Date ______________

Dear Family,

In Chapter 8, your child will learn about variation functions, rational functions, and radical functions.

A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. The variable k is called the **constant of variation**. In a direct variation, as the value of one variable *increases*, the value of the other variable *increases*.

direct variation: $y = 2x$

x	y
-3	-6
1	2
10	20

An **inverse variation** is a relationship between x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. Here, as the value of one variable *increases*, the value of the other variable *decreases*.

inverse variation: $y = \frac{1}{x}$

x	y
$\frac{1}{5}$	5
1	1
10	$\frac{1}{10}$

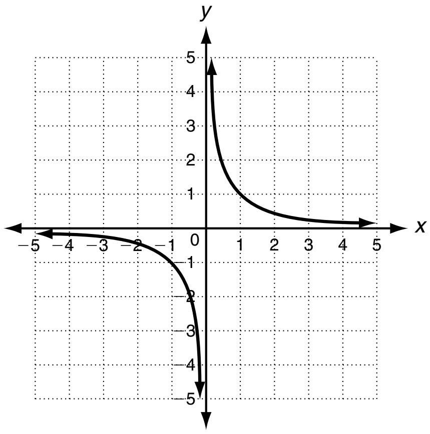
A *rational number* is a number that can be written as a ratio (or quotient) of two integers. For example, $\frac{1}{5}$ is rational because it is the ratio of 1 to 5. In a very general sense, rational numbers can be written as fractions.

A **rational expression** is an expression that can be written as a ratio of two polynomials. For example, $\frac{1}{x}$, $\frac{x+3}{x-7}$, and $\frac{x^2+3x+2}{x+1}$ are rational expressions because the numerators and denominators are polynomials.

Because rational expressions are similar to fractions, you operate with them the same way you operate with fractions.

Operation	How To	Example
Simplify	Divide out common factors in the numerator and denominator.	$\frac{x^2+3x+2}{x^2-1} = \frac{\cancel{(x+1)}(x+2)}{\cancel{(x+1)}(x-1)} = \frac{x+2}{x-1}$
Multiply	Divide out common factors of the numerators and denominators; then multiply numerators and multiply denominators	$\frac{10x-40}{x^2-6x+8} \cdot \frac{x+3}{5x+15} =$ $\frac{(2)\cancel{(5)}\cancel{(x-4)}}{(x-2)\cancel{(x-4)}} \cdot \frac{\cancel{x+3}}{\cancel{5}\cancel{(x+3)}} = \frac{2}{x-2}$
Divide	Multiply by the reciprocal of the divisor.	$\frac{4x^3}{9x^2y} \div \frac{16}{9y^5} = \frac{4x^3}{9x^2y} \cdot \frac{9y^5}{16} = \frac{\cancel{4}x\cancel{(x^2)}}{\cancel{9}\cancel{x^2}\cancel{y}} \cdot \frac{\cancel{9}\cancel{y}(y^4)}{\cancel{4}\cdot 4} = \frac{xy^4}{4}$
Addition / Subtraction	Rewrite both expressions with the least common denominator; then add or subtract the numerators only.	$\frac{x+3}{x^2-4} + \frac{5}{x-2} = \underbrace{\frac{x+3}{(x+2)(x-2)} + \frac{5}{(x-2)}\left(\frac{x+2}{x+2}\right)}_{\text{Use factors of the denominators to find the LCD.}}$ $= \frac{(x+3)+(5x+10)}{(x+2)(x-2)} = \frac{6x+13}{x^2-4}$

A **rational function** is a function whose rule is a rational expression. At right is the graph of $f(x) = \frac{1}{x}$. The special shape of this graph is called a *hyperbola*.



The graphs of many rational functions are **discontinuous** because they have gaps or breaks. Rational functions frequently have **asymptotes**, and some have omitted points called **holes**. The graph of $f(x) = \frac{1}{x}$ is broken into two branches, has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$, and has no holes.

A **radical expression** can be written in the form $\sqrt[n]{a}$. The radical sign $(\sqrt{})$ indicates that you are finding a root of a power, and the **index** n tells you which power. So, the fourth root of 81, or $\sqrt[4]{81}$, is 3 because $3^4 = 81$.

Radical expressions can also be written as **rational exponents**.

$$\sqrt[n]{a} = a^{\frac{1}{n}} \qquad (\sqrt[n]{a})^m = \left(a^{\frac{1}{n}}\right)^m = a^{\frac{m}{n}} \qquad \left(\sqrt[n]{a^m}\right) = (a^m)^{\frac{1}{n}} = a^{\frac{m}{n}}$$

So, $\sqrt[4]{81}$ could also be written as $81^{\frac{1}{4}}$. By using rational exponents, you can apply the properties of exponents (taught in Chapter 1) to simplify radical expressions.

A **radical function** is a function whose rule is a radical expression. For example, $f(x) = \sqrt{x}$ and $f(x) = \sqrt[3]{2x + 4}$ are radical functions.

Your child will also learn how to solve **rational equations** (which contain one or more rational expressions) and **radical equations** (which contain a variable within a radical). You solve a rational equation by multiplying both sides of the equation by the least common denominator of all of the expressions, which eliminates the denominators. You solve a radical equation by raising both sides of the equation to a power that eliminates the radicals. However, these solution processes sometimes result in **extraneous solutions** that do not check in the original equation.

$$\begin{aligned}
\sqrt{x + 18} &= x - 2 \\
\left(\sqrt{x + 18}\right)^2 &= (x - 2)^2 \\
x + 18 &= x^2 - 4x + 4 \\
0 &= x^2 - 5x - 14 \\
0 &= (x - 7)(x + 2) \\
x &= 7 \text{ or } \cancel{x = -2}
\end{aligned}$$

Check $\sqrt{x + 18} = x - 2$

$\sqrt{x + 18}$	$x - 2$
$\sqrt{7 + 18}$	$7 - 2$
$\sqrt{25}$	5
5	5 ✓

$\sqrt{x + 18} = x - 2$

$\sqrt{x + 18}$	$x - 2$
$\sqrt{-2 + 18}$	$-2 -2$
$\sqrt{16}$	-4
4	-4 ▯

Although $x = -2$ appears to be a solution, it does not check. It is an extraneous solution.

For additional resources, visit go.hrw.com and enter the keyword MB7 Parent.