Name ______________________________ Date __________ Class __________

LESSON 7-1

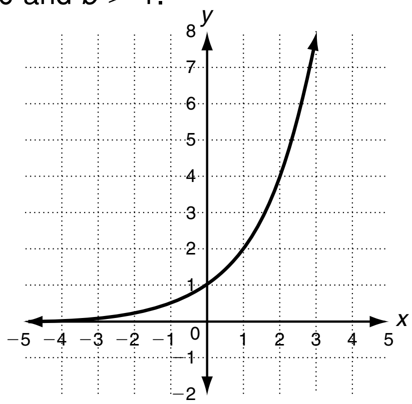
Reading Strategy

Drawing Conclusions

In an exponential function, the variable appears as an exponent: $f(x) = ab^x$, where a is a constant and b is the base. Depending on the value of b, the function either increases (grows) or decreases (decays). You can draw conclusions about the function and its graph based on the value of b.

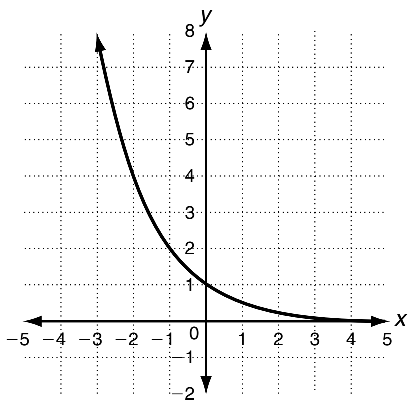
Exponential Growth

An exponential function shows growth if $a > 0$ and $b > 1$.



Exponential Decay

An exponential function shows decay if $a > 0$ and $0 < b < 1$.



1. Complete the table.

x	−2	−1	0	1	2	3
$f(x) = 3^x$	$\frac{1}{9}$					
$f(x) = 0.4^x$						

Use the function $f(x) = 3^x$ for Exercises 2 and 3.

2. Does the function $f(x) = 3^x$ show exponential growth or decay? Explain.

____

3. Is $f(4)$ greater than or less than $f(3)$? Explain how you can draw this conclusion.

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Use the function $f(x) = 0.4^x$ for Exercises 4 and 5.

4. Does the function $f(x) = 0.4^x$ show exponential growth or decay? Explain.

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5. Is $f(-3)$ greater than or less than $f(-2)$? Explain how you can draw this conclusion.

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LESSON 7-1

Reteach

Exponential Functions, Growth, and Decay (continued)

When an initial amount, a, increases or decreases by a constant rate, r, over a number of time periods, t, this formula shows the final amount, $A(t)$.

$$A(t) = a(1 + r)^t$$

- $A(t)$, the final amount, is a function of time, t.
- a is the initial amount.
- Time, t, is measured in years.
- The rate, r, usually is a percent.

An initial amount of $15,000 increases by 12% per year. In how many years will the amount reach $25,000?

Step 1 Identify values for a and r.

$a = \$15{,}000$ $\quad$ $r = 12\% = 0.12$

Step 2 Substitute values for a and r into the formula.

$f(t) = a(1 + r)^t$

$f(t) = 15{,}000(1 + 0.12)^t$

$f(t) = 15{,}000(1.12)^t$ $\quad$ *Simplify.*

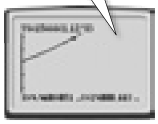
Step 3 Graph the function using a graphing calculator. Modify the scales: [0, 10] and [0, 30,000].

Step 4 Use the graph and the [TRACE] feature on the calculator to find $f(t) = 25{,}000$.

Step 5 Use the graph to approximate the value of t when $f(t) = 25{,}000$.

$t = 4.5$ when $f(t) = 25{,}000$

Remember:
On the graph, x corresponds to t and y corresponds to $f(t)$.



The amount will reach $25,000 in about 4.5 years.

Write an exponential function and graph the function to solve.

3. An initial amount of $40,000 increases by 8% per year. In how many years will the amount reach $60,000?

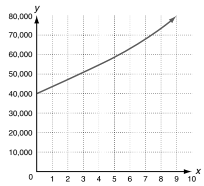
a. $a =$ **40,000**

b. $r =$ **0.08**

c. $f(t) =$ $\mathbf{f(t) = 40{,}000(1.08)^t}$

d. Approximate t when $f(t) = 60{,}000$

$t \approx$ **5.25 yr**



LESSON 7-1

Challenge

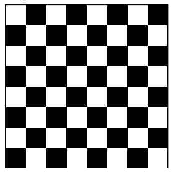
The Vizier and His Wheat

According to legend, Sissa Ben Dahir, the Vizier of the court of King Shirham of India, worked diligently and invented a new game that was called Chess. The King decided to grant Sissa the reward of his choosing. Sissa pondered carefully and requested the following from the King.

One grain of wheat on the first square of the chessboard, two grains of wheat on the second square, four grains on the third square, eight on the fourth square, and so on.

The King thought this was a very modest request and said that he would grant the Vizier's request.

At right is a chessboard with 64 squares.



1. Make a table showing the number of grains of wheat on the first ten squares and the total grains of wheat on squares 1 through n, for $n = 1, 2, 3, \ldots, 10$.

Square n	Grains of Wheat on Square n	Total Grains of Wheat on Board
1	1	1
2	2	3
3	4	7
4	8	15
5	16	31
6	32	63
7	64	127
8	128	255
9	256	511
10	512	1023

2. Using the information from the table, look for a pattern and write an expression for the number of grains of wheat that would be placed on square n. $\quad$ $\mathbf{2^{n-1}}$

3. How many grains of wheat would be placed on the last square? $\quad$ $\mathbf{2^{63} = 9{,}223{,}372{,}036{,}854{,}775{,}808}$

4. Look for a pattern and write a formula for the total number of grains of wheat on the board after wheat has been placed on square n. $\quad$ $\mathbf{2^{n} - 1}$

5. What is the total number of grains of wheat that Sissa received? $\quad$ $\mathbf{2^{64} - 1 = 18{,}446{,}744{,}073{,}709{,}551{,}615}$

6. One grain of wheat weighs approximately 0.000008 kilogram. Find the total weight of wheat the Vizier requested. $\quad$ **147,573,952,589,676 kilograms**

7. In 2000 the world's wheat production was approximately 580 million metric tons. At this rate how many years would it take to fill Sissa's request? One metric ton is 1000 kilograms. $\quad$ **254.4 years**

LESSON 7-1

Problem Solving

Exponential Functions, Growth, and Decay

Justin drove his pickup truck about 22,000 miles in 2004. He read that in 1988 the average residential vehicle traveled about 10,200 miles, which increased by about 2.9% per year through 1994.

1. Write a function for the average mileage, $m(t)$, as a function of t, the time in years since 1988. $\quad$ $\mathbf{m(t) = 10{,}200(1 + 0.029)^t}$

2. Assume that the 2.9% increase is valid through 2008 and use your function to complete the table to show the average annual miles driven.

Year	1988	1992	1996	2000	2004	2008
t	0	4	**8**	**12**	**16**	**20**
$m(t)$	10,200	**11,436**	**12,821**	**14,374**	**16,116**	**18,068**

3. Did Justin drive more or fewer miles than the average residential vehicle driver in 2004? by how much (to the nearest 100 miles)?

He drove more miles; about 5,900 miles more.

4. Later Justin read that the annual mileage for light trucks increased by 7.8% per year from 1988 to 1994.

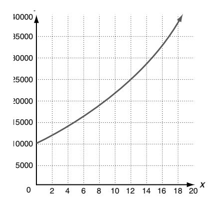
a. Write a function for the average miles driven for a light truck, $n(t)$, as a function of t, the time in years since 1988. He assumes that the average number of miles driven in 1988 was 10,200. $\quad$ $\mathbf{n(t) = 10{,}200(1 + 0.078)^t}$

b. Graph the function. Then use your graph to estimate the average number of miles driven (to the nearest 1000) for a light truck in 2004.

About 34,000 miles

c. Did Justin drive more or fewer miles than the average light truck driver in 2004? by how much?

He drove fewer miles than the average light truck driver by about 12,000 miles.



Justin bought his truck new for $32,000. Its value decreases 9.0% each year. Choose the letter for the best answer.

5. Which function represents the yearly value of Justin's truck?

A $f(x) = 32{,}000(1 + 0.9)^t$

B $f(x) = 32{,}000(1 - 0.9)^t$

C $f(x) = 32{,}000(1 + 0.09)^t$

(D) $f(x) = 32{,}000(1 - 0.09)^t$

6. When will the value of Justin's truck fall below half of what he paid for it?

A In 6 years

(B) In 8 years

C In 10 years

D In 12 years

LESSON 7-1

Reading Strategy

Drawing Conclusions

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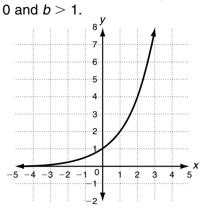

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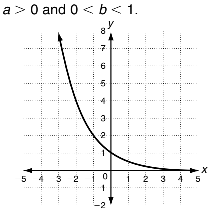

1. Complete the table.

x	−2	−1	0	1	2	3
$f(x) = 3^x$	$\frac{1}{9}$	$\mathbf{\frac{1}{3}}$	**1**	**3**	**9**	**27**
$f(x) = 0.4^x$	**6.25**	**2.5**	**1**	**0.4**	**0.16**	**0.064**

Use the function $f(x) = 3^x$ for Exercises 2 and 3.

2. Does the function $f(x) = 3^x$ show exponential growth or decay? Explain.

Growth; because base, b, is greater than 1

3. Is $f(4)$ greater than or less than $f(3)$? Explain how you can draw this conclusion.

$f(4)$ is greater than $f(3)$ because the function increases as x increases.

Use the function $f(x) = 0.4^x$ for Exercises 4 and 5.

4. Does the function $f(x) = 0.4^x$ show exponential growth or decay? Explain.

Decay; because base, b, is between 0 and 1

5. Is $f(-3)$ greater than or less than $f(-2)$? Explain how you can draw this conclusion.

$f(-3)$ is greater than $f(-2)$ because the function increases as x decreases.