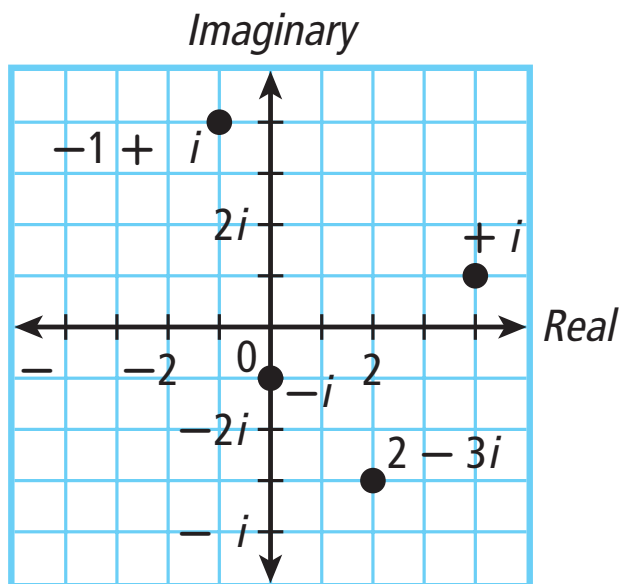# 5-9 Operations with Complex Numbers

## Example 1 Graphing Complex Numbers

**Graph each complex number.**

 A. $2 - 3i$     B. $-1 + 4i$     C. $4 + i$     D. $-i$

## 5-9 Operations with Complex Numbers

### Example 2 Determining the Absolute Value of Complex Numbers

**Find each absolute value.**

**A.** $|3 + 5i|$

$\sqrt{3^2 + 5^2}$

$\sqrt{9 + 25}$

$\sqrt{34}$

**B.** $|-13|$

$|-13 + 0i|$

$\sqrt{(-13)^2 + 0^2}$

$\sqrt{169}$

$13$

**C.** $|-7i|$

$|0 + (-7)i|$

$\sqrt{0^2 + (-7)^2}$

$\sqrt{49}$

$7$

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# 5-9 Operations with Complex Numbers

## Example 3 Adding and Subtracting Complex Numbers

**Add or subtract. Write the result in the form $a + bi$.**

**A.** $(4 + 2i) + (-6 - 7i)$

$(4 - 6) + (2i - 7i)$     *Add real parts and imaginary parts.*

$-2 - 5i$

**B.** $(5 - 2i) - (-2 - 3i)$

$(5 - 2i) + 2 + 3i$     *Distribute.*

$(5 + 2) + (-2i + 3i)$     *Add real parts and imaginary parts.*

$7 + i$

**C.** $(1 - 3i) + (-1 + 3i)$

$(1 - 3i) - 1 + 3i$     *Distribute.*

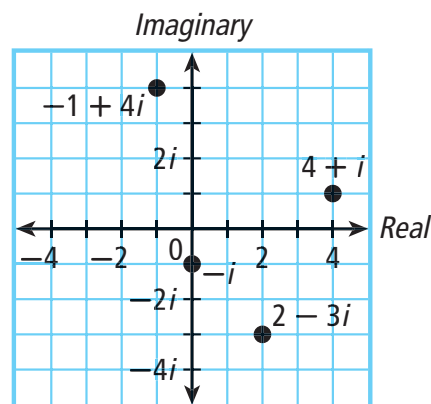$(1 - 1) + (-3i + 3i)$     *Add real parts and imaginary parts.*

$0$

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## 5-9 Operations with Complex Numbers

## Example 4 Adding Complex Numbers on the Complex Plane

**Find $(3 - i) + (2 + 3i)$ by graphing on the complex plane.**

**Step 1** Graph $3 - i$ and $2 + 3i$ on the complex plane. Connect each of these numbers to the origin with a line segment.

*Imaginary*

$-1 + 4i$

$2i$

$4 + i$

*Real*

$-4$   $-2$   $0$   $-i$   $2$   $4$

$-2i$

$2 - 3i$

$-4i$

**Step 2** Draw a parallelogram that has these two line segments as sides. The vertex that is opposite the origin represents the sum of the two complex numbers, $5 + 2i$. Therefore $(3 - i) + (2 + 3i) = 5 + 2i$.

**Check** Add by combining the real parts and combining the imaginary parts.
$(3 - i) + (2 + 3i) = (3 + 2) + (-i + 3i) = 5 + 2i$

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## 5-9 Operations with Complex Numbers

### Example 5 Multiplying Complex Numbers

**Multiply. Write the result in the form *a* + *bi*.**

**A.** $-2i(2 - 4i)$

$-4i + 8i^2$                    *Distribute.*

$-4i + 8(-1)$                  *Use $i^2 = -1$.*

$-8 - 4i$                       *Write in a + bi form.*

**B.** $(3 + 6i)(4 - i)$

$12 + 24i - 3i - 6i^2$          *Multiply.*

$12 + 21i - 6(-1)$             *Use $i^2 = -1$.*

$18 + 21i$                      *Write in a + bi form.*

**C.** $(2 + 9i)(2 - 9i)$

$4 + 18i - 18i - 81i^2$         *Multiply.*

$4 - 81(-1)$                   *Use $i^2 = -1$.*

$85$                            *Write in a + bi form.*

**D.** $(-5i)(6i)$

$-30i^2$                        *Multiply.*

$-30(-1)$                      *Use $i^2 = -1$.*

$30$                            *Write in a + bi form.*

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## 5-9 Operations with Complex Numbers

## Example 6 Evaluating Powers of $i$

**Simplify.**

**A.** $-6i^{14}$

$$-6(i^2)^7 = -6(-1)^7 \qquad \textit{Rewrite } i^{14} \textit{ as a power of } i^2.$$

$$= -6(-1) = 6 \qquad \textit{Simplify.}$$

**B.** $i^{63}$

$$i \cdot i^{62} = i \cdot (i^2)^{31} \qquad \textit{Rewrite as a product of } i \textit{ and an even power of } i.$$

$$= i \cdot (-1)^{31} \qquad \textit{Rewrite } i^{62} \textit{ as a power of } i^2.$$

$$= i \cdot (-1) \qquad \textit{Simplify.}$$

$$= -i$$

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## 5-9 Operations with Complex Numbers

## Example 7 Dividing Complex Numbers

**Simplify.**

**A.** $\dfrac{3 + 10i}{5i}$

$\dfrac{3 + 10i}{5i}\left(\dfrac{-5i}{-5i}\right)$     *Multiply by the conjugate.*

$\dfrac{-15i - 50i^2}{-25i^2}$     *Distribute.*

$\dfrac{-15i + 50}{25}$     *Use $i^2 = -1$.*

$\dfrac{-3i + 10}{5} = 2 - \dfrac{3}{5}i$     *Simplify.*

**B.** $\dfrac{2 + 8i}{4 - 2i}$

$\dfrac{2 + 8i}{4 - 2i}\left(\dfrac{4 + 2i}{4 + 2i}\right)$

$\dfrac{8 + 4i + 32i + 16i^2}{16 - 4i^2}$

$\dfrac{-8 + 36i}{20}$

$\dfrac{-2 + 9i}{5} = -\dfrac{2}{5} + \dfrac{9}{5}i$

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