CHAPTER 7

Vocabulary

The table contains important vocabulary terms from Chapter 7. As you work through the chapter, fill in the page number, definition, and a clarifying example.

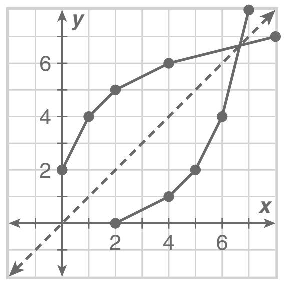
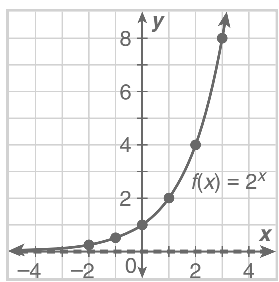
Term	Page	Definition	Clarifying Example
base of an exponential function			
common logarithm			
exponential decay			
exponential equation			
exponential function			
exponential growth			

Vocabulary

The table contains important vocabulary terms from Chapter 7. As you work through the chapter, fill in the page number, definition, and a clarifying example.

Term	Page	Definition	Clarifying Example
base of an exponential function	490	**The value of *b* in a function of the form $f(x) = ab^x$, where *a* and *b* are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.**	**In the function $f(x) = 5(2)^x$, the base is 2.**
common logarithm	506	**A logarithm whose base is 10, denoted $\log_{10}$ or just log.**	**$\log 100 = \log_{10} 100 = 2$, since $10^2 = 100$.**
exponential decay	490	**An exponential function of the form $f(x) = ab^x$ in which $0 < b < 1$. If *r* is the rate of decay, then the function can be written $y = a(1 - r)^t$, where *a* is the initial amount and *t* is the time.**	**$f(x) = \left(\frac{1}{2}\right)^x$**
exponential equation	522	**An equation that contains one or more exponential expressions.**	**$2^{x+1} = 8$**
exponential function	490	**A function of the form $f(x) = ab^x$, where *a* and *b* are real numbers with $a \neq 0$, $b > 0$, and $b \neq 1$.**	**$f(x) = 4 \cdot 3^x$**
exponential growth	490	**An exponential function of the form $f(x) = ab^x$ in which $b > 1$. If *r* is the rate of growth, then the function can be written $y = a(1 + r)^t$, where *a* is the initial amount and *t* is the time.**	**$f(x) = 2^x$**

Term	Page	Definition	Clarifying Example
inverse function			
inverse relation			
logarithm			
logarithmic equation			
logarithmic function			
natural logarithm			

Term	Page	Definition	Clarifying Example
inverse function	499	**The function that results from exchanging the input and output values of a one-to-one function. The inverse of $f(x)$ is denoted $f^{-1}(x)$.**	**Function: $f(x) = x + 6$ Inverse function: $f(x) = x - 6$**
inverse relation	498	**The inverse of the relation consisting of all ordered pairs (x, y) is the set of all ordered pairs (y, x). The graph of an inverse relation is the reflection of the graph of the relation across the line $y = x$.**	
logarithm	505	**The exponent that a specified base must be raised to in order to get a certain value.**	**$\log_2 8 = 3$, because 3 is the power that 2 is raised to in order to get 8; $2^3 = 8$.**
logarithmic equation	523	**An equation that contains a logarithm of a variable.**	**$\log x + 3 = 7$**
logarithmic function	507	**A function of the form $f(x) = \log_b x$, where $b \neq 1$ and $b > 0$, which is the inverse of the exponential function $f(x) = b^x$.**	**$f(x) = \log_2 x$** 
natural logarithm	531	**A logarithm with base *e*, written as ln.**	**$\ln 5 = \log_e 5 \approx 1.6$**

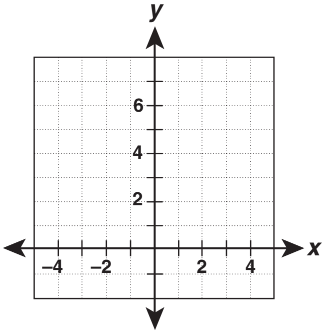
CHAPTER 7

Chapter Review

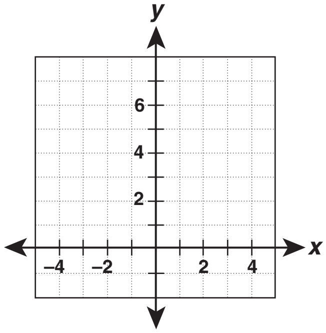
7-1 Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

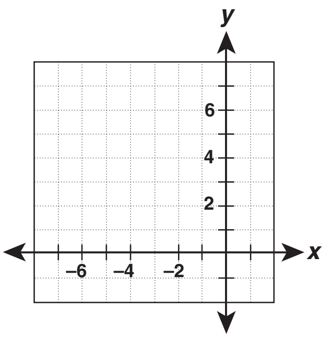
1. $f(x) = \left(\frac{1}{2}\right)^x$



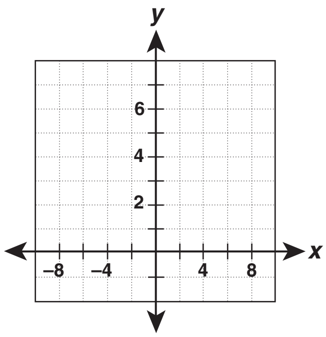
2. $f(x) = \frac{1}{4}(0.25)^x$



3. $f(x) = 12(1.5)^x$



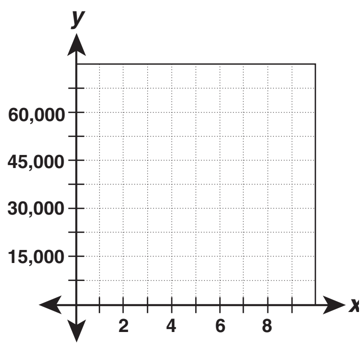
4. $f(x) = 3.2\left(1\frac{1}{4}\right)^x$



5. Suppose that the number of bacteria in a culture was 1200 on Sunday and the number has been increasing at a rate of 75% per day since then.

a. Write a function representing the growth of the culture per day.

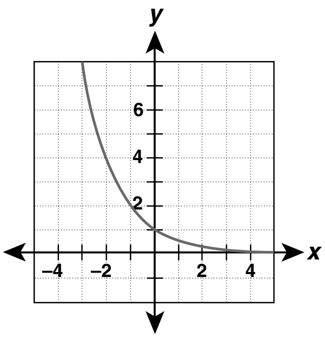
b. Graph the function, and use the graph to predict the number of bacteria in the culture the following Sunday.



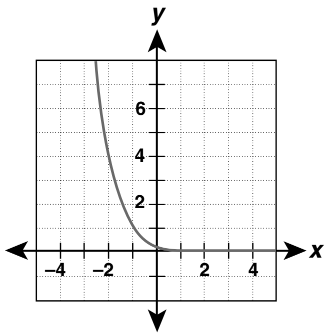
7-1 Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

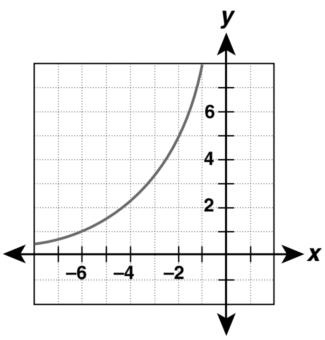
1. $f(x) = \left(\frac{1}{2}\right)^x$ decay



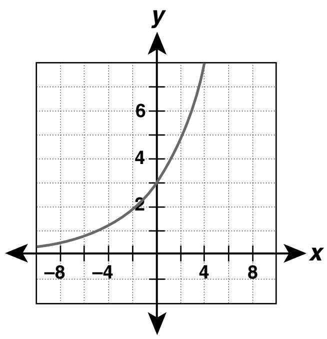
2. $f(x) = \frac{1}{4}(0.25)^x$ decay



3. $f(x) = 12(1.5)^x$ growth



4. $f(x) = 3.2\left(1\frac{1}{4}\right)^x$ growth



5. Suppose that the number of bacteria in a culture was 1200 on Sunday and the number has been increasing at a rate of 75% per day since then.

a. Write a function representing the growth of the culture per day.

$p = 1200(1 + 0.75)^d$

b. Graph the function, and use the graph to predict the number of bacteria in the culture the following Sunday.

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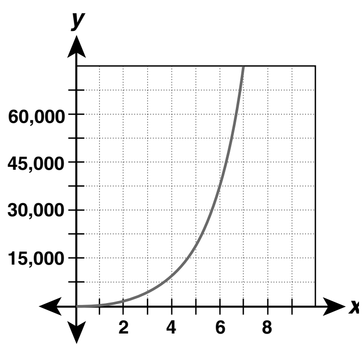
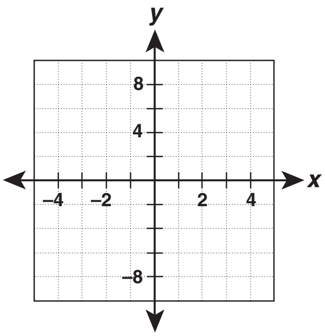


7-2 Inverses of Relations and Functions

Graph each relation. Then graph its inverse.

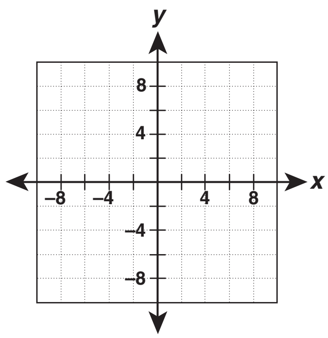
6.

x	-1	0	1	2	3
y	-5	-2	1	4	7



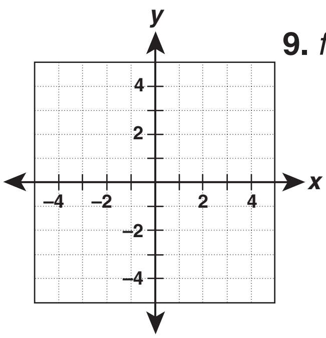
7.

x	-2	-1	0	1	2
y	-8	-1	0	1	8

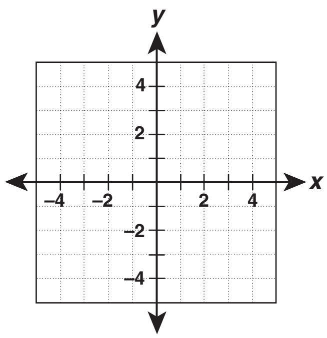


Graph each function. Then write and graph the inverse.

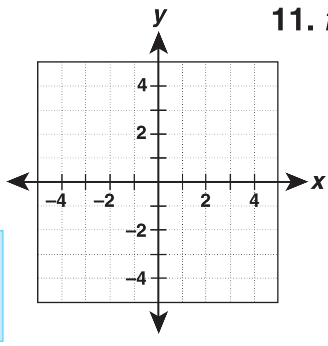
8. $f(x) = x + 3.4$



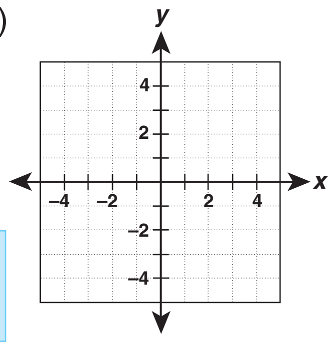
9. $f(x) = \frac{1}{2} - x$



10. $f(x) = 3x - 2$



11. $f(x) = \frac{1}{4}(x - 3)$



12. Junie's washing machine repair bill includes $150 for parts and $45 per hour for labor. Her bill can be expressed as a function of hour x by $f(x) = 150 + 45x$. Find the inverse function. Use it to find the number of hours of labor she was charged if her bill was $262.50.

7-2 Inverses of Relations and Functions

Graph each relation. Then graph its inverse.

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x	-1	0	1	2	3
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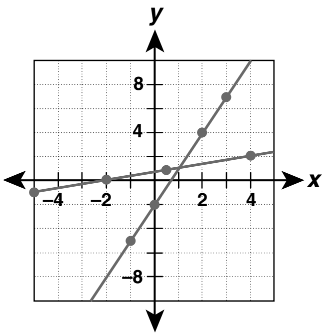
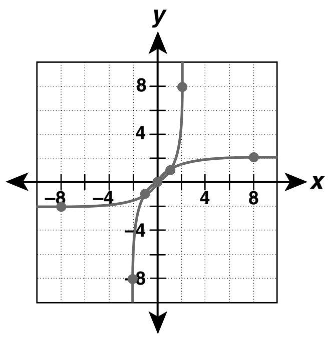


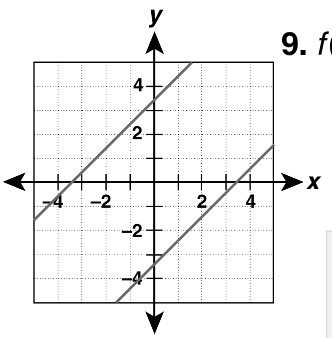
7.

x	-2	-1	0	1	2
y	-8	-1	0	1	8



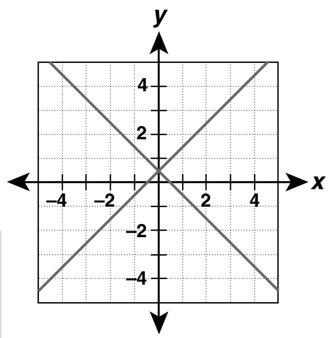
Graph each function. Then write and graph the inverse.

8. $f(x) = x + 3.4$



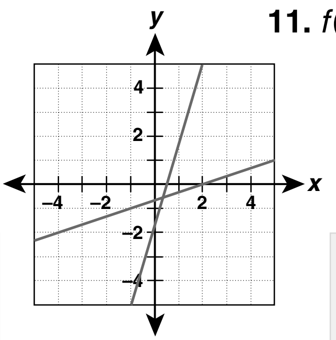
$f(x) = x - 3.4$

9. $f(x) = \frac{1}{2} - x$



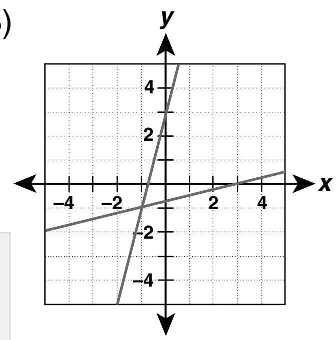
$f(x) = \frac{1}{2} - x$

10. $f(x) = 3x - 2$



$f(x) = \frac{1}{3}x - \frac{2}{3}$

11. $f(x) = \frac{1}{4}(x - 3)$



$y = 4x + 3$

12. Junie's washing machine repair bill includes \$150 for parts and \$45 per hour for labor. Her bill can be expressed as a function of hour x by $f(x) = 150 + 45x$. Find the inverse function. Use it to find the number of hours of labor she was charged if her bill was \$262.50.

$f(x)^{-1} = \frac{x - 150}{45}$; 2.5 hours

7-3 Logarithmic Functions

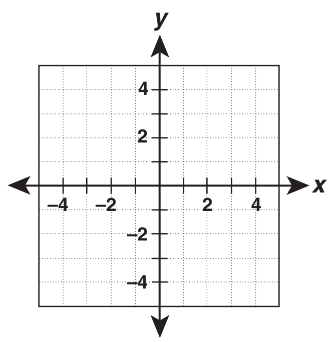
Write the exponential function in logarithmic form.

13. $4^2 = 16$ **14.** $14.3^0 = 1$ **15.** $2^{-3} = 0.125$ **16.** $0.5^x = 8$

Write the logarithmic function in exponential form.

17. $\log_2 512 = 9$ **18.** $\log_{\frac{1}{6}} 36 = -2$ **19.** $\log_{0.45} 0 = 1$ **20.** $\log_e x = 7$

21. Use the given x-values to graph $f(x) = \left(\frac{1}{2}\right)^x$; $x = -2, -1, 0, 1, 2$. Then graph the inverse function.



7-4 Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

22. $\log_2 16 + \log_2 4$ **23.** $\log_3 \frac{1}{27} + \log_3 \frac{1}{81}$ **24.** $\log_{\frac{1}{4}} 64 + \log_{\frac{1}{4}} \left(\frac{1}{16}\right)$

Simplify each expression.

25. $\log_3 243^2$ **26.** $\log_{\frac{1}{6}} 216$ **27.** $5^{\log_5 \frac{1}{25}}$

Evaluate.

28. $\log_{16} 32$ **29.** $\log_{\frac{1}{1000}} 100$ **30.** $\log_{128} 16$

7-3 Logarithmic Functions

Write the exponential function in logarithmic form.

13. $4^2 = 16$ **14.** $14.3^0 = 1$ **15.** $2^{-3} = 0.125$ **16.** $0.5^x = 8$

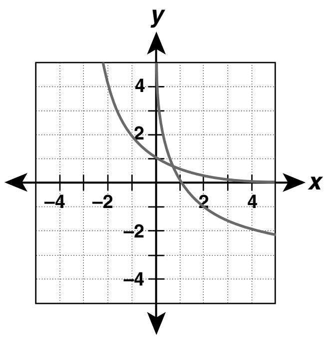
$\log_4 16 = 2$ $\log_{14.3} 1 = 0$ $\log_2 0.125 = -3$ $\log_{0.5} 8 = x$

Write the logarithmic function in exponential form.

17. $\log_2 512 = 9$ **18.** $\log_{\frac{1}{6}} 36 = -2$ **19.** $\log_{0.45} 0 = 1$ **20.** $\log_e x = 7$

$2^9 = 512$ $\left(\frac{1}{6}\right)^{-2} = 36$ $0.45^0 = 1$ $e^7 = x$

21. Use the given x-values to graph $f(x) = \left(\frac{1}{2}\right)^x$; $x = -2, -1, 0, 1, 2$. Then graph the inverse function.



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Express as a single logarithm. Simplify, if possible.

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6 -7 -1

Simplify each expression.

25. $\log_3 243^2$ **26.** $\log_{\frac{1}{6}} 216$ **27.** $5^{\log_5 \frac{1}{25}}$

10 -3 $\frac{1}{25}$

Evaluate.

28. $\log_{16} 32$ **29.** $\log_{\frac{1}{1000}} 100$ **30.** $\log_{128} 16$

$\frac{5}{4}$ $\frac{-2}{3}$ $\frac{4}{7}$

7-5 Exponential and Logarithmic Equations and Inequalities

Solve.

31. $5^x = \frac{1}{625}$

32. $36^{x-2} > 6^{3x}$

33. $10^{2x+1} = 72$

34. $3^{2x-2} = 30$

35. $\log_3(x + 2) \leq 4$

36. $\log_4 x^{\frac{2}{3}} = 3$

37. $\log 25 + \log x = 3$

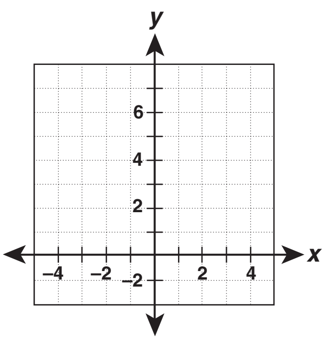
38. $2\log_3 x + \log_3(x - 2) = 2$

39. Suppose you deposit \$1000 into an account that pays 1.8% compounded quarterly. The equation $A = P(1 + r)^n$ gives the amount A in the account after n quarters for an initial investment of P that earns interest at a rate of r. Use logarithms to solve for n to find how long it will take for the account to contain at least \$1200.

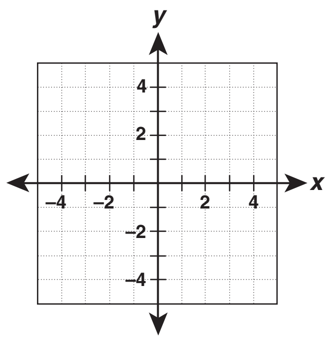
7-6 The Natural Base, *e*

Graph.

40. $f(x) = e^x + 4$



41. $f(x) = 4 - e^x$



7-5 Exponential and Logarithmic Equations and Inequalities

Solve.

31. $5^x = \frac{1}{625}$

$x = -4$

32. $36^{x-2} > 6^{3x}$

$x < -4$

33. $10^{2x+1} = 72$

$x \approx 0.43$

34. $3^{2x-2} = 30$

$x \approx 2.55$

35. $\log_3(x + 2) \leq 4$

$x \leq 79$

36. $\log_4 x^{\frac{2}{3}} = 3$

$x = 512$

37. $\log 25 + \log x = 3$

$x = 40$

38. $2\log_3 x + \log_3(x - 2) = 2$

$x = 3$

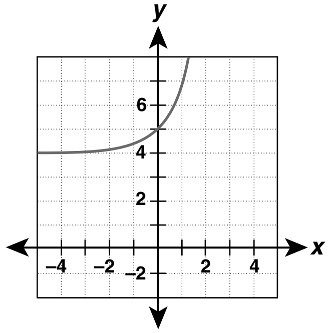
39. Suppose you deposit \$1000 into an account that pays 1.8% compounded quarterly. The equation $A = P(1 + r)^n$ gives the amount A in the account after n quarters for an initial investment of P that earns interest at a rate of r. Use logarithms to solve for n to find how long it will take for the account to contain at least \$1200.

10.2 quarters or 2.6 years

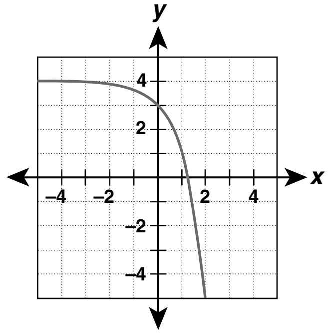
7-6 The Natural Base, *e*

Graph.

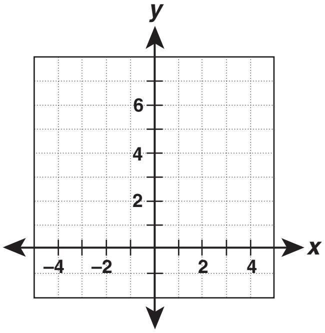
40. $f(x) = e^x + 4$



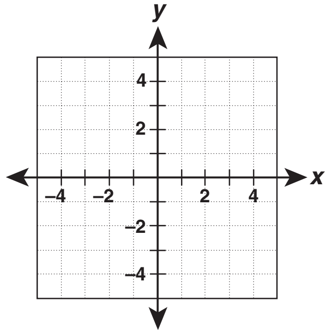
41. $f(x) = 4 - e^x$



42. $f(x) = \frac{e^x}{4}$



43. $f(x) = 4(e^x - 1)$



Simplify.

44. $\ln e^3$

45. $\ln e^{\sqrt{x}}$

46. $e^{\ln(2x-5)}$

47. $\ln e^{x-3}$

48. An accident at a nuclear power plant released 12 grams of radioactive plutonium-239 into the atmosphere. The half-life of plutonium-239 is 24,360 years.

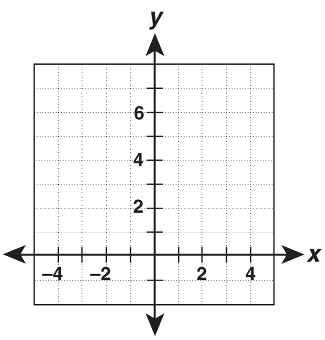
a. Use the formula $\frac{1}{2} = e^{-kt}$ to find the value of the decay constant for plutonium-239.

b. Use the decay function $N_t = N_0e^{-kt}$ to determine how much of the 12 grams of plutonium-239 will remain after 500 years.

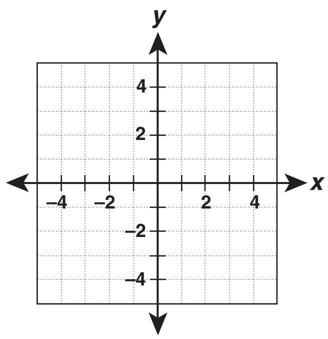
7-7 Transforming Exponential and Logarithmic Functions

Graph the function. Find the *y*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

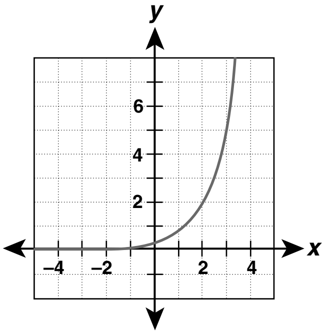
49. $f(x) = 0.2(2^x)$



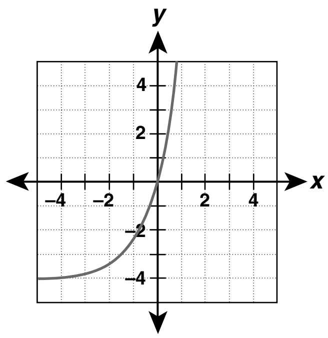
50. $g(x) = e^{(3x)}$



42. $f(x) = \frac{e^x}{4}$



43. $f(x) = 4(e^x - 1)$



Simplify.

44. $\ln e^3$

3

45. $\ln e^{\sqrt{x}}$

$\sqrt{x}$

46. $e^{\ln(2x-5)}$

$2x - 5$

47. $\ln e^{x-3}$

$x - 3$

48. An accident at a nuclear power plant released 12 grams of radioactive plutonium-239 into the atmosphere. The half-life of plutonium-239 is 24,360 years.

a. Use the formula $\frac{1}{2} = e^{-kt}$ to find the value of the decay constant for plutonium-239.

$k \approx 2.85 \times 10^{-5}$

b. Use the decay function $N_t = N_0 e^{-kt}$ to determine how much of the 12 grams of plutonium-239 will remain after 500 years.

11.8 grams

7-7 Transforming Exponential and Logarithmic Functions

Graph the function. Find the *y*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

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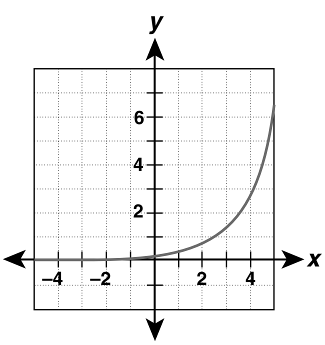
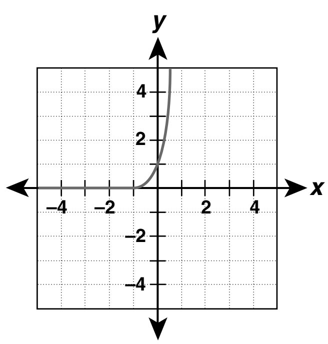


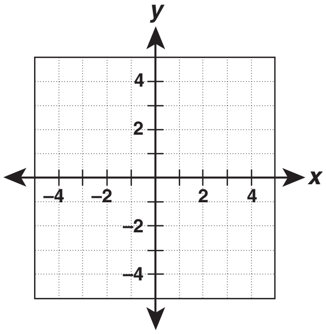
y-intercept: 0.2
asymptote: $y = 0$
The graph has been stretched vertically by a factor of 0.2

50. $g(x) = e^{(3x)}$

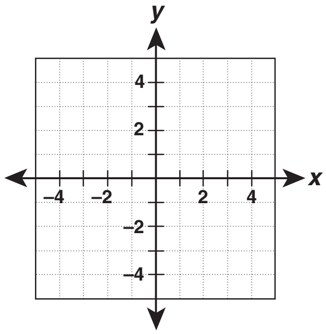


y-intercept: 1
asymptote: $y = 0$
The graph has been horizontally compressed by a factor of 3

51. $h(x) = 2.1\log(x - 2)$



52. $p(x) = -\ln(x + 1)$



Write the transformed function.

53. $f(x) = 2^x$ is reflected across the y-axis and translated 2 units to the right.

7-8 Curve Fitting with Exponential and Logarithmic Models

Determine whether y is an exponential function of *x*. If so, find the constant ratio. Then use exponential regression to find a function that models the data.

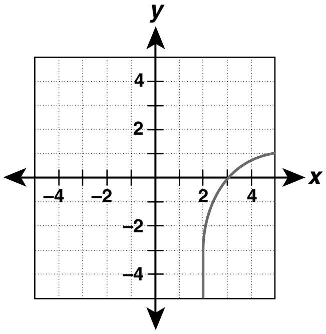
54.

x	0	1	2	3	4	5
y	−1	1	5	11	19	29

55.

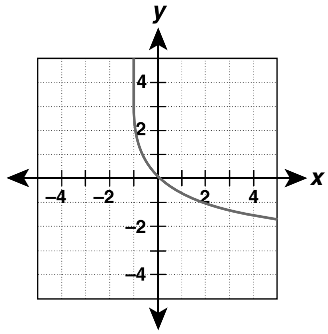
x	0	1	2	3	4	5
y	0.5	1.5	4.5	13.5	40.5	121.5

51. $h(x) = 2.1\log(x - 2)$



y-intercept: none
asymptote: $x = 2.1$
The graph has been translated horizontally 2 units to the right and stretched vertically by a factor of 2.1

52. $p(x) = -\ln(x + 1)$



y-intercept: 0
asymptote: $x = -1$
The graph has been reflected about the *x*-axis, translated 1 unit left.

Write the transformed function.

53. $f(x) = 2^x$ is reflected across the *y*-axis and translated 2 units to the right.

$f(x) = 2^{-(x-2)}$

7-8 Curve Fitting with Exponential and Logarithmic Models

Determine whether y is an exponential function of *x*. If so, find the constant ratio. Then use exponential regression to find a function that models the data.

54.

x	0	1	2	3	4	5
y	−1	1	5	11	19	29

No, it is a quadratic polynomial

55.

x	0	1	2	3	4	5
y	0.5	1.5	4.5	13.5	40.5	121.5

Yes; constant ratio: 3: $f(x) = (0.5)3^x$

Big Ideas

Answer these questions to summarize the important concepts from Chapter 7 in your own words.

1. Explain how to determine if an exponential function is a growth equation or a decay equation.

2. Explain how to find the inverse of a function, if it exists.

3. Explain why the ln $e = 1$ by converting to logarithmic form.

4. Explain how to solve an exponential equation.

For more review of Chapter 7:

- Complete the Chapter 7 Study Guide and Review on pages 554–557 of your textbook.
- Complete the Ready to Go On quizzes on pages 521 and 553 of your textbook.

Big Ideas

Answer these questions to summarize the important concepts from Chapter 7 in your own words.

1. Explain how to determine if an exponential function is a growth equation or a decay equation.

Answers will vary. Possible answer: If the growth factor is greater than one, the function is a growth function. If the growth factor is less than one, the function is a decay function.

2. Explain how to find the inverse of a function, if it exists.

Answers will vary. Possible answer: Interchange *x* and *y* in the original function and solve for *y*.

3. Explain why the ln $e = 1$ by converting to logarithmic form.

Answers will vary. Possible answer: ln $e = 1 \Leftrightarrow e^1 = e$

4. Explain how to solve an exponential equation.

Answers will vary. Possible answer: Convert the exponential equation to logarithmic form and solve.

For more review of Chapter 7:

- Complete the Chapter 7 Study Guide and Review on pages 554–557 of your textbook.
- Complete the Ready to Go On quizzes on pages 521 and 553 of your textbook.