Name ____ Date ___________ Class ____________

SECTION 7A

Ready To Go On? Skills Intervention

7-1 Exponential Functions, Growth, and Decay

Find these vocabulary words in Lesson 7-1 and the Multilingual Glossary.

Vocabulary			
exponential growth	exponential decay	asymptote	base

Graphing Exponential Functions

A. Tell whether the function $f(x) = 2.5^x$ shows growth or decay. Then graph.

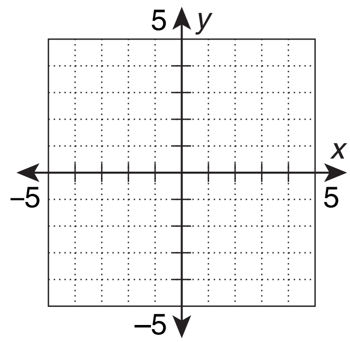
What is the value of the base? ________________

Is the base greater than one or between one and zero? ____________________

Does the function show growth or decay? ________________

Complete the table of values:

x	−3	−2	−1	0	1	2
y	______	______	______	______	______	______



Graph the function using the table of values.

B. Tell whether the function $g(x) = 2(0.75^x)$ shows growth or decay. Then graph.

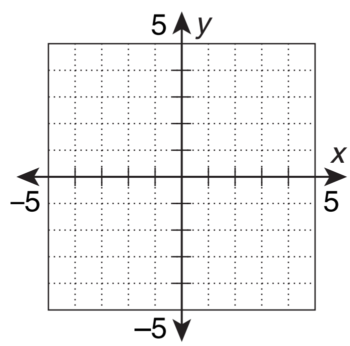
Find the value of the base. ________________

Is the base greater than one or between one and zero? ________________________

Does the function show growth or decay? ________________

Complete the table of values:

x	−3	−2	−1	0	1	2
y	______	______	______	______	______	______



Graph the function using the table of values.

Name ________________________________ Date __________ Class __________

SECTION 7A

Ready To Go On? Problem Solving Intervention

7-1 Exponential Functions, Growth, and Decay

A function of the form $f(x) = ab^x$, where a is greater than 0 and b is greater than 1, is an exponential growth function which increases as x increases. When b is between 0 and 1 the function is called an exponential decay function, which decreases as x decreases.

The value of a new car is $24,500, and its value decreases 9% each year.
a. Write an exponential function representing the value of the car.
b. Graph the function on a calculator.
c. Use the graph to predict when the car's value will fall to $10,000.

Understand the Problem

1. What is the initial value of the car? ______________________

2. Determine whether the function will show growth or decay. ________________

3. Describe the growth factor or decay factor of the car's value. ______________

Make a Plan

4. What do you need to determine? ____

____

5. Let $A(t)$ represent the final value of the car. Write a function to model the value of the car.

Final amount $A(t)$	=	Initial Amount a	·	1 plus rate of increase *or* 1 minus rate of decrease $(1 - r)$	Time t

__________ = __________(________________)$^{\_\_}$

6. Simplify the function in Exercise 5. ______________________

Solve

7. Graph the function in Exercise 6 on your calculator.

8. Use the graph to predict when the value of the car will fall below $10,000. Use the **trace** feature. It will take ___________ years for the car's value to drop to $10,000.

Look Back

9. To check your solution, substitute the solution you predicted for t in Exercise 8 into the equation you wrote in Exercise 6. Let $A(t)$ equal 10,000.

___________ = ___________ (___________)$^{\_\_}$

10. Does your solution make the equation true?___________

Name ______________________________ Date __________ Class __________

SECTION 7A

Ready To Go On? Skills Intervention

7-2 Inverses of Relations and Functions

Find these vocabulary words in Lesson 7-2 and the Multilingual Glossary.

Vocabulary	
inverse relation	inverse function

Writing Inverse Functions by Using Inverse Operations

A. Use inverse operations to write the inverse of $f(x) = 5x + 2$.

$\square = 5x + 2$ Set $y = f(x)$.

$\square = 5\square + 2$ Switch x and y.

$\square - \square = 5\square$ Solve for y.

$\dfrac{\square - \square}{\square} = y$

$y = \dfrac{\square - \square}{\square}$ Write in $y =$ format.

$\square = \dfrac{\square - \square}{\square}$ Write the inverse by substituting $f^{-1}(x)$ for y.

$f^{-1}(x) = \square x - \square$ Simplify.

Check: Since (1, 7) satisfies $f(x)$, does (7, 1) satisfy $f^{-1}(x)$? ______

B. Use inverse operations to write the inverse of $f(x) = \dfrac{2x - 4}{3}$.

$\square = \dfrac{2x - 4}{3}$ Set $y = f(x)$.

$\square = \dfrac{2\square - 4}{3}$ Switch x and y.

$3\square = \square y - 4$ Solve for y.

$\dfrac{3\square + 4}{\square} = y$

$y = \dfrac{3\square + \square}{\square}$ Write in $y =$ format.

$\square = \dfrac{3\square + \square}{\square}$ Write the inverse by substituting $f^{-1}(x)$ for y.

$f^{-1}(x) = \square x + \square$ Simplify.

Check your answer:

Since (2, 0) satisfies $f(x)$, does (0, 2) satisfy $f^{-1}(x)$? ______

Name ______________________________________ Date ___________ Class ____________

SECTION 7A

Ready To Go On? Problem Solving Intervention

7-2 Inverses of Relations and Functions

To write the inverse of a function, switch x and y in the original function and solve for y.

Ruth rents an apartment in the city for a \$550 initial realtor fee and a rate of \$700 per month. The total amount spent on the apartment can be expressed as a function of months, x, by $f(x) = 550 + 700x$. Find the inverse function. Then, use the inverse function to find the number of months Ruth rented the apartment if she spent a total of \$13,150.

Understand the Problem

1. Describe the fees Ruth spent on the apartment. ______________________________

____

Make a Plan

2. What do you need to determine? ____

____

3. Use inverse operations to write the inverse of $f(x)$ that models months as a function of the total amount spent on the apartment.

$\square = 550 + 700x$	Set $y = f(x)$.
$\square = 550 + 700\square$	Switch x and y.
$\square - \square = 700\square$	Solve for y.
$\dfrac{\square - \square}{\square} = y$	
$\square = \dfrac{\square - \square}{\square}$	Write in $y =$ format and substitute $f^{-1}(x)$ for y.
$f^{-1}(x) = \square x - \square$	Simplify.

Solve

4. Evaluate the inverse function for $x = \$13{,}150$.

$f^{-1}(x) = \square x - \square = \square(13{,}150) - \square =$ ______

Ruth rented the apartment for ______ months.

Look Back

5. To check your solution, substitute the number of months into the original function.

$f(x) = 550 + 700(\square) =$ __________

6. Does your solution make the function equal \$13,150? ______

Name ______________________ Date __________ Class __________

SECTION 7A

Ready To Go On? Skills Intervention

7-3 Logarithmic Functions

Find these vocabulary words in Lesson 7-3 and the Multilingual Glossary.

Vocabulary		
logarithm	common logarithm	logarithmic function

Converting from Exponential to Logarithmic Form

Remember a logarithm is an exponent: $b^x = a$; $\log_b a = x$; $3^2 = 9$; $\log_3 9 = 2$

A. Convert $5^3 = 125$ to logarithmic form.

Find the value of the base *b*. ____________

Find the value of the exponent *x*. ____________

Find the value of *a*. ____________

Write in the form $\log_b a = x$. ______________

B. Convert $2^{-1} = 0.5$ to logarithmic form.

Find the value of the base *b*. ____________

Find the value of the exponent *x*. ____________

Find the value of *a*. ____________

Write in the form $\log_b a = x$. ________________

Converting from Logarithmic to Exponential Form

A. Convert $\log_6 1 = 0$ to exponential form.

Find the value of the base *b*. ____________

Find the value of the exponent *x*. ____________

Find the value of *a*. ____________

Write in the form $b^x = a$. ____________

B. Convert $\log_{12} 144 = 2$ to exponential form.

Find the value of the base *b*. ____________

Find the value of the exponent *x*. ____________

Find the value of *a*. ____________

Write in the form $b^x = a$. ____________

Name ______________________ Date __________ Class __________

SECTION 7A

Ready To Go On? Skills Intervention

7-4 Properties of Logarithms

Adding and Subtracting Logarithms

A. Express $\log_6 3 + \log_6 72$ as a single logarithm. Simplify, if possible.

Use the ___________ Property of Logarithms to simplify this expression.

$\log_6 (\square \cdot \square)$ Apply the appropriate property of logarithms.

$\log_6 \square = \square$ Simplify. Think $6^? = 216$.

B. Express $\log_2 224 - \log_2 7$ as a single logarithm. Simplify, if possible.

Use the ___________ Property of Logarithms to simplify this expression.

$\log_2 \left(\frac{\square}{\square}\right)$ Apply the appropriate property of logarithms.

$\log_2 \square = \square$ Simplify. Think $2^? = 32$.

Simplifying Logarithms with Exponents

A. Express $\log 10^4$ as a single logarithm. Simplify, if possible.

Use the ___________ Property of Logarithms to simplify this expression.

$\square \log \square$ Apply the appropriate property of logarithms.

$\square(\square) = \square$ Simplify. Think $10^? = 10$.

B. Express $\log_3 \left(\frac{1}{3}\right)^5$ as a single logarithm. Simplify, if possible.

Use the ___________ Property of Logarithms to simplify this expression.

$\square \log_3 \square$ Apply the appropriate property of logarithms.

$\square(\square) = \square$ Simplify. Think $3^? = \frac{1}{3}$.

Recognizing Inverses

A. Simplify $\log_7 7^{8x-1}$.

Use the ___________ Property of Logarithms to simplify this expression.

$(\square) \log_7 \square$ Apply the appropriate property of logarithms.

$(\square)(\square) = \square$ Simplify. Think $7^? = 7$.

B. Simplify $\log_8 8^{-3x}$.

$(\square) \log_8 \square$ Apply the Inverse Property of Logarithms.

$(\square)(\square) = \square$ Simplify.

Name ______________________________ Date __________ Class __________

SECTION 7A

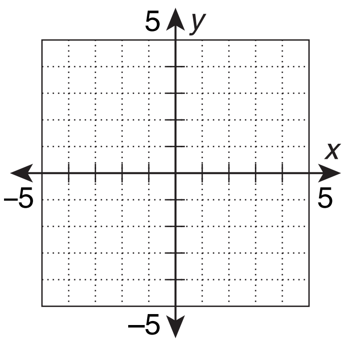
Ready To Go On? Quiz

7-1 Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

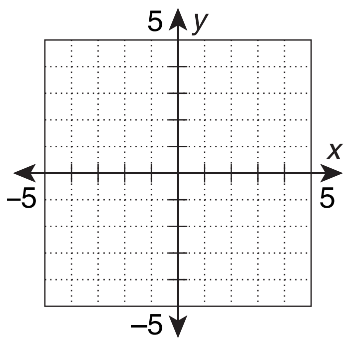
1. $f(x) = 2\left(\frac{1}{4}\right)^x$

____



2. $f(x) = \frac{1}{3}(4)^x$

____



3. The population of a town is 20,000 and, it increases at a rate of 2% per year. Predict the town's population after 5 years.

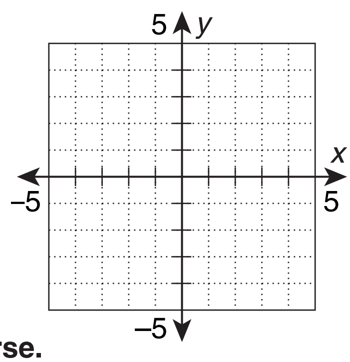
____

7-2 Inverses of Relations and Functions

4. Graph the relation and connect the points.

Then graph its inverse.

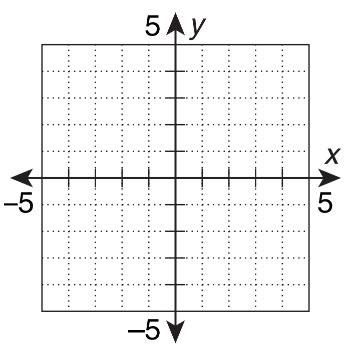
x	0	1	2	3
y	-5	-2	1	4



Graph each function. Then write and graph the inverse.

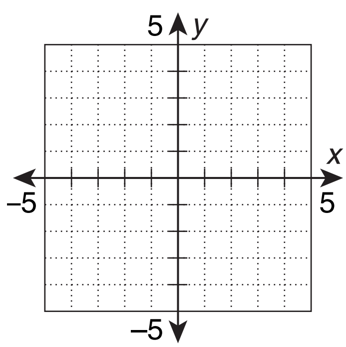
5. $f(x) = \frac{x}{3}$

$f^{-1}(x) =$ __________



6. $f(x) = 4x + 5$

$f^{-1}(x) =$ __________



Name ______________________ Date ____________ Class ____________

SECTION 7A Ready To Go On? Quiz continued

7. Sarah bought a set of bowls for a wedding present. She spent a total of \$37.80, which included a shipping charge of \$6.50 and 5% sales tax. What was the price of the bowls, including tax?

____

7-3 Logarithmic Functions

Write the exponential function in logarithmic form.

8. $3^4 = 81$ ____________

9. $2.5^0 = 1$ ____________

10. $5^{-2} = \frac{1}{25}$ ____________

11. $0.7^x = 0.343$ ____________

Write the logarithmic function in exponential form.

12. $\log_2 128 = 7$ ____________

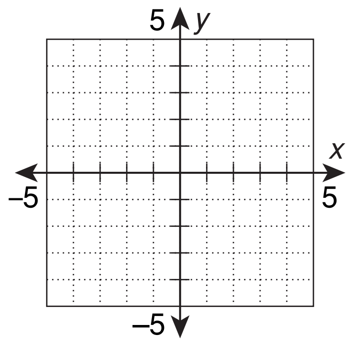
13. $\log_{\frac{1}{5}} 125 = -3$ ____________

14. $\log_{0.16} 1 = 0$ ____________

15. $\log_e x = 2$ ____________

16. Use the given x-values to graph $f(x) = 0.5^x$;

$x = -2, -1, 0, 1, 2$. Then graph the inverse function.



7-4 Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

17. $\log_4 64 + \log_4 \frac{1}{4}$

____

18. $\log_3 29.7 - \log_3 1.1$

____

Simplify each expression.

19. $\log_2 512^4$ ____________

20. $9^{\log_9 0.5}$ ____________

Evaluate.

21. $\log_{\frac{1}{3}} 243$

____

22. $\log_{36} 216$

____

Name ______________________________________ Date ___________ Class ____________

SECTION 7A

Ready To Go On? Enrichment

The Richter Scale

The Richter scale is used to measure the magnitude, or size, of earthquakes. It is a logarithmic function given by the formula:

$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$, where M is the magnitude and E is the number of ergs of energy released.

Some of the largest earthquakes in the world are shown below. Use the table to answer the following questions.

Location	Year	Magnitude
Chile	1960	9.5
Prince William Sound, Alaska	1964	9.2
Coast of Northern Sumatra	2004	9.0
Kamchatka	1952	9.0
Coast of Ecuador	1906	8.8
Northern Sumatra, Indonesia	2005	8.7
Rat Island, Alaska	1965	8.7

1. How much energy was released by the earthquake in Chile?

____

2. How much energy was released by the 2005 earthquake in Northern Sumatra?

____

3. How many times as much energy is released by an earthquake with a magnitude of 9.0 than by an earthquake with a magnitude of 7.0?

____

4. On July 26, 2005, an earthquake in western Montana registered a magnitude of 5.6 on the Richter scale. Find the energy released by the earthquake.

____

Name ______________________ Date __________ Class __________

SECTION 7B

Ready To Go On? Skills Intervention

7-5 Exponential and Logarithmic Equations and Inequalities

Find these vocabulary words in Lesson 7-5 and the Multilingual Glossary.

Vocabulary

exponential equation — logarithmic equation

Solving Exponential Equations

A. Solve $625^x = 5^{x+6}$.

$\square^x = 5^{x+6}$	625 is a power of 5. Rewrite each side with the same base.
$\square^{\square} = 5^{x+6}$	To raise a power to a power, __________ exponents.
$\square = x + 6$	Set the exponents equal.
$x = \square$	Solve for *x*.

B. Solve $425 = 9^{x-1}$.

$\square\, 425 = \square\, 9^{x-1}$	Take the __________ of both sides.
$\log 425 = (\square)\log \square$	Apply the __________ Property of Logarithms.
$\dfrac{\log 425}{\log 9} = (\square)$	Divide both sides by log 9.
$\square = x - 1$	Divide.
$x \approx \square$	Solve for *x*.

Solving Logarithmic Equations

A. Solve $\log_8 x^3 = 4$.

$\square \log_8 x = 4$	Apply the __________ Property of Logarithms.
$\log_8 x = \square$	Divide both sides by __________ to isolate $\log_8 x$.
$x = 8^{(\square)}$	Apply the definition of a logarithm.
$x = (2^3)^{\square}$	
$= 2^{\square} = \square$	Simplify.

B. Solve $\log 500 + \log x = 5$.

$\log(\square) = 5$	Apply the __________ Property of Logarithms.
$\square = 10^{\square}$	Apply the definition of a logarithm.
$x = \dfrac{10^{\square}}{\square} = \square$	Solve for *x*.

Name ________________________________ Date ___________ Class ____________

SECTION 7B

Ready To Go On? Problem Solving Intervention

7-5 Exponential and Logarithmic Equations and Inequalities

You can use exponential functions to predict population growth.

The population of a small French village, currently 1250, grows at a rate of 2% per year. This growth can be expressed by the exponential equation $P = 1250(1 + 0.02)^t$, where P is the population after t years. Find the number of years it will take for the population to exceed 2000.

Understand the Problem

1. Describe the growth of the village's population. ______________________________

____

Make a Plan

2. What do you need to determine? ______________________________________

____

3. Write an inequality that models the situation.

$P > \square$	Define P.
$1250(\square + \square)^t > 2000$	Substitute known values in the equation.

Solve

4. Solve the inequality for t.

$1250(\square + \square)^t > \square$	Write the inequality from Exercise 3.
$\dfrac{1250(\square + \square)^t}{\square} > \dfrac{\square}{\square}$	Divide both sides by 1250.
$(\square + \square)^t > \square$	Simplify.
$\log(\square)^{\square} > \log 1.6$	Take the log of both sides.
$\square \log(\square) > \log 1.6$	Apply the Power Property of Logarithms.
$t > \dfrac{\log 1.6}{\log \square}$	Isolate t.
$t > \square \approx \square$	Solve for t. Round to a whole number.

Beginning in year ______, the village's population will exceed 2000 people.

Look Back

5. To check your solution, substitute your answer for t into the original exponential equation.

$P = 1250(1 + 0.02)^t = 1250(1 + 0.02)^{\square} \approx$ ______

6. Does your solution make the expression exceed 2000? ______

Name ______________________ Date __________ Class __________

SECTION 7B

Ready To Go On? Skills Intervention

7-6 The Natural Base, e

Find these vocabulary words in Lesson 7-6 and the Multilingual Glossary.

Vocabulary		
natural logarithm	natural logarithmic function	half-life

Simplifying Expressions with *e* or ln.

A. Simplify $\ln e^{3x}$.

Apply the __________ Property of Logarithms.

$\ln e^{3x} = \square x \ln \square$

$= 3x(\square)$ Simplify. Think $e^{?} = e$.

$= \square$

B. Simplify $e^{\ln(x+2)}$.

Apply the __________ Property of Exponents.

$e^{\ln(x+2)} =$ __________ Simplify. Think $b^{\log_b x} = x$.

C. Simplify $e^{10\ln x}$.

Apply the reverse of the __________ Property of Logarithms.

$e^{10\ln x} = e^{\ln \square}$

Apply the __________ Property of Exponents.

$e^{\ln \square} = x^{\square}$ Simplify. Think $b^{\log_b x} = x$.

D. Simplify $7\ln e^{0}$.

Apply the __________ Property of Logarithms.

$7\ln e^{0} = 7(\square)\ln \square$

$= 7(\square)(\square)$ Simplify. Think $e^{?} = e$.

$= \square$

E. Simplify $\ln e^{-6t}$.

Apply the __________ Property of Logarithms.

$\ln e^{-6t} = -6\square \ln \square$

$= -6\square(\square)$ Simplify. Think $e^{?} = e$.

$= -\square$

Name ____________________ Date __________ Class __________

SECTION 7B

Ready To Go On? Problem Solving Intervention

7-6 The Natural Base, e

The half-life of a substance is the time it takes for half of the substance to break down or convert to another substance during the process of decay.

Neptunium-239, a radioactive isotope, has a half-life of 2.4 days. Use the decay function $N(t) = N_0e^{-kt}$ to determine the amount of a 100-gram sample that remains after 20 days.

Understand the Problem

1. Describe the decay of neptunium-239. ____________________

2. What do you need to determine? ____________________

Make a Plan

3. Find the decay constant, k, for neptunium-239. Remember that half of the initial quantity will remain after 2.4 days.

$N(t) = N_0e^{-kt}$

$\square = \square e^{-k\square}$ Substitute 1 for N_0, 2.4 for t, and $\frac{1}{2}$ for $N(t)$.

$\ln\square = \square e^{-k\square}$ Simplify and take the natural log of both sides.

$\ln\square = \square k\ln e$ Apply the Power Property of Logarithms.

$\frac{\ln\square}{\square} = k$ Simplify and isolate k.

$k \approx \square$ Solve for k. Round to 4 decimal places.

Solve

4. Write the decay function using your value for k and solve for $N(t)$.

$N(t) = N_0e^{-kt}$

$N(t) = \square e^{-\square(\square)}$ Substitute 100 for N_0, ____ for t, and your value for k.

$N(t) \approx \square$ Solve for $N(t)$.

Look Back

5. To check your solution, substitute your answers for $N(t)$ and k into the decay function.

$N(t) = N_0e^{-kt}$

$0.31 = 100e^{-0.2888t} \rightarrow t \approx$ _____

6. Do your answers for $N(t)$ and k result in t equaling 20 days? _____

Name ________________________________ Date __________ Class ___________

SECTION 7B

Ready To Go On? Skills Intervention

7-7 Transforming Exponential and Logarithmic Functions

Find these vocabulary words in Lesson 7-7 and the Multilingual Glossary.

Vocabulary	
transformation	parent function

Writing Transformed Exponential Functions

A. $f(x) = 7^x$ is translated 4 units left and stretched vertically by a factor of 5.

To translate a function 4 units horizontally to the left should you add or subtract 4 from x? ______

$f(x) = \square$ — Start by identifying the parent function.

$f(x) = 7^{\square}$ — To translate 4 units left, replace x with $x + 4$.

$f(x) = \square \cdot 7^{\square}$ — Stretch vertically by multiplying by 5.

B. $f(x) = 11^x$ is horizontally compressed by a factor of $\frac{1}{4}$ and reflected across the y-axis.

To reflect a function across the y-axis, should you change the sign on the coefficient or the exponent? __________

$f(x) = \square$ — Start by identifying the parent function.

$f(x) = 11^{\square}$ — Horizontally compress by multiplying x by 4.

$f(x) = 11^{\square}$ — Reflect across the y-axis by replacing x with $-x$.

Writing Transformed Logarithmic Functions

A. $f(x) = \log_2 x$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 5 units down.

To translate a function 5 units down, should you add or subtract 5 from x? ________

$f(x) = \square$ — Start by identifying the parent function.

$f(x) = \square \log_2 x$ — Vertically compress by multiplying the right side by $\frac{1}{3}$.

$f(x) = \square \log_2 x - \square$ — To translate 5 units down, subtract 5 from the right side.

B. $f(x) = \ln x$ is translated 1 unit right and reflected across the x-axis.

$f(x) = \square$ — Start by identifying the parent function.

$f(x) = \ln(\square)$ — To translate 1 unit right, replace x with $x - 1$.

$f(x) = \square \ln(\square)$ — Reflect across the x-axis by multiplying the right side by -1.

Name ______________________________________ Date ___________ Class ____________

SECTION 7B

Ready To Go On? Skills Intervention

7-8 Curve Fitting with Exponential and Logarithmic Models

Find this vocabulary word in Lesson 7-8 and the Multilingual Glossary.

Vocabulary
exponential regression

Identifying Exponential Data

A. Determine whether *f* is an exponential function of *x*. If so, find the constant ratio.

x	-1	0	1	2	3	4
$f(x)$	$\frac{1}{3}$	1	3	9	27	81

For linear functions, _________________ are constant.

For exponential functions, the ______ of each *y*-value and the previous value is constant.

Using the table of values:

a. Find the first differences.

____

b. Find the ratios of the $f(x)$ terms.

____

Is the function linear or exponential? _________________

If the function is exponential, what is the constant ratio? _________________

Use linear or exponential regression to find a function that models the data.

$f(x) =$ ______

B. Determine whether *f* is an exponential function of *x*. If so, find the constant ratio.

x	-1	0	1	2	3	4
$f(x)$	-1.5	1	3.5	6	8.5	11

Using the table of values:

a. Find the first differences.

____

b. Find the ratios of the $f(x)$ terms.

____

Is the function linear or exponential? _________________

If the function is exponential, what is the constant ratio? _________________

Use linear or exponential regression to find a function that models the data.

$f(x) =$ _____ + _____

Name ______________________ Date __________ Class __________

SECTION 7B

Ready To Go On? Quiz

7-5 Exponential and Logarithmic Equations and Inequalities

Solve.

1. $81 = 3^{x-4}$ ____________

2. $\log_4(x - 6) = 3$ ____________

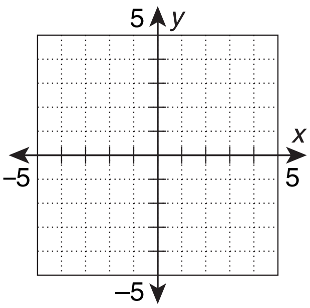
3. $900 = 5^{x-1}$ ____________

4. $\log 50x - \log 2 = 3$ ____________

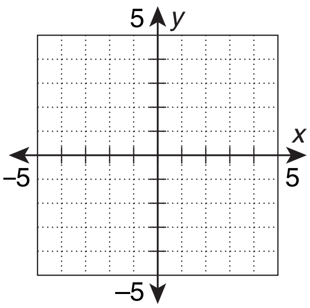
5. A lottery winner can choose a prize of either $500,000 or one penny on the first day, quadruple that (4 cents) on the second day, and so on for 30 days. On what day would the lottery winner receive more than the original $500,000 prize? ____________

7-6 The Natural Base, *e*

6. Graph $f(x) = 2 - e^x$.



7. Graph $f(x) = e^x + \frac{1}{2}$.



Simplify.

8. $\ln e^{\frac{1}{3}}$ ____________

9. $e^{\ln(2x+1)}$ ____________

10. $e^{7\ln x}$ ____________

11. $\ln e^{x + 12y}$ ____________

12. $\ln e^{0.7}$ ____________

13. $\ln e^{-0.6x}$ ____________

14. What is the total amount after 5 years for an investment of $2000 invested at 4% compounded continuously? ____________

15. Use the decay function $N_t = N_0e^{-kt}$ to determine how much of 20 grams of carbon-14 will remain after 500 years. Carbon-14's half-life is 5730 years. ____________

Name ______________________ Date __________ Class __________

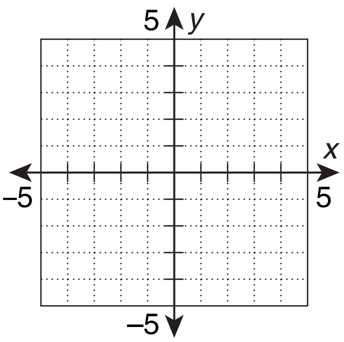
SECTION 7B

Ready To Go On? Quiz continued

7-7 Transforming Exponential and Logarithmic Functions

Graph the function. Find the *y*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

16. $g(x) = -\log(x + 1)$



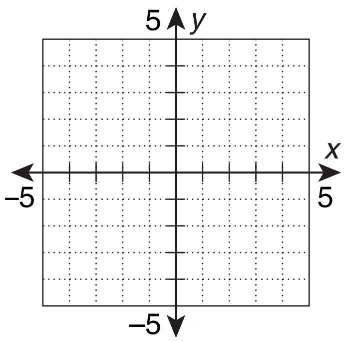
y-intercept: ______________________

asymptote: ______________________

transformation: ______________________

____

17. $h(x) = e^{\frac{x}{3}} - 2$



y-intercept: ______________________

asymptote: ______________________

transformation: ______________________

____

Write the transformed function.

18. $f(x) = 5^x$ is vertically stretched by 4 and reflected across the *y*-axis.

____

19. $f(x) = \ln(3x)$ is horizontally compressed by $\frac{1}{4}$ and vertically translated 1 unit up.

____

7-8 Curve Fitting with Exponential and Logarithmic Models

Determine whether *y* is an exponential function of *x*. If so, find the constant ratio and use exponential regression to find a function that models the data.

20.

x	0	1	2	3	4	5
y	2	6	18	54	162	486

____

21.

x	−2	−1	0	1	2	3
y	−10	−5	0	5	15	30

____

Name ______________________________ Date __________ Class __________

SECTION 7B

Ready To Go On? Enrichment

Compounding Continuously

When investing money at a compounded interest rate, the interest is paid on the original principal and on the accumulated interest. Recall that compound interest is computed using the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

As n increases, the interest approaches that of *continuously* compounded interest. The formula for *continuously* compounded interest is:

$$A = Pe^{rt}$$

Compare compound interest intervals by completing the table.

1. Emmett invests $2000 for 10 years at a 5% compounded interest rate. Complete the table.

Principal (P)	Rate (r)	Time in years (t)	Compound Interval (n)	Amount (A)
2000	5%	10	Semi-annually $n = 2$	
2000	5%	10	Quarterly $n = 4$	
2000	5%	10	Monthly $n = 12$	
2000	5%	10	Daily $n = 365$	
2000	5%	10	Continuously	

2. Which interval earns more money for an investment? ______________________

Answer each of the following questions.

3. How long will it take an investment of $1000 to triple in value if it is invested at a rate of 8% compounded monthly?

____

4. How long will it take an investment of $2500 to double in value if it is invested at a rate of 6% compounded continuously?

____

5. Carolyn has $6825.70 in her savings account. She invested her money at a 4% interest rate for 5 years compounded continuously. How much did she originally invest?

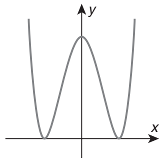
____

SECTION 6B

Ready To Go On? Quiz continued

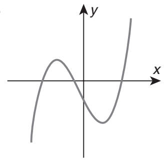
Identify whether the function graphed has an odd or even degree and a positive or negative leading coefficient.

10.



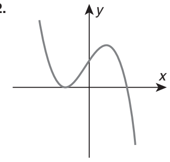
Even; positive

11.



Odd; positive

12.



Odd; negative

6-8 Transforming Polynomial Functions

Let $f(x) = x^4 - 2x^2 - 5$. Write a function $g(x)$ that performs each transformation.

13. Reflect $f(x)$ across the x-axis.

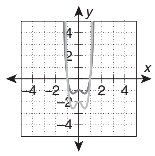
$g(x) = -x^4 + 2x^2 + 5$

14. Reflect $f(x)$ across the y-axis.

$g(x) = x^4 - 2x^2 - 5$

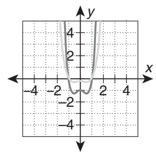
Let $f(x) = 3x^4 - 2x^2 - 1$. Graph $f(x)$ and $g(x)$ on the same coordinate plane. Describe $g(x)$ as a transformation of $f(x)$.

15. $g(x) = 2f(x)$



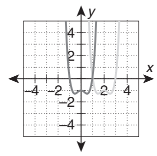
Vertical stretch

16. $g(x) = \frac{1}{4}f(x)$



Horizontal stretch

17. $g(x) = f(x - 2)$



Shift 2 units right

6-9 Curve Fitting with Polynomial Models

18. The table shows the population of a bacteria colony over time. Write a polynomial function for the data. $P(h) = -0.25h^4 + 19.5h^3 - 38.75h^2 + 119.5h - 12$

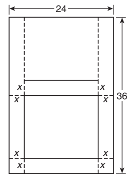
Time (h)	1	2	3	4	5
Number of bacteria	88	224	504	1030	1898

SECTION 6B

Ready To Go On? Enrichment

Packaging

A box with a lid is to be made by cutting a 24-inch by 36-inch piece of cardboard along the dotted lines as shown in the figure and folding the flaps up along the sides. What is the maximum volume of the box? What value of x will produce a box of maximum value? What are the dimensions of the box?



1. Determine the dimensions of the box. $(x) \times (24 - 2x) \times (18 - x)$

2. Write an equation for the volume of the box. $V = 2x^3 - 60x^2 + 432x$

3. Use the Table feature on a graphing calculator with △Tbl = 1 to fill in the chart.

X	Y_1
1	374
2	640
3	810
4	896
5	910
6	864
7	770

4. For what interval of x does the maximum volume appear to exist? $4 < x < 6$

5. Reset the table by setting TblStart to the lower value of the interval in Exercise 4 and △Tbl to 0.1. In what interval of x does the maximum volume appear to exist? $4.6 < x < 4.8$

6. Reset the table by setting TblStart to the lower value of the interval in Exercise 5 and △Tbl to 0.05. What appears to be the value of x that produces the maximum volume? 4.7

7. What is the maximum volume of the box? 912.646 cubic inches

8. What are the dimensions of the box? 4.7 in. × 14.6 in. × 13.3 in.

SECTION 7A

Ready To Go On? Skills Intervention

7-1 Exponential Functions, Growth, and Decay

Find these vocabulary words in Lesson 7-1 and the Multilingual Glossary.

Vocabulary			
exponential growth	exponential decay	asymptote	base

Graphing Exponential Functions

A. Tell whether the function $f(x) = 2.5^x$ shows growth or decay. Then graph.

What is the value of the base? 2.5

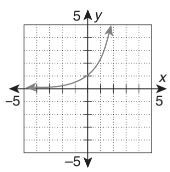
Is the base greater than one or between one and zero? Greater than one

Does the function show growth or decay? Growth

Complete the table of values:

x	−3	−2	−1	0	1	2
y	0.06	0.16	0.04	1	2.5	6.25

Graph the function using the table of values.



B. Tell whether the function $g(x) = 2(0.75^x)$ shows growth or decay. Then graph.

Find the value of the base. 0.75

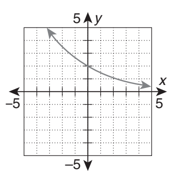
Is the base greater than one or between one and zero? Between one and zero

Does the function show growth or decay? Decay

Complete the table of values:

x	−3	−2	−1	0	1	2
y	4.74	3.56	2.67	2	1.5	1.13

Graph the function using the table of values.



SECTION 7A

Ready To Go On? Problem Solving Intervention

7-1 Exponential Functions, Growth, and Decay

A function of the form $f(x) = ab^x$, where a is greater than 0 and b is greater than 1, is an exponential growth function which increases as x increases. When b is between 0 and 1 the function is called an exponential decay function, which decreases as x decreases.

The value of a new car is $24,500, and its value decreases 9% each year.
a. Write an exponential function representing the value of the car.
b. Graph the function on a calculator.
c. Use the graph to predict when the car's value will fall to $10,000.

Understand the Problem

1. What is the initial value of the car? $24,500

2. Determine whether the function will show growth or decay. Decay

3. Describe the growth factor or decay factor of the car's value. $1 - r = 1 - 0.09 = 0.91$

Make a Plan

4. What do you need to determine? A function representing the decay of the value of the car and when the value will fall to $10,000.

5. Let $A(t)$ represent the final value of the car. Write a function to model the value of the car.

Final amount $A(t)$ = Initial Amount a · 1 plus rate of increase *or* 1 minus rate of decrease $(1 - r)$ ^ Time t

$A(t) = 24{,}500\,(1 - 0.09)^t$

6. Simplify the function in Exercise 5. $A(t) = 24{,}500(0.91)^t$

Solve

7. Graph the function in Exercise 6 on your calculator.

8. Use the graph to predict when the value of the car will fall below $10,000. Use the **trace** feature. It will take about 9.5 years for the car's value to drop to $10,000.

Look Back

9. To check your solution, substitute the solution you predicted for t in Exercise 8 into the equation you wrote in Exercise 6. Let $A(t)$ equal 10,000.

$10{,}000 = 24{,}500\,(0.91)^{9.5}$

10. Does your solution make the equation true? Yes

SECTION 7A

Ready To Go On? Skills Intervention

7-2 Inverses of Relations and Functions

Find these vocabulary words in Lesson 7-2 and the Multilingual Glossary.

Vocabulary	
inverse relation	inverse function

Writing Inverse Functions by Using Inverse Operations

A. Use inverse operations to write the inverse of $f(x) = 5x + 2$.

$\boxed{y} = 5x + 2$	Set $y = f(x)$.
$\boxed{x} = 5\boxed{y} + 2$	Switch x and y.
$\boxed{x} - \boxed{2} = 5\boxed{y}$	Solve for y.
$\dfrac{\boxed{x} - \boxed{2}}{\boxed{5}} = y$	
$y = \dfrac{\boxed{x} - \boxed{2}}{\boxed{5}}$	Write in $y =$ format.
$\boxed{f^{-1}(x)} = \dfrac{\boxed{x} - \boxed{2}}{\boxed{5}}$	Write the inverse by substituting $f^{-1}(x)$ for y.
$f^{-1}(x) = \boxed{\frac{1}{5}}x - \boxed{\frac{2}{5}}$	Simplify.

Check: Since (1, 7) satisfies $f(x)$, does (7, 1) satisfy $f^{-1}(x)$? Yes

B. Use inverse operations to write the inverse of $f(x) = \dfrac{2x - 4}{3}$.

$\boxed{y} = \dfrac{2x - 4}{3}$	Set $y = f(x)$.
$\boxed{x} = \dfrac{2\boxed{y} - 4}{3}$	Switch x and y.
$3\boxed{x} = \boxed{2}y - 4$	Solve for y.
$\dfrac{3\boxed{x} + 4}{\boxed{2}} = y$	
$y = \dfrac{3\boxed{x} + \boxed{4}}{\boxed{2}}$	Write in $y =$ format.
$\boxed{f^{-1}(x)} = \dfrac{3\boxed{x} + \boxed{4}}{\boxed{2}}$	Write the inverse by substituting $f^{-1}(x)$ for y.
$f^{-1}(x) = \boxed{\frac{3}{2}}x + \boxed{2}$	Simplify.

Check your answer:

Since (2, 0) satisfies $f(x)$, does (0, 2) satisfy $f^{-1}(x)$? Yes

SECTION 7A

Ready To Go On? Problem Solving Intervention

7-2 Inverses of Relations and Functions

To write the inverse of a function, switch x and y in the original function and solve for y.

Ruth rents an apartment in the city for a \$550 initial realtor fee and a rate of \$700 per month. The total amount spent on the apartment can be expressed as a function of months, x, by $f(x) = 550 + 700x$. Find the inverse function. Then, use the inverse function to find the number of months Ruth rented the apartment if she spent a total of \$13,150.

Understand the Problem

1. Describe the fees Ruth spent on the apartment. She spent an initial fee of \$550 and a monthly fee of \$700.

Make a Plan

2. What do you need to determine? The inverse function of $f(x)$ and how many months result in a total amount spent of \$13,150.
3. Use inverse operations to write the inverse of $f(x)$ that models months as a function of the total amount spent on the apartment.

$\boxed{y} = 550 + 700x$	Set $y = f(x)$.
$\boxed{x} = 550 + 700\boxed{y}$	Switch x and y.
$\boxed{x} - \boxed{550} = 700\boxed{y}$	Solve for y.
$\dfrac{\boxed{x} - \boxed{550}}{\boxed{700}} = y$	
$\boxed{f^{-1}(x)} = \dfrac{\boxed{x} - \boxed{550}}{\boxed{700}}$	Write in $y =$ format and substitute $f^{-1}(x)$ for y.
$f^{-1}(x) = \boxed{\frac{1}{700}}x - \boxed{\frac{11}{14}}$	Simplify.

Solve

4. Evaluate the inverse function for $x = \$13{,}150$.

$f^{-1}(x) = \boxed{\frac{1}{700}}x - \boxed{\frac{11}{14}} = \boxed{\frac{1}{700}}(13{,}150) - \boxed{\frac{11}{14}} = $ 18

Ruth rented the apartment for 18 months.

Look Back

5. To check your solution, substitute the number of months into the original function.

$f(x) = 550 + 700(\boxed{18}) = $ 13,150

6. Does your solution make the function equal \$13,150? Yes

SECTION 7A

Ready To Go On? Skills Intervention

7-3 Logarithmic Functions

Find these vocabulary words in Lesson 7-3 and the Multilingual Glossary.

Vocabulary		
logarithm	common logarithm	logarithmic function

Converting from Exponential to Logarithmic Form

Remember a logarithm is an exponent: $b^x = a$, $\log_b a = x$; $3^2 = 9$, $\log_3 9 = 2$

A. Convert $5^3 = 125$ to logarithmic form.

Find the value of the base b. 5

Find the value of the exponent x. 3

Find the value of a. 125

Write in the form $\log_b a = x$. $\log_5 125 = 3$

B. Convert $2^{-1} = 0.5$ to logarithmic form.

Find the value of the base b. 2

Find the value of the exponent x. -1

Find the value of a. 0.5

Write in the form $\log_b a = x$. $\log_2 0.5 = -1$

Converting from Logarithmic to Exponential Form

A. Convert $\log_6 1 = 0$ to exponential form.

Find the value of the base b. 6

Find the value of the exponent x. 0

Find the value of a. 1

Write in the form $b^x = a$. $6^0 = 1$

B. Convert $\log_{12} 144 = 2$ to exponential form.

Find the value of the base b. 12

Find the value of the exponent x. 2

Find the value of a. 144

Write in the form $b^x = a$. $12^2 = 144$

SECTION 7A

Ready To Go On? Skills Intervention

7-4 Properties of Logarithms

Adding and Subtracting Logarithms

A. Express $\log_6 3 + \log_6 72$ as a single logarithm. Simplify, if possible.

Use the Product Property of Logarithms to simplify this expression.

$\log_6(\boxed{3} \cdot \boxed{72})$	Apply the appropriate property of logarithms.
$\log_6 \boxed{216} = \boxed{3}$	Simplify. Think $6^? = 216$.

B. Express $\log_2 224 - \log_2 7$ as a single logarithm. Simplify, if possible.

Use the Quotient Property of Logarithms to simplify this expression.

$\log_2\left(\dfrac{\boxed{224}}{\boxed{7}}\right)$	Apply the appropriate property of logarithms.
$\log_2 \boxed{32} = \boxed{5}$	Simplify. Think $2^? = 32$.

Simplifying Logarithms with Exponents

A. Express $\log 10^4$ as a single logarithm. Simplify, if possible.

Use the Power Property of Logarithms to simplify this expression.

$\boxed{4}\log\boxed{10}$	Apply the appropriate property of logarithms.
$\boxed{4}(\boxed{1}) = \boxed{4}$	Simplify. Think $10^? = 10$.

B. Express $\log_3\left(\frac{1}{3}\right)^5$ as a single logarithm. Simplify, if possible.

Use the Power Property of Logarithms to simplify this expression.

$\boxed{5}\log_3\boxed{\frac{1}{3}}$	Apply the appropriate property of logarithms.
$\boxed{5}(\boxed{-1}) = \boxed{-5}$	Simplify. Think $3^? = \frac{1}{3}$.

Recognizing Inverses

A. Simplify $\log_7 7^{8x-1}$.

Use the Inverse Property of Logarithms to simplify this expression.

$(\boxed{8x-1})\log_7\boxed{7}$	Apply the appropriate property of logarithms.
$(\boxed{8x-1})(\boxed{1}) = \boxed{8x-1}$	Simplify. Think $7^? = 7$.

B. Simplify $\log_8 8^{-3x}$.

$(\boxed{-3x})\log_8\boxed{8}$	Apply the Inverse Property of Logarithms.
$(\boxed{-3x})(\boxed{1}) = \boxed{-3x}$	Simplify.

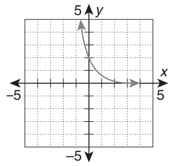
SECTION 7A

Ready To Go On? Quiz

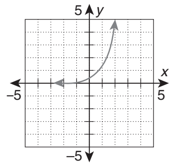
7-1 Exponential Functions, Growth, and Decay

Tell whether the function shows growth or decay. Then graph.

1. $f(x) = 2\left(\frac{1}{4}\right)^x$ Decay



2. $f(x) = \frac{1}{3}(4)^x$ Growth

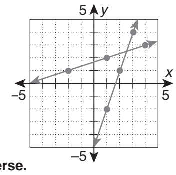


3. The population of a town is 20,000 and, it increases at a rate of 2% per year. Predict the town's population after 5 years.
$p = 20{,}000(1 + 0.02)^5 \approx 22{,}082$ people

7-2 Inverses of Relations and Functions

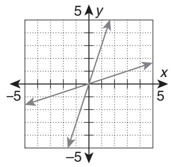
4. Graph the relation and connect the points.
Then graph its inverse.

x	0	1	2	3
y	−5	−2	1	4

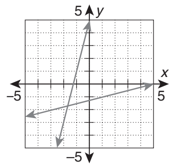


Graph each function. Then write and graph the inverse.

5. $f(x) = \frac{x}{3}$
$f^{-1}(x) =$ $3x$



6. $f(x) = 4x + 5$
$f^{-1}(x) =$ $\frac{x}{4} - \frac{5}{4}$



SECTION 7A

Ready To Go On? Quiz continued

7. Sarah bought a set of bowls for a wedding present. She spent a total of $37.80, which included a shipping charge of $6.50 and 5% sales tax. What was the price of the bowls, including tax?
$c = 1.05(p + 6.50);\ p = \$29.50$

7-3 Logarithmic Functions

Write the exponential function in logarithmic form.

8. $3^4 = 81$ $\log_3 81 = 4$

9. $2.5^0 = 1$ $\log_{2.5} 1 = 0$

10. $5^{-2} = \frac{1}{25}$ $\log_5 \frac{1}{25} = -2$

11. $0.7^x = 0.343$ $\log_{0.7} .343 = x$

Write the logarithmic function in exponential form.

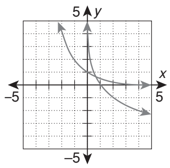
12. $\log_2 128 = 7$ $2^7 = 128$

13. $\log_{\frac{1}{5}} 125 = -3$ $\left(\frac{1}{5}\right)^{-3} = 125$

14. $\log_{0.16} 1 = 0$ $0.16^0 = 1$

15. $\log_e x = 2$ $e^2 = x$

16. Use the given x-values to graph $f(x) = 0.5^x$;
$x = -2, -1, 0, 1, 2$. Then graph the inverse function.



7-4 Properties of Logarithms

Express as a single logarithm. Simplify, if possible.

17. $\log_4 64 + \log_4 \frac{1}{4}$
$\log_4 16 = 2$

18. $\log_3 29.7 - \log_3 1.1$
$\log_3 27 = 3$

Simplify each expression.

19. $\log_2 512^4$ 36

20. $9^{\log_9 0.5}$ 0.5

Evaluate.

21. $\log_{\frac{1}{3}} 243$
-5

22. $\log_{36} 216$
$\frac{3}{2}$

SECTION 7A

Ready To Go On? Enrichment

The Richter Scale

The Richter scale is used to measure the magnitude, or size, of earthquakes. It is a logarithmic function given by the formula:

$M = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$, where M is the magnitude and E is the number of ergs of energy released.

Some of the largest earthquakes in the world are shown below. Use the table to answer the following questions.

Location	Year	Magnitude
Chile	1960	9.5
Prince William Sound, Alaska	1964	9.2
Coast of Northern Sumatra	2004	9.0
Kamchatka	1952	9.0
Coast of Ecuador	1906	8.8
Northern Sumatra, Indonesia	2005	8.7
Rat Island, Alaska	1965	8.7

1. How much energy was released by the earthquake in Chile?
1.12×10^{26} ergs

2. How much energy was released by the 2005 earthquake in Northern Sumatra?
7.08×10^{24} ergs

3. How many times as much energy is released by an earthquake with a magnitude of 9.0 than by an earthquake with a magnitude of 7.0?
1000 times

4. On July 26, 2005, an earthquake in western Montana registered a magnitude of 5.6 on the Richter scale. Find the energy released by the earthquake.
1.58×10^{20} ergs

SECTION 7B

Ready To Go On? Skills Intervention

7-5 Exponential and Logarithmic Equations and Inequalities

Find these vocabulary words in Lesson 7-5 and the Multilingual Glossary.

Vocabulary	
exponential equation	logarithmic equation

Solving Exponential Equations

A. Solve $625^x = 5^{x+6}$.

$(5^4)^x = 5^{x+6}$ — 625 is a power of 5. Rewrite each side with the same base.

$5^{4x} = 5^{x+6}$ — To raise a power to a power, multiply exponents.

$4x = x + 6$ — Set the exponents equal.

$x = 2$ — Solve for x.

B. Solve $425 = 9^{x-1}$.

$\log 425 = \log 9^{x-1}$ — Take the log of both sides.

$\log 425 = (x - 1)\log 9$ — Apply the Power Property of Logarithms.

$\frac{\log 425}{\log 9} = (x - 1)$ — Divide both sides by log 9.

$2.75 = x - 1$ — Divide.

$x \approx 3.75$ — Solve for x.

Solving Logarithmic Equations

A. Solve $\log_8 x^3 = 4$.

$3\log_8 x = 4$ — Apply the Power Property of Logarithms.

$\log_8 x = \frac{4}{3}$ — Divide both sides by 3 to isolate $\log_8 x$.

$x = 8^{\left(\frac{4}{3}\right)}$ — Apply the definition of a logarithm.

$x = (2^3)^{\frac{4}{3}}$

$= 2^4 = 16$ — Simplify.

B. Solve $\log 500 + \log x = 5$.

$\log(500x) = 5$ — Apply the Product Property of Logarithms.

$500x = 10^5$ — Apply the definition of a logarithm.

$x = \frac{10^5}{500} = 200$ — Solve for x.

SECTION 7B

Ready To Go On? Problem Solving Intervention

7-5 Exponential and Logarithmic Equations and Inequalities

You can use exponential functions to predict population growth.

The population of a small French village, currently 1250, grows at a rate of 2% per year. This growth can be expressed by the exponential equation $P = 1250(1 + 0.02)^t$, where P is the population after t years. Find the number of years it will take for the population to exceed 2000.

Understand the Problem

1. Describe the growth of the village's population. The village's population grows at a rate of 2% per year.

Make a Plan

2. What do you need to determine? When the village's population will exceed 2000 people.

3. Write an inequality that models the situation.

$P >$ 2000	Define P.
$1250(1 + 0.02)^t > 2000$	Substitute known values in the equation.

Solve

4. Solve the inequality for t.

$1250(1 + 0.02)^t > 2000$	Write the inequality from Exercise 3.
$\frac{1250(1 + 0.02)^t}{1250} > \frac{2000}{1250}$	Divide both sides by 1250.
$(1 + 0.02)^t > 1.6$	Simplify.
$\log(1.02)^t > \log 1.6$	Take the log of both sides.
$t\log(1.02) > \log 1.6$	Apply the Power Property of Logarithms.
$t > \frac{\log 1.6}{\log 1.02}$	Isolate t.
$t > 23.73 \approx 24$	Solve for t. Round to a whole number.

Beginning in year 24, the village's population will exceed 2000 people.

Look Back

5. To check your solution, substitute your answer for t into the original exponential equation.
   $P = 1250(1 + 0.02)^t = 1250(1 + 0.02)^{24} \approx$ 2011
6. Does your solution make the expression exceed 2000? Yes

SECTION 7B

Ready To Go On? Skills Intervention

7-6 The Natural Base, e

Find these vocabulary words in Lesson 7-6 and the Multilingual Glossary.

Vocabulary		
natural logarithm	natural logarithmic function	half-life

Simplifying Expressions with *e* or ln.

A. Simplify $\ln e^{3x}$.

Apply the Power Property of Logarithms.

$\ln e^{3x} = 3x\ln e$	
$= 3x(1)$	Simplify. Think $e^? = e$.
$= 3x$	

B. Simplify $e^{\ln(x+2)}$.

Apply the Inverse Property of Exponents.

$e^{\ln(x+2)} = x + 2$	Simplify. Think $b^{\log_b x} = x$.

C. Simplify $e^{10\ln x}$.

Apply the reverse of the Power Property of Logarithms.

$e^{10\ln x} = e^{\ln x^{10}}$

Apply the Inverse Property of Exponents.

$e^{\ln x^{10}} = x^{10}$	Simplify. Think $b^{\log_b x} = x$.

D. Simplify $7\ln e^0$.

Apply the Power Property of Logarithms.

$7\ln e^0 = 7(0)\ln e$	
$= 7(0)(1)$	Simplify. Think $e^? = e$.
$= 0$	

E. Simplify $\ln e^{-6t}$.

Apply the Power Property of Logarithms.

$\ln e^{-6t} = -6t\ln e$	
$= -6t(1)$	Simplify. Think $e^? = e$.
$= -6t$	

SECTION 7B

Ready To Go On? Problem Solving Intervention

7-6 The Natural Base, e

The half-life of a substance is the time it takes for half of the substance to break down or convert to another substance during the process of decay.

Neptunium-239, a radioactive isotope, has a half-life of 2.4 days. Use the decay function $N(t) = N_0e^{-kt}$ to determine the amount of a 100-gram sample that remains after 20 days.

Understand the Problem

1. Describe the decay of neptunium-239. Half of the isotope decays every 2.4 days.
2. What do you need to determine? How much of 100 grams of neptunium-239 remains after 20 days.

Make a Plan

3. Find the decay constant, k, for neptunium-239. Remember that half of the initial quantity will remain after 2.4 days.

$N(t) = N_0e^{-kt}$	
$\frac{1}{2} = 1e^{-k(2.4)}$	Substitute 1 for N_0, 2.4 for t, and $\frac{1}{2}$ for $N(t)$.
$\ln\frac{1}{2} = \ln e^{-k(2.4)}$	Simplify and take the natural log of both sides.
$\ln\frac{1}{2} = -2.4k\ln e$	Apply the Power Property of Logarithms.
$\frac{\ln 0.5}{-2.4} = k$	Simplify and isolate k.
$k \approx 0.2888$	Solve for k. Round to 4 decimal places.

Solve

4. Write the decay function using your value for k and solve for $N(t)$.

$N(t) = N_0e^{-kt}$	
$N(t) = 100e^{-0.2888(20)}$	Substitute 100 for N_0, 20 for t, and your value for k.
$N(t) \approx 0.31$	Solve for $N(t)$.

Look Back

5. To check your solution, substitute your answers for $N(t)$ and k into the decay function.
   $N(t) = N_0e^{-kt}$
   $0.31 = 100e^{-0.2888t} \rightarrow t \approx$ 20
6. Do your answers for $N(t)$ and k result in t equaling 20 days? Yes

SECTION 7B

Ready To Go On? Skills Intervention

7-7 Transforming Exponential and Logarithmic Functions

Find these vocabulary words in Lesson 7-7 and the Multilingual Glossary.

Vocabulary	
transformation	parent function

Writing Transformed Exponential Functions

A. $f(x) = 7^x$ is translated 4 units left and stretched vertically by a factor of 5.

To translate a function 4 units horizontally to the left should you add or subtract 4 from x? Add

$f(x) = 7^x$	Start by identifying the parent function.
$f(x) = 7^{x+4}$	To translate 4 units left, replace x with $x + 4$.
$f(x) = 5 \cdot 7^{x+4}$	Stretch vertically by multiplying by 5.

B. $f(x) = 11^x$ is horizontally compressed by a factor of $\frac{1}{4}$ and reflected across the y-axis.

To reflect a function across the y-axis, should you change the sign on the coefficient or the exponent? Exponent

$f(x) = 11^x$	Start by identifying the parent function.
$f(x) = 11^{4x}$	Horizontally compress by multiplying x by 4.
$f(x) = 11^{-4x}$	Reflect across the y-axis by replacing x with $-x$.

Writing Transformed Logarithmic Functions

A. $f(x) = \log_2 x$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 5 units down.

To translate a function 5 units down, should you add or subtract 5 from x? Add

$f(x) = \log_2 x$	Start by identifying the parent function.
$f(x) = \frac{1}{3}\log_2 x$	Vertically compress by multiplying the right side by $\frac{1}{3}$.
$f(x) = \frac{1}{3}\log_2 x - 5$	To translate 5 units down, subtract 5 from the right side.

B. $f(x) = \ln x$ is translated 1 unit right and reflected across the x-axis.

$f(x) = \ln x$	Start by identifying the parent function.
$f(x) = \ln(x - 1)$	To translate 1 unit right, replace x with $x - 1$.
$f(x) = -1\ln(x - 1)$	Reflect across the x-axis by multiplying the right side by -1.

SECTION 7B

Ready To Go On? Skills Intervention

7-8 Curve Fitting with Exponential and Logarithmic Models

Find this vocabulary word in Lesson 7-8 and the Multilingual Glossary.

Vocabulary
exponential regression

Identifying Exponential Data

A. Determine whether *f* is an exponential function of *x*. If so, find the constant ratio.

x	−1	0	1	2	3	4
$f(x)$	$\frac{1}{3}$	1	3	9	27	81

For linear functions, first differences are constant.

For exponential functions, the ratio of each y-value and the previous value is constant.

Using the table of values:

a. Find the first differences.

$+\frac{2}{3}$; +2; +6; +18; +54

b. Find the ratios of the $f(x)$ terms.

3; 3; 3; 3; 3

Is the function linear or exponential? Exponential

If the function is exponential, what is the constant ratio? 3

Use linear or exponential regression to find a function that models the data.

$f(x) = 3^x$

B. Determine whether *f* is an exponential function of *x*. If so, find the constant ratio.

x	−1	0	1	2	3	4
$f(x)$	−1.5	1	3.5	6	8.5	11

Using the table of values:

a. Find the first differences.

+2.5; +2.5; +2.5; +2.5; +2.5

b. Find the ratios of the $f(x)$ terms.

−0.67; 3.5; 1.7; 1.42; 1.29

Is the function linear or exponential? Linear

If the function is exponential, what is the constant ratio? Not applicable

Use linear or exponential regression to find a function that models the data.

$f(x) = 2.5x + 1$

SECTION 7B

Ready To Go On? Quiz

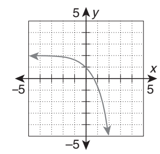
7-5 Exponential and Logarithmic Equations and Inequalities

Solve.

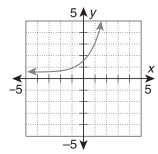
1. $81 = 3^{x-4}$ — $x = 8$
2. $\log_4(x - 6) = 3$ — $x = 70$
3. $900 = 5^{x-1}$ — $x \approx 5.23$
4. $\log 50x - \log 2 = 3$ — $x = 40$
5. A lottery winner can choose a prize of either $500,000 or one penny on the first day, quadruple that (4 cents) on the second day, and so on for 30 days. On what day would the lottery winner receive more than the original $500,000 prize? Day 14

7-6 The Natural Base, *e*

6. Graph $f(x) = 2 - e^x$.



7. Graph $f(x) = e^x + \frac{1}{2}$.



Simplify.

8. $\ln e^{\frac{1}{3}}$ — $\frac{1}{3}$
9. $e^{\ln(2x+1)}$ — $2x + 1$
10. $e^{7\ln x}$ — x^7
11. $\ln e^{x + 12y}$ — $x + 12y$
12. $\ln e^{0.7}$ — 0.7
13. $\ln e^{-0.6x}$ — $-0.6x$
14. What is the total amount after 5 years for an investment of $2000 invested at 4% compounded continuously? $2442.81
15. Use the decay function $N_t = N_0 e^{-kt}$ to determine how much of 20 grams of carbon-14 will remain after 500 years. Carbon-14's half-life is 5730 years. ≈18.83 grams

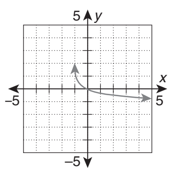
SECTION 7B

Ready To Go On? Quiz continued

7-7 Transforming Exponential and Logarithmic Functions

Graph the function. Find the *y*-intercept and asymptote. Describe how the graph is transformed from the graph of the parent function.

16. $g(x) = -\log(x + 1)$

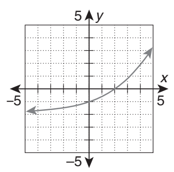


y-intercept: 0

asymptote: $x = -1$

transformation: Shifted one unit left; reflected across x-axis

17. $h(x) = e^{\frac{x}{3}} - 2$



y-intercept: 0

asymptote: $y = -2$

transformation: Shifted 2 units down; horizontally stretched by 3

Write the transformed function.

18. $f(x) = 5^x$ is vertically stretched by 4 and reflected across the y-axis. $f(x) = 4(5^{-x})$
19. $f(x) = \ln(3x)$ is horizontally compressed by $\frac{1}{4}$ and vertically translated 1 unit up. $f(x) = \ln(12x) + 1$

7-8 Curve Fitting with Exponential and Logarithmic Models

Determine whether *y* is an exponential function of *x*. If so, find the constant ratio and use exponential regression to find a function that models the data.

20.

x	0	1	2	3	4	5
y	2	6	18	54	162	486

Yes; constant ratio: 3; $f(x) = 2(3^x)$

21.

x	−2	−1	0	1	2	3
y	−10	−5	0	5	15	30

No

SECTION 7B

Ready To Go On? Enrichment

Compounding Continuously

When investing money at a compounded interest rate, the interest is paid on the original principal and on the accumulated interest. Recall that compound interest is computed using the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

As n increases, the interest approaches that of *continuously* compounded interest. The formula for *continuously* compounded interest is:

$$A = Pe^{rt}$$

Compare compound interest intervals by completing the table.

1. Emmett invests $2000 for 10 years at a 5% compounded interest rate. Complete the table.

Principal (P)	Rate (r)	Time in years (t)	Compound Interval (n)	Amount (A)
2000	5%	10	Semi-annually $n = 2$	$3277.23
2000	5%	10	Quarterly $n = 4$	$3287.24
2000	5%	10	Monthly $n = 12$	$3294.02
2000	5%	10	Daily $n = 365$	$3297.33
2000	5%	10	Continuously	$3297.44

2. Which interval earns more money for an investment? Compounding continuously

Answer each of the following questions.

3. How long will it take an investment of $1000 to triple in value if it is invested at a rate of 8% compounded monthly? $t \approx 13.73$ years
4. How long will it take an investment of $2500 to double in value if it is invested at a rate of 6% compounded continuously? $t \approx 11.55$ years
5. Carolyn has $6825.70 in her savings account. She invested her money at a 4% interest rate for 5 years compounded continuously. How much did she originally invest? $5588.41