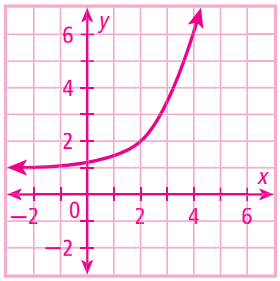
7-6 The Natural Base, e

Example 1 Graphing Exponential Functions

Graph $f(x) = e^{x-2} + 1$.

Make a table. Because e is irrational, the table values are rounded to the nearest tenth.



x	−2	−1	0	1	2	3	4
$f(x) = e^{x-2} + 1$	1.0	1.0	1.1	1.4	2	3.7	8.4

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Example 2 Simplifying Expression with e or ln

Simplify.

A. $\ln e^{0.15t}$

$\ln e^{0.15t} = 0.15t$

B. $e^{3\ln(x+1)}$

$e^{3\ln(x+1)} = e^{\ln(x+1)^3}$

$= (x+1)^3$

C. $\ln e^{2x} + \ln e^{x}$

$\ln e^{2x} + \ln e^{x}$

$= 2x + x = 3x$

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Example 3 Economics Application

What is the total amount for an investment of $500 invested at 5.25% for 40 years and compounded continuously?

$A = Pe^{rt}$

$A = 500e^{0.0525(40)}$ *Substitute 500 for P, 0.0525 for r, and 40 for t.*

$A \approx 4083.08$ *Use the e^x key on a calculator.*

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500e^(.0525*40)
        4083.084956
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The total amount is $4083.08.

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Example 4 Paleontology Application

Plutonium-239 (Pu-239) has a half-life of 24,110 years. How long does it take for a 1 g sample of Pu-239 to decay to 0.1 g?

Step 1 Find the decay constant for plutonium-239.

$N(t) = N_0 e^{-kt}$ *Use the natural decay function.*

$\frac{1}{2} = 1e^{-k(24{,}110)}$ *Substitute 1 for N_0, 24,110 for t, and $\frac{1}{2}$ for $N(t)$ because half of the initial quantity will remain.*

$\ln \frac{1}{2} = \ln e^{-24{,}110k}$ *Simplify and take the ln of both sides.*

$\ln 2^{-1} = -24{,}110k$ *Write $\frac{1}{2}$ as 2^{-1}, and simplify the right side.*

$-\ln 2 = -24{,}110k$ *$\ln 2^{-1} = -1 \ln 2 = -\ln 2$*

$k = \frac{\ln 2}{24{,}110} \approx 0.000029$

Step 2 Write the decay function and solve for *t*.

$N(t) = N_0 e^{-0.000029t}$ *Substitute 0.000029 for k.*

$0.1 = 1e^{-0.000029t}$ *Substitute 1 for N0 and 0.01 for $N(t)$.*

$\ln 0.1 = \ln e^{-0.000029t}$ *Take the ln of both sides.*

$\ln 0.1 = -0.000029t$ *Simplify.*

$t = -\frac{\ln 0.1}{0.000029} \approx 80{,}000$

It takes approximately 80,000 years to decay.