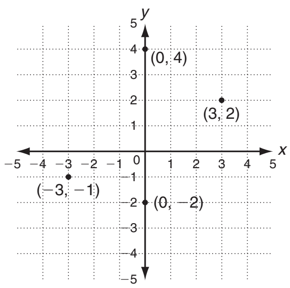
Name ______________________ Date ___________ Class ___________

LESSON 5-9

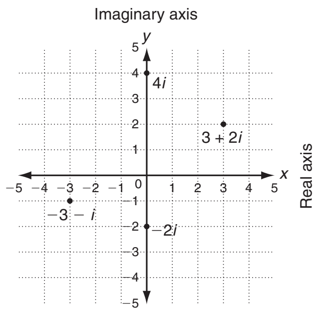
Reading Strategy
Use a Model

Complex numbers can be graphed on a **complex plane.** Use the coordinate plane as a model. In a complex plane, the horizontal axis represents real numbers, and the vertical axis represents imaginary numbers.

The ordered pairs of numbers $(0, -2)$, $(-3, -1)$, $(0, 4)$, and $(3, 2)$ can be graphed on the coordinate grid.

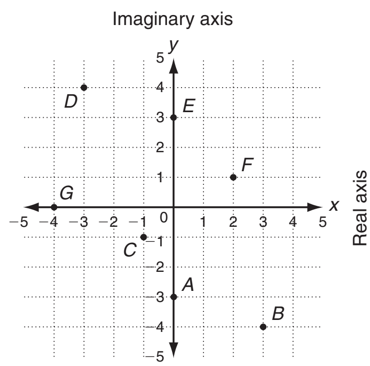


The complex numbers $-2i$, $-3 - i$, $4i$, and $3 + 2i$ can be graphed on the complex plane.



Answer each question.

1. Identify the location of each point on the complex plane below.



   a. *A* ______________________
   b. *B* ______________________
   c. *C* ______________________
   d. *D* ______________________
   e. *E* ______________________
   f. *F* ______________________
   g. *G* ______________________

2. Describe the location of the complex number $5 + \sqrt{-4}$ in the complex plane.

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3. How far from the origin is $-1 + i$? Explain how you know.

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4. Explain why the complex numbers $2 + 3i$ and $2 - 3i$ are the same distance from the origin.

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LESSON 5-9

Reteach

Operations with Complex Numbers (continued)

To add or subtract complex numbers, add the real parts and then add the imaginary parts.

$(3 - 2i) + (4 + 5i)$

$(3 + 4) + (-2i + 5i)$

$7 + 3i$

First, group to add the real parts and the imaginary parts. This is similar to adding like terms.

$(4 - i) - (-2 + 6i)$

$(4 - i) + 2 - 6i$

$(4 + 2) + (-i - 6i)$

$6 - 7i$

Remember to distribute when subtracting. Then group to add the real parts and the imaginary parts.

Use the Distributive Property to multiply complex numbers. Remember that $i^2 = -1$.

$3i(2 - i)$	
$6i - 3i^2$	*Distribute.*
$6i - 3(-1)$	*Use $i^2 = -1$.*
$3 + 6i$	*Write in the form $a + bi$.*
$(4 + 2i)(5 - i)$	
$20 + 10i - 4i - 2i^2$	*Multiply.*
$20 + 6i - 2(-1)$	*Combine imaginary parts and use $i^2 = -1$.*
$22 + 6i$	*Combine real parts.*

Add, subtract, or multiply. Write the result in the form $a + bi$.

12. $(6 + i) + (3 - 2i)$
$(6 + 3) + (i - 2i)$
$9 - i$

13. $(9 - 3i) - (2 + i)$
$(9 - 3i) + (-2 - i)$
$7 - 4i$

14. $(3 + i)(2 + 2i)$
$6 + 2i + 6i + 2i^2$
$4 + 8i$

15. $(2 - 4i) + (1 - 4i)$
$3 - 8i$

16. $(1 - 7i) - (1 - 5i)$
$-2i$

17. $5i(4 + 3i)$
$-15 + 20i$

18. $(6 - 5i) + (-5i - 6)$
$10i$

19. $(2 - i)(3i + 2)$
$7 + 8i$

20. $(2 + 4i)^2$
$-12 + 16i$

LESSON 5-9

Challenge

Order of Operations with Complex Numbers

The real number system is a subset of the complex number system and both systems share many properties. However, there are properties of one system that may not apply in the other system.

Exercises 1–3 are performed in the set of real numbers.

1. In the expression $\sqrt{a} \cdot \sqrt{b}$ there are square root operations and multiplication. Which operation should be done first according to the order of operations?
Square roots should be simplified first.

2. Evaluate $\sqrt{3} \cdot \sqrt{12}$ and $\sqrt{3 \cdot 12}$.
6; 6

3. What do you notice about the two answers? Will this result always happen? What does that say about the order of operations?
The answers are the same. Yes; this will always be true in the system of real numbers. The order of operations can be reversed in this case.

For nonnegative real numbers a and b, $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$.
Is the equation true when a and b are imaginary numbers?

Answer the following questions about complex numbers.

4. Evaluate $\sqrt{-3} \cdot \sqrt{-12}$ and $\sqrt{(-3) \cdot (-12)}$.
$\sqrt{3}i \cdot \sqrt{12}i = \sqrt{3} \cdot \sqrt{12} \cdot i^2 = \sqrt{36}i^2 = 6 \cdot -1 = -6; \sqrt{36} = 6$

5. What do you notice about your two answers? Is this the same as Exercise 3?
The answers are different. The order of operations cannot be changed in this case.

6. Write a general rule for the product of radicals when using complex numbers.
Possible answer: When multiplying radicals that have negative radicands, first simplify the radical using the imaginary number *i*, and then find the product.

Evaluate and simplify.

7. $\sqrt{-8} \cdot \sqrt{-128}$
-32

8. $\sqrt{-3} \cdot \sqrt{-2} \cdot \sqrt{-6} \cdot \sqrt{-4}$
12

9. $(\sqrt{-5})^2$
-5

10. $\sqrt{-2} \cdot \sqrt{-90} \cdot \sqrt{-5}$
$-30i$

11. $\sqrt{-3} \cdot \sqrt{12}$
$6i$

12. $(\sqrt{-2})^5$
$4i\sqrt{2}$

LESSON 5-9

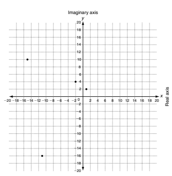
Problem Solving

Operations with Complex Numbers

Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where c is a constant. Hannah suggests they make their own fractal pattern using this formula, where $c = 1$ and $Z_1 = 1 + 2i$.

1. Complete the table to show values of n and Z_n.

n	$Z_{n+1} = (Z_n)^2 + c$	Z_n
1	$Z_1 = 1 + 2i$	$Z_1 = 1 + 2i$
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 =$ **$-2 + 4i$**
3	$Z_3 =$ **$(-2 + 4i)^2$** $+ 1$	$Z_3 =$ **$-11 - 16i$**
4	$Z_4 =$ **$(-11 - 16i)^2$** $+ 1$	$Z_4 =$ **$-134 + 352i$**



2. Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.
$(-13, -35i)$; possible answer: this point cannot be generated using the given formula.

Choose the letter for the best answer.

3. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?
A The imaginary part is always twice the real part.
B The real and imaginary parts become equal.
C The real part becomes zero.
(D) The imaginary part becomes zero.

4. Hannah changes the formula to $Z_{n+1} = \frac{1}{(Z_n)^2} + c$. Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of Z_2?
A $0.48 - 0.16i$
(B) $0.88 - 0.16i$
C $1.2 - 0.4i$
D $2.2 - 0.4i$

5. Aoki takes Hannah's new formula, leaves $c = 1$, and sets $Z_1 = \frac{1}{1 + 2i}$. What is the value of Z_3?
(A) $Z_3 = -11 - 16i$
B $Z_3 = 2 + 2i$
C $Z_3 = 0.48 - 0.16i$
D $Z_3 = 147.4 + i$

6. Hannah reverts to $Z_{n+1} = (Z_n)^2 + c$. She sets $Z_1 = i$ and $c = i$. Which statement is NOT true?
A Z_n flip-flops between $(-1 + i)$ and $(-i)$.
B The coefficient of i never reaches 2.
(C) The imaginary part becomes zero.
D On a graph $Z_1 - Z_3$ create a triangle.

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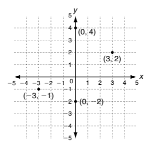

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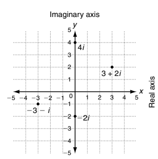
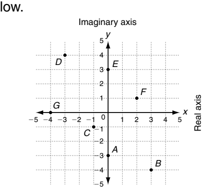


Answer each question.

1. Identify the location of each point on the complex plane below.
a. A **$-3i$**
b. B **$3 - 4i$**
c. C **$-1 - i$**
d. D **$-3 + 4i$**
e. E **$3i$**
f. F **$2 + i$**
g. G **-4**



2. Describe the location of the complex number $5 + \sqrt{-4}$ in the complex plane.
$5 + \sqrt{-4} = 5 + 2i$; located 5 units to the right and two units up

3. How far from the origin is $-1 + i$? Explain how you know.
$\sqrt{2}$; the point $(-1 + i)$ is one vertex of a right triangle with vertices at the origin and $(-1 + 0i)$. Each leg of the triangle equals 1. Using the Pythagorean Theorem, $1^2 + 1^2 = c^2$, $c^2 = 2$, $c = \sqrt{2}$.

4. Explain why the complex numbers $2 + 3i$ and $2 - 3i$ are the same distance from the origin.
The real value is the same for both, and $3i$ and $-3i$ are the same distance from the real number axis. So the distances to the origin are corresponding sides on congruent triangles.