Name ______________________________________ Date ___________ Class ____________

LESSON **5-9**

Problem Solving

Operations with Complex Numbers

Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where c is a constant. Hannah suggests they make their own fractal pattern using this formula, where $c = 1$ and $Z_1 = 1 + 2i$.

1. Complete the table to show values of n and Z_n.

n	$Z_{n+1} = (Z_n)^2 + c$	Z_n
1	$Z_1 = 1 + 2i$	$Z_1 = 1 + 2i$
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 = \mathbf{-2 + 4i}$
3	$Z_3 = (\underline{\qquad\qquad})^2 + 1$	$Z_3 =$
4	$Z_4 = (\underline{\qquad\qquad})^2 + 1$	$Z_4 =$

Choose the letter for the best answer.

2. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?

A The real and imaginary parts become equal.

B The real part becomes zero.

C The imaginary part becomes zero.

3. Hannah changes the formula to $Z_{n+1} = \frac{1}{(Z_n)^2} + c$. Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of Z_2?

A $0.48 - 0.16i$

B $0.88 - 0.16i$

C $1.2 - 0.4i$

4. Aoki takes Hannah's new formula, leaves $c = 1$, and sets $Z_1 = \frac{1}{1 + 2i}$. What is the value of Z_3?

A $Z_3 = -11 - 16i$

B $Z_3 = 2 + 2i$

C $Z_3 = 0.48 - 0.16i$

5. Hannah reverts to $Z_{n+1} = (Z_n)^2 + c$. She sets $Z_1 = i$ and $c = i$. Which statement is NOT true?

A Z_n flip-flops between $(-1 + i)$ and $(-i)$.

B The coefficient of i never reaches 2.

C The imaginary part becomes zero.

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2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 = -2 + 4i$
3	$Z_3 = (\underline{-2 + 4i})^2 + 1$	$Z_3 = -11 - 16i$
4	$Z_4 = (\underline{-11 - 16i})^2 + 1$	$Z_4 = -134 + 352i$

Choose the letter for the best answer.

2. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?

- **A** The real and imaginary parts become equal.
- **B** The real part becomes zero.
- [✓ **C**] The imaginary part becomes zero.

3. Hannah changes the formula to $Z_{n+1} = \dfrac{1}{(Z_n)^2} + c$. Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of Z_2?

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- **A** Z_n flip-flops between $(-1 + i)$ and $(-i)$.
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- [✓ **C**] The imaginary part becomes zero.