LESSON 7-6

Success for English Language Learners

The Natural Base, e

Steps for Success

Step I Introduce the lesson using the following procedures.

- Have students discuss the definition of *e*, including the fact that it is irrational. Remind students that an irrational number is a number that does not terminate. Write out the value for *e* as 2.7182818284590 . . . .
- Discuss natural logarithms with students, including the use of the abbreviation ln. A natural logarithm is a logarithm with a base of *e*. Tell students that natural logarithms have the same properties as other logarithms.

Step II Teach the lesson.

- Have students note that when they simplify an expression with *e* or ln, they use the same properties as they did with other logarithms.
- Emphasize to students that $f(x) = \ln x$ is the inverse of $f(x) = e^x$. Remind students that the inverse can mean "the opposite" and can be a process that undoes an operation.

Step III Ask English Language Learners to complete the worksheet for this lesson.

- Point out that Example 1 in the student textbook is supported by Problem 1 on the worksheet. Help students visualize the process by which the table was created by asking them to substitute values of *x* into the function.
- Point out that Example 3 in the student textbook is supported by Problem 2 on the worksheet. Explain that students should refer to the beginning of the lesson for the meanings of the variables.
- Think and Discuss supports the problems on the worksheet.

Making Connections

- To tie the lesson regarding *e* and natural logarithms to other numbers that occur in nature, review π as the ratio of the circumference of a circle to its diameter. You may also want to review a Fibonacci sequence, which is a natural pattern found in nature with 1,1, 2, 3, 5, 8,13, 21, ...

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Problem 1

Graph $f(x) = e^x + 2$.

Input x	Output $f(x) = e^x + 2$
-3	2.0
-2	2.1
-1	2.4
0	3
1	4.7
2	9.4
3	22.1

The values of x are input and $f(x)$ is the output. So e is treated in the function simply as a number.

Use the ordered pairs from this input and output to draw the graph.

Problem 2

What is the total amount for an investment of $1000 invested at 5% for 10 years compounded continuously?

r is the annual interest rate.

t is the time in years.

A is the total amount.

$$A = Pe^{rt}$$

P is the principal.

e is a number, approximately 2.71.

$$A = 1000e^{0.05(10)}$$

$$A \approx 1648.72$$

The total amount is $1648.72.

Think and Discuss

1. How would you work with an irrational number in a function?

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2. What would you do if you had to give the value of e?

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3. How do you know what operation to do first when evaluating $A = Pe^{rt}$?

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Lesson 7-5

1. Use the properties of logarithms or use the base of the logarithm as the base for both sides.
2. If the bases are equal, the exponents must be equal.
3. The Order of Operations says do exponents first.

Lesson 7-6

1. Operators work on an irrational number the same way that they work on any other number.
2. If you had to give a value of *e*, you would have to round it.
3. The Order of Operations says do exponents first.

Lesson 7-7

1. It is of the form $f(x) = a^x + k$.
2. It is of the form $f(x) = a^{(x + k)}$.
3. If it is of the form $f(x) = a \cdot b^x$, it is a vertical transformation. If it is of the form $f(x) = b^{(a \cdot x)}$, it is a horizontal transformation.

Lesson 7-8

1. It has a constant ratio of *y*-values for equally spaced *x*-values.
2. If the ratios are close to constant, an exponential function may be appropriate.
3. You would use an exponential regression.

CHAPTER 8

Lesson 8-1

1. A graph of direction variation passes through the origin.
2. Direct variation is a function in the form $y = kx$, in which *y* varies directly as *x*.
3. Inverse variation is a function in the form $y = \frac{k}{x}$, in which *y* varies inversely as *x*.

Lesson 8-2

1. You can apply the Quotient of Powers Property to subtract exponents that are being divided.
2. Factoring out -1 will give you an expression in which x^2 is positive. You may be able to factor the expression with x^2 and then divide out common factors.
3. When you have a result that has division by 0, it is considered undefined.

Lesson 8-3

1. You know an *x*-value is undefined when it equals 0.
2. Add or subtract the numerators, but leave the denominators as the same.
3. The least common multiple is the smallest amount divisible by each expression.

Lesson 8-4

1. You know a rational function is translated to the left by the value of h in $f(x) = \frac{1}{x - 3}$.
2. You know the vertical asymptote at the line $x = h$ in the form $f(x) = \frac{a}{x - h} + k$.
3. You know the horizontal asymptote at the line $y = k$ in the form $f(x) = \frac{a}{x - h} + k$.

Lesson 8-5

1. You can multiply by the LCD. Then simplify.
2. Check solutions in the equation to make sure that they make it true.
3. Jason and Lacy have separate rates and rates that involve working together. You can add their rates of working together to find Lacy's separate rate.