations of square-root functions are summarized below.

$|a| \rightarrow$ vertical stretch or compression factor
$a < 0 \rightarrow$ reflection across the x-axis

$h \rightarrow$ horizontal translation

$$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$$

$|b| \rightarrow$ horizontal stretch or compression factor
$b < 0 \rightarrow$ reflection across the y-axis

$k \rightarrow$ vertical translation

Transforming Square Root Functions

Use the description to write the transformed function *g*. Then identify its domain and range.

A. $f(x) = \sqrt{x}$ is translated 2 units left and 1 unit up.

$f(x) =$ ____ Start by identifying the parent function.

$g(x) = \sqrt{x + \square}$ To translate 2 units left, replace x with $x +$ ____.

$g(x) = \sqrt{x + \square} + \square$ To translate 1 unit up, ____ 1 to the right side.

Domain of g: $\{x \mid$ ________ $\}$

Range of g: $\{y \mid$ ________ $\}$

B. $f(x) = \sqrt[3]{x}$ is horizontally compressed by a factor of $\frac{1}{2}$ and reflected across the y-axis.

$f(x) = \sqrt[3]{x}$ Start by identifying the parent function.

$g(x) = \sqrt[3]{\square x}$ Horizontally compress by multiplying x by ____.

$g(x) = \sqrt[3]{-2\square}$ Reflect across the y-axis by replacing x with ____.

Domain of g: $\{x \mid$ ________ $\}$

Range of g: $\{y \mid$ ________ $\}$

C. $f(x) = \sqrt{x}$ is vertically stretched by a factor of 4 and translated 5 units down.

$f(x) = \sqrt{\square}$ Start by identifying the parent function.

$g(x) =$ ___ $\sqrt{x}$ Stretch vertically by multiplying by 4.

$g(x) =$ ___ $\sqrt{x} - 5$ To translate 5 units down, ________ 5 from the right side.

Domain of g: $\{x \mid$ ________ $\}$

Range of g: $\{y \mid$ ________ $\}$

Name ______________________________________ Date ___________ Class ____________

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-7 Radical Functions

A radical function that is vertically stretched can be represented by $af(x)$.

On Mars, the function $f(x) = 4.8\sqrt{x}$ approximates an object's downward velocity in feet per second as the object hits the ground after bouncing x feet in height. The corresponding function for Earth is stretched vertically by a factor of $\frac{5}{3}$. Write the corresponding function g for Earth, and use it to estimate how fast an object will hit the Earth's surface after a bounce of 30 feet in height. Round to the nearest tenth.

Understand the Problem

1. How is the function for Mars translated to represent an objects downward velocity on Earth?

____

Make a Plan

2. What do you need to determine? ______________________________________

____

3. First, determine the transformed function $g(x)$.

$g(x) = \_\_\_\_(4.8\sqrt{x})$ Stretch vertically by multiplying by ____.

$g(x) = \_\_\_\sqrt{x}$ Simplify.

Solve

4. Find the value of g for a bounce of $x = 30$ feet.

$g(x) = 8\sqrt{\square}$ Substitute 30 for x.

$g(x) \approx$ ______ Simplify.

5. The object will hit the Earth's surface with a downward velocity of about ______ ft/s.

Look Back

6. To check your solution, find the value of $f(x)$ for a bounce 30 feet in height on Mars.

$f(x) = 4.8\sqrt{x} = 4.8\sqrt{\square} \approx$ ______ ft/s

Then, multiply this value by a factor of $\frac{5}{3}$.

Is $\frac{5}{3}$ (______) $\approx$ 43.8 ft/s? ______

7. Is your solution in Exercise 5 equal to the value in Exercise 6? ______

Name ______________________________ Date __________ Class __________

SECTION 8B

Ready To Go On? Skills Intervention

8-8 Solving Radical Equations and Inequalities

Find these vocabulary words in Lesson 8-8 and the Multilingual Glossary.

Vocabulary	
radical equation	radical inequality

Solving Equations Containing One Radical

Solve $-7\sqrt[3]{18x - 1} = -14$.

$-7\sqrt[3]{18x - 1} = -14$

$\dfrac{-7\sqrt[3]{18x - 1}}{\square} = \dfrac{-14}{\square}$ Isolate the radical by dividing both sides by -7.

$\sqrt[3]{18x - 1} = \_\_\_\_$ Simplify.

$(\sqrt[3]{18x - 1})– = 2–$ Cube both sides of the equation.

$18x - 1 = \_\_\_\_$ Simplify.

$18x = \_\_\_\_$ Isolate the x-term.

$x = \_\_\_\_$ Solve for x.

Check: Substitute the value of x into the original equation.

$-7\sqrt[3]{18x - 1} = -14$

$-7\sqrt[3]{18\left(\frac{1}{2}\right) - 1} = -14$

$-7(2) = -14$

$-14 = -14$

Solving Equations with Extraneous Solutions

Solve $\sqrt{x + 14} = x + 2$.

$(\sqrt{x + 14})– = (x + 2)–$ Square both sides of the equation.

$x + 14 = \_\_\_\_ + 4x + \_\_\_\_$ Simplify.

$0 = x^2 + \_\_\_\_ - \_\_\_\_$ Write in standard form.

$0 = (x + \_\_\_\_)(x - \_\_\_\_)$ Factor.

$x = \_\_\_\_$ or $x = \_\_\_\_$ Solve for x.

Check for extraneous solutions.

Substitute each value of x into the original equation.

Substitute $x = -5$.

$\sqrt{x + 14} = x + 2$

$\sqrt{-5 + 14} = -5 + 2$

$\sqrt{\square} = -3$

$3 \; \square \; -3$

Substitute $x = 2$.

$\sqrt{x + 14} = x + 2$

$\sqrt{2 + 14} = 2 + 2$

$\sqrt{\square} = 4$

$4 \; \square \; 4$

Because $x = \_\_\_\_$ is extraneous, $x = \_\_\_\_$ is the only solution.

Name ______________________________ Date __________ Class ___________

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-8 Solving Radical Equations and Inequalities

A radical equation contains a variable within a radical. Raising each side of an equation to an even power may introduce extraneous solutions. Remember to check each solution in the original equation.

The speed s in miles per hours that a car is traveling when it goes into a skid can be estimated by the formula $S = 5.5\sqrt{D(F + f)}$, where F is the coefficient of friction, f is the superelevation and D is the length of the skid marks in feet. After an accident, a driver claims to have been traveling the speed limit of 55 mph. The coefficient of friction under the accident conditions was 0.75 and the superelevation was 0.05. How long should the skid marks actually measure if the driver was in fact going 55 mph?

Understand the Problem

1. What variable are you being asked to determine? ______________________

2. What do each of the variables stand for in the formula?

   $S =$ ________ $F =$ ____________________ $f =$ ______________

Make a Plan

3. What variable are you going to solve for? ____ $S =$ _____

4. What number represents each variable in the formula? $F =$ _____

   $f =$ _____

Solve

5. Substitute known values into the formula and solve.

$S = 5.5\sqrt{D(F + f)}$

$55 = 5.5\sqrt{D(\_\_\_\_\_ + 0.05)}$ Substitute.

$55 = 5.5\sqrt{D(\_\_\_\_\_)}$ Simplify under the radical.

$\_\_\_\_ = \sqrt{D(0.8)}$ Divide both sides by 5.5.

$\_\_\_\_\_ = 0.8D$ Square both sides.

$\_\_\_\_\_ = D$ Solve for D.

6. The skid marks should measure ______ feet.

Look Back

7. Use the value for D from Exercise 5 and solve for S.

   Does your answer check? ______

$S = 5.5\sqrt{D(F + f)}$

$S = 5.5\sqrt{\_\_\_(0.75 + 0.05)}$ Substitute.

$S = 5.5\sqrt{\_\_\_}$ Simplify.

$S =$ ___

Name ______________________ Date ____________ Class ____________

SECTION **8B**

Ready To Go On? Quiz

8-6 Radical Expressions and Rational Exponents

Simplify each expression. Assume that all variables are positive.

1. $\sqrt{48x^6y^3}$

____

2. $\sqrt[6]{\dfrac{y^2}{8}}$

____

Write each expression in radical form, and simplify.

3. $25^{\frac{3}{2}}$

____

4. $81^{\frac{3}{4}}$

____

Write each expression by using rational exponents.

5. $\sqrt[4]{7^5}$

____

6. $(\sqrt[3]{-125})^4$

____

7. A town has a population of 14,500 people. The population doubles every 12 years and can be represented by the function $p(t) = 14{,}500 \cdot 2^{\frac{t}{12}}$, where t is the time in years. What is the town's population after 40 years?

____

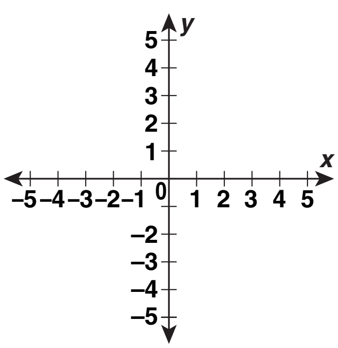
8-7 Radical Functions

Graph each function, and identify its domain and range.

8. $f(x) = -\sqrt{x} + 3$

domain: ________________

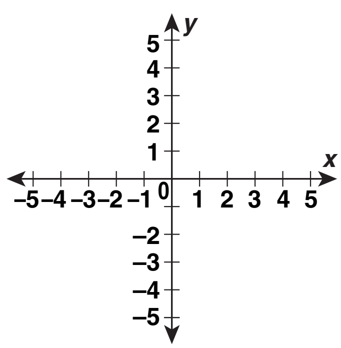
range: ________________



9. $f(x) = 2\sqrt[3]{x + 1}$

domain: ________________

range: ________________



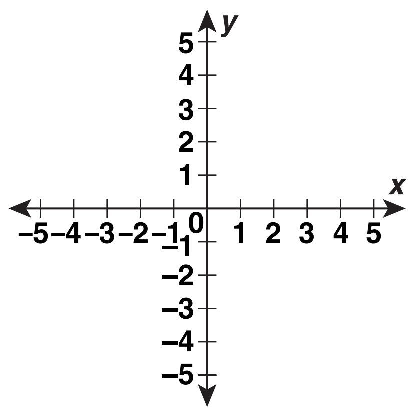
SECTION 8B

Ready To Go On? Quiz continued

10. Water is draining from a pool into two pipes. The speed in feet per second at which water flows through the first pipe is given by $f(x) = \sqrt{36(x - 1)}$, where x is the depth of the water in the pipe. The corresponding function for the second pipe is a translation 2 units up and 3 units left. Write the corresponding function g and estimate the speed at which water flows through the second pipe when the water is 0.5 feet deep.

____

11. Graph the inequality $y \geq -\sqrt{x + 1}$.



8-8 Solving Radical Equations and Inequalities

Solve each equation.

12. $\sqrt{x - 4} = x - 6$

____

13. $2\sqrt[3]{x - 3} = \sqrt[3]{4x}$

____

14. The formula $s = \sqrt{22d}$ relates the speed, s, of a car in miles per hour to the distance, d, in feet that the car travels as it brakes to a stop. Police measure the length of a car's skid marks and then use this formula to determine the speed the car was traveling. What is the length of a car's skid marks if it was traveling 65 mph?

____

Solve each inequality.

15. $\sqrt[3]{3x} > -3$

____

16. $\sqrt{x + 2} - 8 \leq 5$

____

Name ______________________________ Date __________ Class ___________

SECTION 8B

Ready To Go On? Enrichment

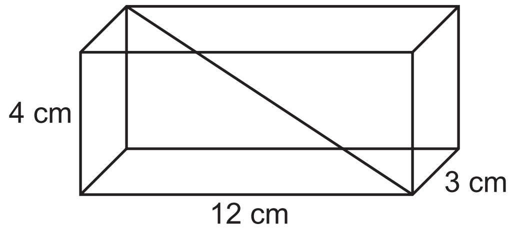
Distance Between Opposite Vertices of a Rectangular Prism

There is a relationship involving the distance between opposite vertices of a rectangular prism.

The length of the diagonal of a rectangular prism can be solved by applying the Pythagorean Theorem *twice*. An alternate way to determine the length of the diagonal d is to apply the formula:

$$d = \sqrt{\ell^2 + w^2 + h^2}$$

where ℓ is the length, w is the width, and h is the height of the prism. Consider the prism:

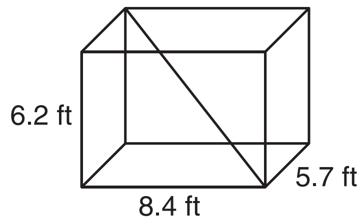


$d = \sqrt{12^2 + 3^2 + 4^2}$
$= 13$ cm
The diagonal is 13 cm.

Solve.

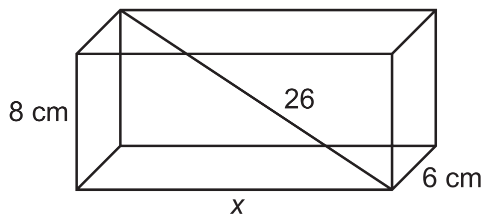
1. Find the length of the diagonal of the prism.

____



2. What is the length of the prism shown at the right?

____



3. A prism has a length of 12 in. and a width of 9 in. What is the height of the prism if its diagonal measures 27 inches?

____

4. The length, width, and height of a prism are tripled. What effect will this have on the length of the diagonal of the prism? Test your answer by tripling the dimensions of the prism in Exercise 2.

____

SECTION 8A
Ready To Go On? Skills Intervention
8-1 Variation Functions

Find these vocabulary words in Lesson 8-1 and the Multilingual Glossary.

Vocabulary		
direct variation	constant of variation	joint variation
inverse variation	combined variation	

Writing Variation Functions

A. Given: y varies directly as x, and $y = 36$ when $x = 4.5$. Write the variation function.

$y =$ kx — y varies directly as x.

36 $= k$(4.5) — Substitute 36 for y and 4.5 for x.

8 $= k$ — Solve for the constant of variation k.

$y =$ 8x — Write the variation function by using the value of k.

Check: Substitute the original values of x and y into the equation.

Does $36 = 8(4.5)$? Yes

B. Given: y varies inversely as x, and $y = 8.4$ when $x = 5$. Write the variation function.

$y = \frac{k}{x}$ — y varies inversely as x.

$8.4 = \frac{k}{5}$ — Substitute 8.4 for y and 5 for x.

42 $= k$ — Solve for the constant of variation k.

$y = \frac{42}{x}$ — Write the variation function by using the value of k.

Check: Substitute the original values of x and y into the equation.

Does $8.4 = \frac{42}{5}$? Yes

C. Given: y varies jointly as x and z, and $y = 52$ when $x = 16$ and $z = 13$. Write the variation function.

$y =$ kxz — y varies jointly as x and z.

52 $= k$(16)(13) — Substitute 52 for y, 16 for x, and 13 for z.

$\frac{52}{208} = k$ — Solve for the constant of variation k.

0.25 $= k$

$y =$ 0.25xz — Write the variation function by using the value of k.

Check: Does $52 = 0.25(16)(13)$? Yes

SECTION 8A
Ready To Go On? Problem Solving Intervention
8-1 Variation Functions

Inverse variation describes a situation in which one quantity increases and the other decreases.

The time t that it takes for a group of volunteers to clean up the park after an event varies inversely as the number of volunteers v. If 15 volunteers can clean up the park in 35 working hours, how many volunteers would be needed to clean up the park in 25 working hours?

Understand the Problem

1. What are you being asked to do? Determine how many volunteers are needed to clean up the park.
2. What type of variation is presented in the problem? Inverse
3. What information are you given? 15 volunteers take 35 hours to clean

Make a Plan

4. What is the general form for determining the variation constant? $t = \frac{k}{v}$
5. What do you need to determine first? The variation constant, k
6. What is t? 35 What is v? 15

Solve

7. Substitute known values to determine k.

$t = \frac{k}{v}$

$35 = \frac{k}{15}$

$525 = k$

8. Use the value for k and let $t = 25$ to determine how many volunteers are needed. $t = \frac{k}{v}$

$25 = \frac{525}{v} \Rightarrow 25v = 525 \Rightarrow v = 21$

9. The park could be cleaned up in 25 hours if there were 21 volunteers.

Look Back

10. Complete the table to check your answer.

Volunteers	15	16	17	18	19	20	21
Hours	35	32.8	30.9	29.2	27.6	26.3	25
Constant	525	525	525	525	525	525	525

11. How many volunteers are needed to clean the park in 25 hours? 21

Does your answer match Exercise 9? Yes

SECTION 8A
Ready To Go On? Skills Intervention
8-2 Multiplying and Dividing Rational Expressions

Find this vocabulary word in Lesson 8-2 and the Multilingual Glossary.

Vocabulary
rational expression

Simplifying Rational Expressions
Simplify. Identify any x-values for which the expression is undefined.

A. $\frac{8x^2}{4x^3 + 8x}$

$\frac{4x(2x)}{4x(x^2 + 2)}$ — Factor out the greatest common factor, 4x.

$\frac{2x}{(x^2 + 2)}$ — Divide out the common factor. Simplify.

The expression is undefined at $x =$ 0 because this value makes the denominator equal zero.

B. $\frac{x^2 - 4}{x^2 + 3x - 10}$

$\frac{(x - 2)(x + 2)}{(x + 5)(x - 2)}$ — Factor the numerator and denominator.

$\frac{(x + 2)}{(x + 5)}$ — Divide out the common factor. Simplify.

The expression is undefined at $x =$ −5 and $x =$ 2.

Dividing Rational Expressions
Divide. Assume that all expressions are defined.

$\frac{x^2 - 9}{12x^3} \div \frac{x^2 + 6x + 9}{6x^3 + 24x^2}$

$\frac{x^2 - 9}{12x^3} \cdot \frac{6x^3 + 24x^2}{x^2 + 6x + 9}$ — Rewrite as multiplication by the reciprocal.

$\frac{(x - 3)(x + 3)}{12x^3} \cdot \frac{6x^2(x + 4)}{(x + 3)(x + 3)}$ — Factor the numerators and denominators.

$\frac{(x - 3)(x + 4)}{2x(x - 3)}$ — Divide out the common factor. Simplify.

SECTION 8A
Ready To Go On? Skills Intervention
8-3 Adding and Subtracting Rational Expressions

Find this vocabulary word in Lesson 8-3 and the Multilingual Glossary.

Vocabulary
complex fraction

Adding Rational Expressions with Like Denominators
Add. Identify any x-values for which the expression is undefined.

$\frac{7x - 3}{x^2 + 4} + \frac{-6x + 11}{x^2 + 4}$ — Notice these rational expressions have *like* denominators.

$\frac{7x - 3 - 6x + 11}{x^2 + 4}$ — Add the numerators.

$\frac{x + 8}{x^2 + 4}$ — Combine like terms.

The expression is undefined at $x =$ no real values because these values make the denominator equal zero. In other words, the expression is always defined.

Subtracting Rational Expressions
Subtract. Identify any x-values for which the expression is undefined.

$\frac{2x^2 - 8}{x^2 - 16} - \frac{x + 1}{x + 4}$

$\frac{2x^2 - 8}{(x - 4)(x + 4)} - \frac{x + 1}{x + 4}$ — Factor the denominators. The LCD is $(x - 4)(x + 4)$.

$\frac{2x^2 - 8}{(x - 4)(x + 4)} - \frac{x + 1}{x + 4} \cdot \frac{(x - 4)}{(x - 4)}$ — Make the 2nd expression's denominator equal to the LCD by multiplying the rational expression by a form of 1.

$\frac{2x^2 - 8 - (x + 1)(x - 4)}{(x - 4)(x + 4)}$ — Subtract the numerators.

$\frac{2x^2 - 8 - (x^2 - 3x - 4)}{(x - 4)(x + 4)}$ — Multiply the binomials in the numerator.

$\frac{2x^2 - 8 - x^2 + 3x + 4}{(x - 4)(x + 4)}$ — Distribute the negative sign.

$\frac{x^2 + 3x - 4}{(x - 4)(x + 4)}$ — Simplify the numerator. Write it in standard form.

$\frac{(x + 4)(x - 1)}{(x - 4)(x + 4)}$ — Factor the numerator.

$\frac{(x - 1)}{(x - 4)}$ — Divide out common factors and simplify.

The expression is undefined at $x =$ −4 and $x =$ 4 because these values make the denominator equal zero.

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-3 Adding and Subtracting Rational Expressions

Rational expressions can be used to determine average speed.

A ferryboat full of passengers averages 40 mi/h traveling to its destination and 50 mi/h on the return trip with no passengers. What is the ferryboat's average speed for the entire trip? Round to the nearest tenth.

Understand the Problem

1. Describe the ferryboat's speed for the trip. The boat travels 40 mi/h for a given distance and then 50 mi/h for the same distance.

Make a Plan

2. What do you need to determine? The boat's average speed for the entire trip.
3. Use the formula distance = rate × time ($d = rt$) to determine the average speed for the entire trip. Let d represent the one-way distance.

Total distance: $2d$	Both legs of the trip have a distance of d.
Time to the destination: $t_1 = \frac{d}{40}$	Use the formula $t = \frac{d}{r}$.
Time on the return trip: $t_2 = \frac{d}{50}$	Use the formula $t = \frac{d}{r}$.
Total time: $t = \frac{d}{40} + \frac{d}{50}$	Add the time for both legs.
Average speed: $r = \dfrac{2d}{\frac{d}{40} + \frac{d}{50}}$	Use average speed = $\dfrac{\text{total distance}}{\text{total time}}$

Solve

4. Multiply the terms in the average speed formula by the LCD, 200, and simplify.

$$\frac{2d(200)}{\frac{d}{40}(200) + \frac{d}{50}(200)} = \frac{400d}{5d + 4d} = \frac{400d}{9d} \approx 44.4 \text{ mi/h}$$

5. The ferryboat's average speed is 44.4 mi/h.

Look Back

6. To check your solution, substitute the average speed from Exercise 5 into the average speed equation and choose a distance for d, for example 100 miles.

$$\text{Average speed} = \frac{2(100)}{\frac{100}{40} + \frac{100}{50}} \approx 44.4 \text{ mi/h}$$

7. Does your solution make sense? Yes

SECTION 8A

Ready To Go On? Skills Intervention

8-4 Rational Functions

Find these vocabulary words in Lesson 8-4 and the Multilingual Glossary.

Vocabulary
rational function   discontinuous function   continuous function

Graphing Rational Functions with Vertical Asymptotes
Identify the zeros and asymptotes of each function. Then graph.

A. $f(x) = \dfrac{2x^3}{x^2 - 25}$

$f(x) = \dfrac{2x^3}{(x-5)(x+5)}$ Factor the denominator.

Zero(s): 0   The numerator is zero when x = 0.

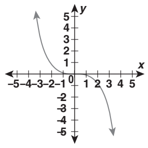
Vertical asymptote(s): x = −5 and x = 5   The denominator is zero when x = ±5.

The degree of p (the numerator) is greater than the degree of q (the denominator).

Horizontal asymptote(s): None

Complete the table of values:

x	−5	−3	−1	0	1	3	5
y	und	3.38	0.08	0	−0.08	−3.38	und



Graph the function using the table of values.

B. $f(x) = \dfrac{3x^2 - 3}{x^2 - 9}$

$f(x) = \dfrac{3(x-1)(x+1)}{(x-3)(x+3)}$ Factor the numerator and denominator.

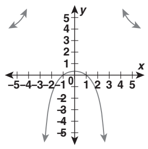
Zero(s): −1 and 1   The numerator is zero when x = ±1.

Vertical asymptote(s): x = −3 and x = 3   The denominator is zero when x = ±3.

Horizontal asymptote(s): 3   Degree of p = degree of q.

Complete the table of values:

x	−5	−3	−1	0	1	3	5
y	4.5	und	0	0.33	0	und	4.5



Graph the function using the table of values.

SECTION 8A

Ready To Go On? Skills Intervention

8-5 Solving Rational Equations and Inequalities

Find these vocabulary words in Lesson 8-5 and the Multilingual Glossary.

Vocabulary
rational equation   extraneous solution   rational inequality

Solving Rational Equations

Solve the equation $\frac{18}{y} = 11 - y$.

The Least Common Denominator (LCD) is: y.

$\frac{18}{y}(y) = 11(y) - y(y)$	Multiply each term by the LCD.
$18 = 11y - y^2$	Simplify. Note that $y \neq 0$.
$y^2 - 11y + 18 = 0$	Write in standard form.
$(y - 9)(y - 2) = 0$	Factor.
$y - 9 = 0$ or $y - 2 = 0$	Apply the Zero Product Property.
$y = 9$ or $y = 2$	Solve for y.

Check your answer: Substitute the solutions for y into the original equation.

$\frac{18}{y} = 11 - y$

$\frac{18}{9} = 11 - 9 \rightarrow 2 = 2$ ✓ $\frac{18}{2} = 11 - 2 \rightarrow 9 = 9$ ✓

Extraneous Solutions

Solve the equation $\frac{4}{x^2 - 4} = \frac{1}{x + 2} + \frac{1}{x - 2}$.

The Least Common Denominator (LCD) is: $(x-2)(x+2)$.

$\frac{4}{x^2-4}(x-2)(x+2) = \frac{1}{x+2}(x-2)(x+2) + \frac{1}{x-2}(x-2)(x+2)$	Multiply each term by the LCD.
$4 = (x - 2) + (x + 2)$	Divide out common factors.
$4 = 2x$	Note that $x \neq \pm 2$. Simplify.
$2 = x$	Solve for x.

The solution x = 2 is extraneous because it makes the denominators of the original equation equal to 0. Therefore, the equation has no solution.

SECTION 8A

Ready To Go On? Problem Solving Intervention

8-5 Solving Rational Equations and Inequalities

Ruthann canoes 5 miles upstream and 5 miles downstream a river. Her entire trip takes 6 hours. Given, in still water, Ruthann paddles at an average speed of 2 mi/h, what is the average speed of the river's current? Round to the nearest tenth.

Understand the Problem

1. What is the total distance Ruthann canoed? 10 miles What was her average speed? 2 mi/hr
2. What do you need to determine? The average speed of the current

Make a Plan

3. Use the formula distance = rate × time ($d = rt$) to complete the table. Let c represent the speed of the current.

Direction	Distance (mi)	Average Speed (mi/h)	Time (h)
Upstream	5	$2 - c$	$\frac{5}{2 - c}$
Downstream	5	$2 + c$	$\frac{5}{2 + c}$

4. Complete: Total time = time upstream + time downstream

$$6 = \frac{5}{2 - c} + \frac{5}{2 + c}$$

Solve

5. Multiply the terms in the equation from Exercise 4 by the LCD and simplify.

$6(2 - c)(2 + c) = \left(\frac{5}{2 - c}\right)(2 - c)(2 + c) + \left(\frac{5}{2 + c}\right)(2 - c)(2 + c)$

$6(2 - c)(2 + c) = 5(2 + c) + 5(2 - c)$ Simplify.

$6(4 - c^2) = (10 + 5c) + (10 - 5c)$ Use the Distributive Property.

$24 - 6c^2 = 20 \rightarrow 4 = 6c^2 \rightarrow \frac{4}{6} = c^2 \rightarrow \pm 0.8 = c$

The speed of the current is 0.8 mi/h. (Its speed cannot be negative).

Look Back

6. To check your solution, substitute the current's speed into the equation from Exercise 4. $6 = \frac{5}{2 - 0.8} + \frac{5}{2 + 0.8}$ Does your solution make sense? Yes

SECTION 8A Ready To Go On? Quiz

8-1 Variation Functions

1. The price, P, paid for tomatoes varies directly as its weight, w, in pounds. If the price of 1.5 pounds of tomatoes is \$2.97, what is the price of 4.99 pounds of tomatoes?

\$9.88

2. The simple interest, I, in dollars earned on a certain investment amount varies jointly as the interest rate, r, and the time, t. I = \$124.80 when $r = 4\%$ and $t = 2$ years. Find t when I = \$300.30 and $r = 5.5\%$.

3.5 years

8-2 Multiplying and Dividing Rational Expressions

Simplify. Identify any x-values for which the expression is undefined.

3. $\frac{12x}{4x^2 + 8x}$

$\frac{3}{x+2}$; $x \neq -2$ and $x \neq 0$

4. $\frac{x^2 - 6x + 9}{x^2 - 9}$

$\frac{x-3}{x+3}$; $x \neq \pm 3$

Multiply or divide. Assume that all expressions are defined.

5. $\frac{7x - 7}{x^2 - x - 2} \cdot \frac{x + 1}{14x - 14}$

$\frac{1}{2(x-2)}$ or $\frac{1}{2x-4}$

6. $\frac{16x^2 - 1}{x^3y^2} \div \frac{4x^2 + 3x - 1}{x^2y + xy}$

$\frac{4x+1}{x^2y}$

8-3 Adding and Subtracting Rational Expressions

Add or subtract. Identify any x-values for which the expression is undefined.

7. $\frac{x + 3}{x^2 + 7x + 6} + \frac{x + 1}{x + 6}$

$\frac{x^2 + 3x + 4}{(x+6)(x+1)}$; $x \neq -6$ and $x \neq -1$

8. $\frac{1}{x + 5} - \frac{x}{x - 5}$

$\frac{-(x^2 + 4x + 5)}{(x+5)(x-5)}$; $x \neq \pm 5$

9. Matt ran an average speed of 8.3 meters per second during the first lap of a race and an average speed of 7.45 meters per second during the second lap. What was Matt's average speed for the entire race? Round to the nearest hundredth.

7.85 m/s

SECTION 8A Ready To Go On? Quiz continued

8-4 Rational Functions

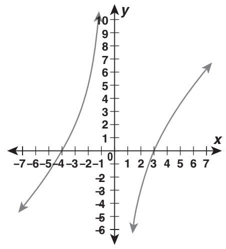
Identify the zeroes and asymptotes of each function. Then graph.

10. $f(x) = \frac{x^2 + x - 12}{x}$

zero(s): $-4; 3$

vertical asymptote(s): $x = 0$

horizontal asymptote(s): none

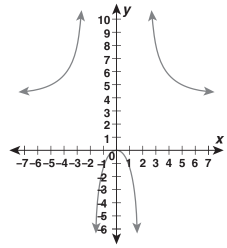


11. $f(x) = \frac{4x^2 + x}{x^2 - 4}$

zero(s): $-\frac{1}{4}; 0$

vertical asymptote(s): $x = -2; x = 2$

horizontal asymptote(s): $y = 4$



8-5 Solving Rational Equations and Inequalities

Solve each equation.

12. $x - \frac{10}{x} = -3$

$x = -5; 2$

13. $\frac{x - 1}{x - 12} = \frac{2x - 13}{x - 12}$

No solution

14. Brian can lay a foundation for a house in 10 hours. Together, Brian and Robin can lay a foundation in 6.5 hours. How long will it take Robin to lay a foundation when working alone?

About 18.6 hours

SECTION 8A Ready To Go On? Enrichment

Continued Fractions

Lesson 8-3 introduced *complex fractions.* A complex fraction contains one or more fractions in its numerator, its denominator or both, as shown below.

$$\frac{\frac{4}{3}}{x+5} \qquad \frac{2 + \frac{1}{x}}{x - 7}$$

Continued fractions are more complicated. The two fractions below are continued fractions.

Finite: $4 + \cfrac{1}{5 + \cfrac{1}{6 + \cfrac{1}{2}}}$

Infinite: $1 + \cfrac{1}{3 + \cfrac{1}{3 + \cfrac{1}{3 + \ldots}}}$

Since each numerator is 1, both examples are *simple* continued fractions. A *finite* simple continued fraction represents a rational number, and an *infinite* simple continued fraction represents an irrational number.

Find a simple fraction for each continued fraction.

1. $7 + \cfrac{1}{6 + \cfrac{1}{5 + \cfrac{1}{4}}}$

$\frac{931}{130}$

2. $1 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2 + \cfrac{1}{2}}}}$

$\frac{41}{29}$

3. $1 + \cfrac{1}{1 + \cfrac{1}{2 + \cfrac{1}{1 + \cfrac{1}{2}}}}$

$\frac{19}{11}$

Exercise 1 above can also be written in the condensed form [7, 6, 5, 4]. Write the simple continued fractions in expanded form.

4. [2, 3, 5, 6]

$2 + \cfrac{1}{3 + \cfrac{1}{5 + \cfrac{1}{6}}}$

5. $[2, \overline{2, 4}]$

Hint: This one is infinite. Write the first several terms.

$2 + \cfrac{1}{2 + \cfrac{1}{4 + \cfrac{1}{2 + \cfrac{1}{4 \ldots}}}}$

SECTION 8B Ready To Go On? Skills Intervention

8-6 Radical Expressions and Rational Exponents

Find these vocabulary words in Lesson 8-6 and the Multilingual Glossary.

Vocabulary	
rational exponent	index

Writing Expressions in Radical Form

Write each expression in radical form, and simplify.

There are two methods to re-writing this expression: 1) evaluate the root first or 2) evaluate the power first

A. $(-64)^{\frac{2}{3}}$

$\left(\sqrt[3]{-64}\right)^2$ Write with a radical.

$\left(\sqrt[3]{-4^3}\right)^2$ Factor 64 into a perfect cube.

$(-4)^2$ Evaluate the root.

16 Evaluate the power.

B. $(9)^{\frac{5}{2}}$

$\left(\sqrt[2]{9}\right)^5$ Write with a radical.

$\left(\sqrt[2]{3^2}\right)^5$ Factor 9 into a perfect square.

$(3)^5$ Evaluate the root.

243 Evaluate the power.

Writing Expressions by using Rational Exponents

Write each expression by using rational exponents.

A. $(\sqrt[4]{100})^3$

$\left(100^{\frac{1}{4}}\right)^3$ Rewrite the root as a rational exponent.

$100^{\frac{1}{4}(3)}$ Apply the Power of a Power Property of Rational Exponents.

$100^{\frac{3}{4}}$ Simplify the exponent.

B. $(\sqrt[5]{-2})^4$

$\left((-2)^{\frac{1}{5}}\right)^4$ Rewrite the root as a rational exponent.

$(-2)^{\frac{1}{5}(4)}$ Apply the Power of a Power Property of Rational Exponents.

$(-2)^{\frac{4}{5}}$ Simplify the exponent.

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-6 Radical Expressions and Rational Exponents

A rational exponent is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written using rational exponents.

The initial amount deposited in a savings account is \$2500. The amount a in dollars in the account after y years can be represented by the function $a(y) = 2500\left(2^{\frac{y}{24}}\right)$. To the nearest dollar, what will the amount in the account be after 8 years?

Understand the Problem

1. What variable are you being asked to solve for in the formula? a
2. What does \$2500 represent? The initial deposit

Make a Plan

3. What does y represent in the formula? Years
4. How many years is the money in the account? 8

Solve

5. Substitute known values into the formula and solve.

$a(y) = 2500\left(2^{\frac{y}{24}}\right)$

$a(y) = 2500\left(2^{\frac{8}{24}}\right)$ Substitute.

$a(y) = 2500\left(2^{\frac{1}{3}}\right)$ Simplify the fraction.

$a(y) =$ 3149.80 Use a calculator.

$a(y) =$ 3150 Round.

6. The amount in the account, to the nearest whole dollar, after 8 years will be \$3150.

Look Back

7. Using a graphing calculator and the formula $a(y) = 2500\left(2^{\frac{y}{24}}\right)$ to complete the table.

Year	0	2	4	6	8	10
Amount in Account	\$2500	\$2649	\$ 2806	\$ 2973	\$ 3150	\$ 3337

8. Does the amount in Exercise 6 match the amount in the table for year 8? Yes

SECTION 8B

Ready To Go On? Skills Intervention

8-7 Radical Functions

Find these vocabulary words in Lesson 8-7 and the Multilingual Glossary.

Vocabulary
radical function square-root function

Transformations of square-root functions are summarized below.

$|a|$ → vertical stretch or compression factor
$a < 0$ → reflection across the x-axis

h → horizontal translation

$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$

$|b|$ → horizontal stretch or compression factor
$b < 0$ → reflection across the y-axis

k → vertical translation

Transforming Square Root Functions
Use the description to write the transformed function g. Then identify its domain and range.

A. $f(x) = \sqrt{x}$ is translated 2 units left and 1 unit up.

$f(x) = \sqrt{x}$ Start by identifying the parent function.

$g(x) = \sqrt{x + 2}$ To translate 2 units left, replace x with $x +$ 2.

$g(x) = \sqrt{x + 2} + 1$ To translate 1 unit up, add 1 to the right side.

Domain of g: $\{x \mid x \geq -2\}$

Range of g: $\{y \mid y \geq 1\}$

B. $f(x) = \sqrt[3]{x}$ is horizontally compressed by a factor of $\frac{1}{2}$ and reflected across the y-axis.

$f(x) = \sqrt[3]{x}$ Start by identifying the parent function.

$g(x) = \sqrt[3]{2x}$ Horizontally compress by multiplying x by 2.

$g(x) = \sqrt[3]{-2x}$ Reflect across the y-axis by replacing x with $-x$.

Domain of g: $\{x \mid x \in \text{Reals}\}$

Range of g: $\{y \mid y \in \text{Reals}\}$

C. $f(x) = \sqrt{x}$ is vertically stretched by a factor of 4 and translated 5 units down.

$f(x) = \sqrt{x}$ Start by identifying the parent function.

$g(x) = 4\sqrt{x}$ Stretch vertically by multiplying by 4.

$g(x) = 4\sqrt{x} - 5$ To translate 5 units down, subtract 5 from the right side.

Domain of g: $\{x \mid x \geq 0\}$

Range of g: $\{y \mid y \geq -5\}$

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-7 Radical Functions

A radical function that is vertically stretched can be represented by $af(x)$.

On Mars, the function $f(x) = 4.8\sqrt{x}$ approximates an object's downward velocity in feet per second as the object hits the ground after bouncing x feet in height. The corresponding function for Earth is stretched vertically by a factor of $\frac{5}{3}$. Write the corresponding function g for Earth, and use it to estimate how fast an object will hit the Earth's surface after a bounce of 30 feet in height. Round to the nearest tenth.

Understand the Problem

1. How is the function for Mars translated to represent an objects downward velocity on Earth?
$f(x)$ is stretched vertically by $\frac{5}{3}$.

Make a Plan

2. What do you need to determine? $g(x)$ and the downward velocity of an object with a bounce height of 30 ft

3. First, determine the transformed function $g(x)$.

$g(x) = \frac{5}{3}(4.8\sqrt{x})$ Stretch vertically by multiplying by $\frac{5}{3}$.

$g(x) = 8\sqrt{x}$ Simplify.

Solve

4. Find the value of g for a bounce of $x = 30$ feet.

$g(x) = 8\sqrt{30}$ Substitute 30 for x.

$g(x) \approx 43.8$ Simplify.

5. The object will hit the Earth's surface with a downward velocity of about 43.8 ft/s.

Look Back

6. To check your solution, find the value of $f(x)$ for a bounce 30 feet in height on Mars.

$f(x) = 4.8\sqrt{x} = 4.8\sqrt{30} \approx 26.29$ ft/s

Then, multiply this value by a factor of $\frac{5}{3}$.

Is $\frac{5}{3}(26.29) \approx 43.8$ ft/s? Yes

7. Is your solution in Exercise 5 equal to the value in Exercise 6? Yes

SECTION 8B

Ready To Go On? Skills Intervention

8-8 Solving Radical Equations and Inequalities

Find these vocabulary words in Lesson 8-8 and the Multilingual Glossary.

Vocabulary
radical equation radical inequality

Solving Equations Containing One Radical

Solve $-7\sqrt[3]{18x - 1} = -14$.

$-7\sqrt[3]{18x - 1} = -14$

$\frac{-7\sqrt[3]{18x - 1}}{-7} = \frac{-14}{-7}$ Isolate the radical by dividing both sides by -7.

$\sqrt[3]{18x - 1} = 2$ Simplify.

$(\sqrt[3]{18x - 1})^3 = 2^3$ Cube both sides of the equation.

$18x - 1 = 8$ Simplify.

$18x = 9$ Isolate the x-term.

$x = \frac{1}{2}$ Solve for x.

Check: Substitute the value of x into the original equation.

$-7\sqrt[3]{18x - 1} = -14$

$-7\sqrt[3]{18\left(\frac{1}{2}\right) - 1} = -14$

$-7(2) = -14$

$-14 = -14$

Solving Equations with Extraneous Solutions

Solve $\sqrt{x + 14} = x + 2$.

$(\sqrt{x + 14})^2 = (x + 2)^2$ Square both sides of the equation.

$x + 14 = x^2 + 4x + 4$ Simplify.

$0 = x^2 + 3x - 10$ Write in standard form.

$0 = (x + 5)(x - 2)$ Factor.

$x = -5$ or $x = 2$ Solve for x.

Check for extraneous solutions.

Substitute each value of x into the original equation.

Substitute $x = -5$.	Substitute $x = 2$.
$\sqrt{x + 14} = x + 2$	$\sqrt{x + 14} = x + 2$
$\sqrt{-5 + 14} = -5 + 2$	$\sqrt{2 + 14} = 2 + 2$
$\sqrt{9} = -3$	$\sqrt{16} = 4$
$3 \neq -3$	$4 = 4$

Because $x = -5$ is extraneous, $x = 2$ is the only solution.

SECTION 8B

Ready To Go On? Problem Solving Intervention

8-8 Solving Radical Equations and Inequalities

A radical equation contains a variable within a radical. Raising each side of an equation to an even power may introduce extraneous solutions. Remember to check each solution in the original equation.

The speed s in miles per hours that a car is traveling when it goes into a skid can be estimated by the formula $S = 5.5\sqrt{D(F + f)}$, where F is the coefficient of friction, f is the superelevation and D is the length of the skid marks in feet. After an accident, a driver claims to have been traveling the speed limit of 55 mph. The coefficient of friction under the accident conditions was 0.75 and the superelevation was 0.05. How long should the skid marks actually measure if the driver was in fact going 55 mph?

Understand the Problem

1. What variable are you being asked to determine? **The length of skid marks**
2. What do each of the variables stand for in the formula?
   S = **Speed** F = **Coefficient of friction** f = **Superelevation**

Make a Plan

3. What variable are you going to solve for? **D** S = **55**
4. What number represents each variable in the formula? F = **0.75**, f = **0.05**

Solve

5. Substitute known values into the formula and solve.

$S = 5.5\sqrt{D(F + f)}$

$55 = 5.5\sqrt{D(\mathbf{0.75} + 0.05)}$ Substitute.

$55 = 5.5\sqrt{D(\mathbf{0.80})}$ Simplify under the radical.

$\mathbf{10} = \sqrt{D(0.8)}$ Divide both sides by 5.5.

$\mathbf{100} = 0.8D$ Square both sides.

$\mathbf{125} = D$ Solve for D.

6. The skid marks should measure **125** feet.

Look Back

7. Use the value for D from Exercise 5 and solve for S. Does your answer check? **Yes**

$S = 5.5\sqrt{D(F + f)}$

$S = 5.5\sqrt{\mathbf{125}(0.75 + 0.05)}$ Substitute.

$S = 5.5\sqrt{\mathbf{100}}$ Simplify.

$S = \mathbf{55}$

SECTION 8B

Ready To Go On? Quiz

8-6 Radical Expressions and Rational Exponents

Simplify each expression. Assume that all variables are positive.

1. $\sqrt{48x^6y^3}$ **$4x^3y\sqrt{3y}$**
2. $\sqrt[6]{\frac{y^2}{8}}$ **$\frac{\sqrt{2}(\sqrt[3]{y})}{2}$**

Write each expression in radical form, and simplify.

3. $25^{\frac{3}{2}}$ **$\sqrt{25}^3 = 125$**
4. $81^{\frac{3}{4}}$ **$\sqrt[4]{81}^3 = 27$**

Write each expression by using rational exponents.

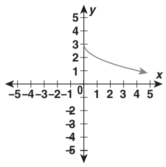
5. $\sqrt[4]{7^5}$ **$7^{\frac{5}{4}}$**
6. $(\sqrt[3]{-125})^4$ **$(-125)^{\frac{4}{3}}$**

7. A town has a population of 14,500 people. The population doubles every 12 years and can be represented by the function $p(t) = 14{,}500 \cdot 2^{\frac{t}{12}}$, where t is the time in years. What is the town's population after 40 years?
   About 146,151 people

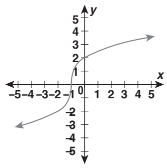
8-7 Radical Functions

Graph each function, and identify its domain and range.

8. $f(x) = -\sqrt{x} + 3$
   domain: **$\{x \mid x \geq 0\}$**
   range: **$\{y \mid y \leq 3\}$**



9. $f(x) = 2\sqrt[3]{x + 1}$
   domain: **$\{x \mid x \in R\}$**
   range: **$\{y \mid y \in R\}$**

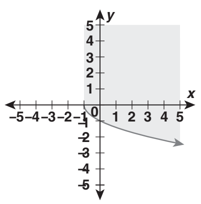


SECTION 8B

Ready To Go On? Quiz continued

10. Water is draining from a pool into two pipes. The speed in feet per second at which water flows through the first pipe is given by $f(x) = \sqrt{36(x - 1)}$, where x is the depth of the water in the pipe. The corresponding function for the second pipe is a translation 2 units up and 3 units left. Write the corresponding function g and estimate the speed at which water flows through the second pipe when the water is 0.5 feet deep.
    $g(x) = \sqrt{36(x + 2)} + 2$; ≈ 11.5 ft/s

11. Graph the inequality $y \geq -\sqrt{x + 1}$.



8-8 Solving Radical Equations and Inequalities

Solve each equation.

12. $\sqrt{x - 4} = x - 6$ **$x = 8$**
13. $2\sqrt[3]{x - 3} = \sqrt[3]{4x}$ **$x = 6$**

14. The formula $s = \sqrt{22d}$ relates the speed, s, of a car in miles per hour to the distance, d, in feet that the car travels as it brakes to a stop. Police measure the length of a car's skid marks and then use this formula to determine the speed the car was traveling. What is the length of a car's skid marks if it was traveling 65 mph?
    about 192 feet

Solve each inequality.

15. $\sqrt[3]{3x} > -3$ **$x > -9$**
16. $\sqrt{x + 2} - 8 \leq 5$ **$-2 \leq x \leq 167$**

SECTION 8B

Ready To Go On? Enrichment

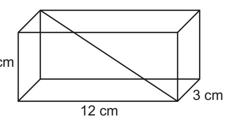
Distance Between Opposite Vertices of a Rectangular Prism

There is a relationship involving the distance between opposite vertices of a rectangular prism.

The length of the diagonal of a rectangular prism can be solved by applying the Pythagorean Theorem *twice*. An alternate way to determine the length of the diagonal d is to apply the formula:

$$d = \sqrt{\ell^2 + w^2 + h^2}$$

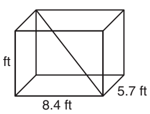
where ℓ is the length, w is the width, and h is the height of the prism. Consider the prism:



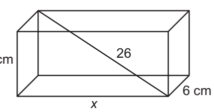
$d = \sqrt{12^2 + 3^2 + 4^2}$
$= 13$ cm
The diagonal is 13 cm.

Solve.

1. Find the length of the diagonal of the prism.
   ≈11.9 ft



2. What is the length of the prism shown at the right?
   24 cm



3. A prism has a length of 12 in. and a width of 9 in. What is the height of the prism if its diagonal measures 27 inches?
   22.4 in.

4. The length, width, and height of a prism are tripled. What effect will this have on the length of the diagonal of the prism? Test your answer by tripling the dimensions of the prism in Exercise 2.
   The length of the diagonal is also tripled.