Name _____ Date _____ Class _____

TEKS 2A.2.A

LESSON 5-9

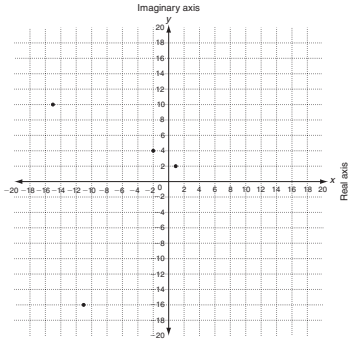
Problem Solving

Operations with Complex Numbers

Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where c is a constant. Hannah suggests they make their own fractal pattern using this formula, where $c = 1$ and $Z_1 = 1 + 2i$.

1. Complete the table to show values of n and Z_n.

n	$Z_{n+1} = (Z_n)^2 + c$	Z_n
1	$Z_1 = 1 + 2i$	$Z_1 = 1 + 2i$
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 =$
3	$Z_3 = (\_\_\_\_\_)^2 + 1$	$Z_3 =$
4	$Z_4 = (\_\_\_\_\_)^2 + 1$	$Z_4 =$



2. Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.

Choose the letter for the best answer.

3. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?

A The imaginary part is always twice the real part.

B The real and imaginary parts become equal.

C The real part becomes zero.

D The imaginary part becomes zero.

4. Hannah changes the formula to $Z_{n+1} = \frac{1}{(Z_n)^2} + c$. Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of Z_2?

A $0.48 - 0.16i$

B $0.88 - 0.16i$

C $1.2 - 0.4i$

D $2.2 - 0.4i$

5. Aoki takes Hannah's new formula, leaves $c = 1$, and sets $Z_1 = \frac{1}{1 + 2i}$. What is the value of Z_3?

A $Z_3 = -11 - 16i$

B $Z_3 = 2 + 2i$

C $Z_3 = 0.48 - 0.16i$

D $Z_3 = 147.4 + i$

6. Hannah reverts to $Z_{n+1} = (Z_n)^2 + c$. She sets $Z_1 = i$ and $c = i$. Which statement is NOT true?

A Z_n flip-flops between $(-1 + i)$ and $(-i)$.

B The coefficient of i never reaches 2.

C The imaginary part becomes zero.

D On a graph $Z_1 - Z_3$ create a triangle.

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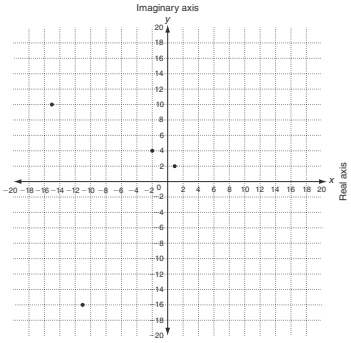
2	$Z_2 = (1 + 2i)^2 + 1$	$Z_2 = -2 + 4i$
3	$Z_3 = (-2 + 4i)^2 + 1$	$Z_3 = -11 - 16i$
4	$Z_4 = (-11 - 16i)^2 + 1$	$Z_4 = -134 + 352i$



2. Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.

$(-13, -35i)$; possible answer: this point cannot be generated using the given formula.

Choose the letter for the best answer.

3. Aoki creates a second pattern by changing the value of c to 3. What happens to Z_n as n increases?

A The imaginary part is always twice the real part.

B The real and imaginary parts become equal.

C The real part becomes zero.

(D) The imaginary part becomes zero.

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