Name ______________________ Date ___________ Class ___________

LESSON 5-9

# Challenge

## *Order of Operations with Complex Numbers*

The real number system is a subset of the complex number system and both systems share many properties. However, there are properties of one system that may not apply in the other system.

**Exercises 1–3 are performed in the set of real numbers.**

**1.** In the expression $\sqrt{a} \cdot \sqrt{b}$ there are square root operations and multiplication. Which operation should be done first according to the order of operations? ______________________

**2.** Evaluate $\sqrt{3} \cdot \sqrt{12}$ and $\sqrt{3 \cdot 12}$. ______________________

**3.** What do you notice about the two answers? Will this result always happen? What does that say about the order of operations?

______________________________________________

For nonnegative real numbers *a* and *b*, $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$.
Is the equation true when a and b are imaginary numbers?

**Answer the following questions about complex numbers.**

**4.** Evaluate $\sqrt{-3} \cdot \sqrt{-12}$ and $\sqrt{(-3) \cdot (-12)}$.

______________________________________________

**5.** What do you notice about your two answers? Is this the same as Exercise 3?

______________________________________________

**6.** Write a general rule for the product of radicals when using complex numbers.

______________________________________________

**Evaluate and simplify.**

**7.** $\sqrt{-8} \cdot \sqrt{-128}$ ______________________

**8.** $\sqrt{-3} \cdot \sqrt{-2} \cdot \sqrt{-6} \cdot \sqrt{-4}$ ______________________

**9.** $(\sqrt{-5})^2$ ______________________

**10.** $\sqrt{-2} \cdot \sqrt{-90} \cdot \sqrt{-5}$ ______________________

**11.** $\sqrt{-3} \cdot \sqrt{12}$ ______________________

**12.** $(\sqrt{-2})^5$ ______________________

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## Reteach

### Operations with Complex Numbers (continued)

To add or subtract complex numbers, add the real parts and then add the imaginary parts.

$(3 - 2i) + (4 + 5i)$

$(3 + 4) + (-2i + 5i)$

$7 + 3i$

First, group to add the real parts and the imaginary parts. This is similar to adding like terms.

$(4 - i) - (-2 + 6i)$

$(4 - i) + 2 - 6i$

$(4 + 2) + (-i - 6i)$

$6 - 7i$

Remember to distribute when subtracting. Then group to add the real parts and the imaginary parts.

Use the Distributive Property to multiply complex numbers. Remember that $i^2 = -1$.

| | |
|---|---|
| $3i(2 - i)$ | |
| $6i - 3i^2$ | *Distribute.* |
| $6i - 3(-1)$ | *Use $i^2 = -1$.* |
| $3 + 6i$ | *Write in the form $a + bi$.* |

| | |
|---|---|
| $(4 + 2i)(5 - i)$ | |
| $20 + 10i - 4i - 2i^2$ | *Multiply.* |
| $20 + 6i - 2(-1)$ | *Combine imaginary parts and use $i^2 = -1$.* |
| $22 + 6i$ | *Combine real parts.* |

**Add, subtract, or multiply. Write the result in the form $a + bi$.**

**12.** $(6 + i) + (3 - 2i)$
$(6 + 3) + (i - 2i)$
**$9 - i$**

**13.** $(9 - 3i) - (2 + i)$
$(9 - 3i) + (-2 - i)$
**$7 - 4i$**

**14.** $(3 + i)(2 + 2i)$
$6 + 2i + 6i + 2i^2$
**$4 + 8i$**

**15.** $(2 - 4i) + (1 - 4i)$ **$3 - 8i$**

**16.** $(1 - 7i) - (1 - 5i)$ **$-2i$**

**17.** $5i(4 + 3i)$ **$-15 + 20i$**

**18.** $(6 - 5i) + (-5i - 6)$ **$10i$**

**19.** $(2 - i)(3i + 2)$ **$7 + 8i$**

**20.** $(2 + 4i)^2$ **$-12 + 16i$**




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## Challenge

### Order of Operations with Complex Numbers

The real number system is a subset of the complex number system and both systems share many properties. However, there are properties of one system that may not apply in the other system.

**Exercises 1–3 are performed in the set of real numbers.**

**1.** In the expression $\sqrt{a} \cdot \sqrt{b}$ there are square root operations and multiplication. Which operation should be done first according to the order of operations?
**Square roots should be simplified first.**

**2.** Evaluate $\sqrt{3} \cdot \sqrt{12}$ and $\sqrt{3 \cdot 12}$.
**6; 6**

**3.** What do you notice about the two answers? Will this result always happen? What does that say about the order of operations?
**The answers are the same. Yes; this will always be true in the system of real numbers. The order of operations can be reversed in this case.**

For nonnegative real numbers *a* and *b*, $\sqrt{a} \cdot \sqrt{b} = \sqrt{a \cdot b}$.
Is the equation true when a and b are imaginary numbers?

**Answer the following questions about complex numbers.**

**4.** Evaluate $\sqrt{-3} \cdot \sqrt{-12}$ and $\sqrt{(-3) \cdot (-12)}$.
**$\sqrt{3}i \cdot \sqrt{12}i = \sqrt{3} \cdot \sqrt{12} \cdot i^2 = \sqrt{36}i^2 = 6 \cdot -1 = -6; \sqrt{36} = 6$**

**5.** What do you notice about your two answers? Is this the same as Exercise 3?
**The answers are different. The order of operations cannot be changed in this case.**

**6.** Write a general rule for the product of radicals when using complex numbers.
**Possible answer: When multiplying radicals that have negative radicands, first simplify the radical using the imaginary number *i*, and then find the product.**

**Evaluate and simplify.**

**7.** $\sqrt{-8} \cdot \sqrt{-128}$ **$-32$**

**8.** $\sqrt{-3} \cdot \sqrt{-2} \cdot \sqrt{-6} \cdot \sqrt{-4}$ **$12$**

**9.** $(\sqrt{-5})^2$ **$-5$**

**10.** $\sqrt{-2} \cdot \sqrt{-90} \cdot \sqrt{-5}$ **$-30i$**

**11.** $\sqrt{-3} \cdot \sqrt{12}$ **$6i$**

**12.** $(\sqrt{-2})^5$ **$4i\sqrt{2}$**




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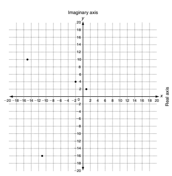
## Problem Solving

### Operations with Complex Numbers

**Hannah and Aoki are designing fractals. Aoki recalls that many fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where *c* is a constant. Hannah suggests they make their own fractal pattern using this formula, where $c = 1$ and $Z_1 = 1 + 2i$.**

**1.** Complete the table to show values of *n* and $Z_n$.

| *n* | $Z_{n+1} = (Z_n)^2 + c$ | $Z_n$ |
|---|---|---|
| 1 | $Z_1 = 1 + 2i$ | $Z_1 = 1 + 2i$ |
| 2 | $Z_2 = (1 + 2i)^2 + 1$ | $Z_2 =$ **$-2 + 4i$** |
| 3 | $Z_3 = ($**$-2 + 4i$**$)^2 + 1$ | $Z_3 =$ **$-11 - 16i$** |
| 4 | $Z_4 = ($**$-11 - 16i$**$)^2 + 1$ | $Z_4 =$ **$-134 + 352i$** |




**2.** Four points are shown on the complex plane. Which point is not part of the fractal pattern they have created? Explain.
**$(-13, -35i)$; possible answer: this point cannot be generated using the given formula.**

**Choose the letter for the best answer.**

**3.** Aoki creates a second pattern by changing the value of *c* to 3. What happens to $Z_n$ as *n* increases?
**A** The imaginary part is always twice the real part.
**B** The real and imaginary parts become equal.
**C** The real part becomes zero.
**(D)** The imaginary part becomes zero.

**4.** Hannah changes the formula to $Z_{n+1} = \frac{1}{(Z_n)^2} + c$. Leaving $c = 1$ and $Z_1 = 1 + 2i$, what is the value of $Z_2$?
**A** $0.48 - 0.16i$
**(B)** $0.88 - 0.16i$
**C** $1.2 - 0.4i$
**D** $2.2 - 0.4i$

**5.** Aoki takes Hannah's new formula, leaves $c = 1$, and sets $Z_1 = \frac{1}{1 + 2i}$. What is the value of $Z_3$?
**(A)** $Z_3 = -11 - 16i$
**B** $Z_3 = 2 + 2i$
**C** $Z_3 = 0.48 - 0.16i$
**D** $Z_3 = 147.4 + i$

**6.** Hannah reverts to $Z_{n+1} = (Z_n)^2 + c$. She sets $Z_1 = i$ and $c = i$. Which statement is NOT true?
**A** $Z_n$ flip-flops between $(-1 + i)$ and $(-i)$.
**B** The coefficient of *i* never reaches 2.
**(C)** The imaginary part becomes zero.
**D** On a graph $Z_1 - Z_3$ create a triangle.

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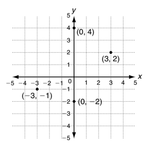
## Reading Strategy

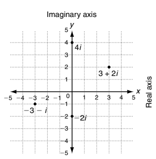
### Use a Model

Complex numbers can be graphed on a **complex plane.** Use the coordinate plane as a model. In a complex plane, the horizontal axis represents real numbers, and the vertical axis represents imaginary numbers.

The ordered pairs of numbers $(0, -2)$, $(-3, -1)$, $(0, 4)$, and $(3, 2)$ can be graphed on the coordinate grid.

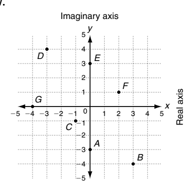



The complex numbers $-2i$, $-3 - i$, $4i$, and $3 + 2i$ can be graphed on the complex plane.




**Answer each question.**

**1.** Identify the location of each point on the complex plane below.




**a.** *A* **$-3i$**
**b.** *B* **$3 - 4i$**
**c.** *C* **$-1 - i$**
**d.** *D* **$-3 + 4i$**
**e.** *E* **$3i$**
**f.** *F* **$2 + i$**
**g.** *G* **$-4$**

**2.** Describe the location of the complex number $5 + \sqrt{-4}$ in the complex plane.
**$5 + \sqrt{-4} = 5 + 2i$; located 5 units to the right and two units up**

**3.** How far from the origin is $-1 + i$? Explain how you know.
**$\sqrt{2}$; the point $(-1 + i)$ is one vertex of a right triangle with vertices at the origin and $(-1 + 0i)$. Each leg of the triangle equals 1. Using the Pythagorean Theorem, $1^2 + 1^2 = c^2$, $c^2 = 2$, $c = \sqrt{2}$.**

**4.** Explain why the complex numbers $2 + 3i$ and $2 - 3i$ are the same distance from the origin.
**The real value is the same for both, and $3i$ and $-3i$ are the same distance from the real number axis. So the distances to the origin are corresponding sides on congruent triangles.**