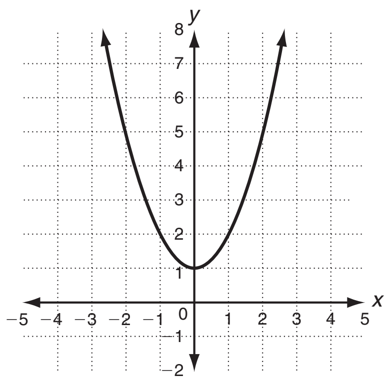
Name ______________________________ Date __________ Class __________

LESSON 5-7

Reading Strategy

Analyze Information

You can graph quadratic inequalities just as you can graph linear inequalities. The solution of a quadratic inequality is a region in the plane. The graph of $y = x^2 + 1$ is shown below. Its curve describes the boundary between two regions.



The graph can show four different inequalities:

$y \geq x^2 + 1$
$y > x^2 + 1$
$y < x^2 + 1$
$y \leq x^2 + 1$

Analyze the graph using the inequalities shown above.

1. Which inequalities include the boundary as part of their solution?

____

2. Which inequality describes just the region outside the parabola?
Is (0, 1) a solution of this inequality? Explain why or why not.

____

3. Describe the region represented by $y > x^2 + 1$.

____

4. The points (2, 10) and (3, 10) are in the solution region of which inequality?
Write another solution of this inequality.

____

5. How would you change the graph to show that the boundary line is not included in the solution region?

____

LESSON 5-7

Reteach

Solving Quadratic Inequalities (continued)

You can use algebra to solve quadratic inequalities.

Solve the inequality $x^2 - 2x - 5 \le 3$.

Step 1 Write the related equation. $x^2 - 2x - 5 = 3$

Step 2 Solve the equation.

$x^2 - 2x - 8 = 0$

$(x - 4)(x + 2) = 0$ — Write the equation in standard form. Then factor to solve for x.

$(x - 4) = 0$ or $(x + 2) = 0$

$x = 4$ or $x = -2$ — These solutions are called **critical values.**

Step 3 Use the critical values to write three intervals.

Intervals: $x \le -2$, $-2 \le x \le 4$, $x \ge 4$

Step 4 Using the inequality, test a value for x in each interval.

$x^2 - 2x - 5 \le 3$

$x \le -2$: Try -3. $(-3)^2 - 2(-3) - 5 \le 3$?

$10 \le 3$ False.

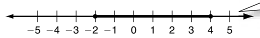
$-2 \le x \le 4$: Try 0. $(0)^2 - 2(0) - 5 \le 3$?

$-5 \le 3$ True.

$x \ge 4$: Try 5. $(5)^2 - 2(5) - 5 \le 3$?

$10 \le 3$ False.

Step 5 Shade the solution on a number line.



Use closed circles when the inequality is $\le$ or $\ge$.

Use open circles when the inequality is $<$ or $>$.

Solve each inequality. Graph the solution on the number line.

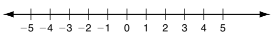
3. $x^2 - 2x + 1 \ge 4$

Solve: $x^2 - 2x -$ 3 = 0.

Critical values: −1, 3

Test x-values: −2, 0, 4

$x \le -1$ or $x \ge 3$



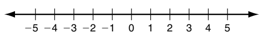
4. $x^2 + x + 4 < 6$

Solve: $x^2 + x - 2 = 0$

Critical values: −2, 1

Test x-values: −3, 0, 2

$-2 < x < 1$



LESSON 5-7

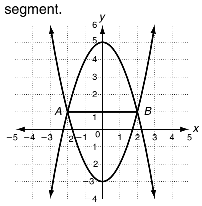
Challenge

Areas Defined by Inequalities

The area inside a parabola bounded by a horizontal line segment is given by the formula $A = \frac{2}{3}bh$, where b is the length of the line segment and h is the vertical distance from the vertex of the parabola to the line segment.

Consider the region bounded by the curves $y = 5 - x^2$ and $y = x^2 - 3$. This region is shown in the graph at right.

To find the area of the region bounded by the curves, you need to know the length of the horizontal line segment AB.



1. Adapt the substitution method for systems of linear equations to find the coordinates of the intersection points of the parabolas. What are the coordinates of A and B?

(−2, 1), (2, 1)

2. What is the length of line segment AB? 4 units

3. Find the area enclosed by each parabola and line segment AB. Use this data to find the area bounded between the two curves.

The area enclosed between the segment and each parabola is $\frac{32}{3}$ square units so the area bounded by both parabolas is $\frac{64}{3}$ square units.

For Exercises 4–6 use this system of inequalities: $\begin{cases} y \ge x^2 - 5 \\ y \le 2x^2 - 4 \\ y \le 4 \end{cases}$

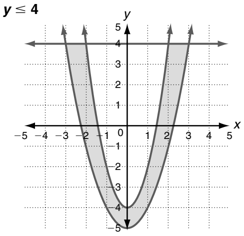
4. Graph the system of inequalities and shade the intersection of the three regions.

5. Identify the points of intersection of the parabolas and the line $y = 4$.

(−3, 4), (−2, 4), (2, 4), (3, 4)

6. Find the area enclosed by the three inequalities.

Area $= \frac{108}{3} - \frac{64}{3} = \frac{44}{3}$ square units



LESSON 5-7

Problem Solving

Solving Quadratic Inequalities

The manager at Travel Tours is proposing a fall tour to Australia and New Zealand. He works out the details and finds that the profit P for x persons is $P(x) = -28x^2 + 1400x - 3496$. The owner of Travel Tours has decided that the tour will be canceled if the profit is less than $10,000.

1. a. Write an inequality that you could use to find the number of people needed to make the tour possible.

$-28x^2 + 1400x - 3496 \ge 10{,}000$

b. Solve the related equation to find the critical values.

$x = 13.04, 36.96$

c. Test an x-value in each interval.

x-value	Evaluate	$P \ge 10{,}000$?
10	$-28(10)^2 + 1400(10) - 3496$	no
30	13,304	yes
40	7704	no

d. How many people will Travel Tours need to make the tour possible?

From 14 to 36 people

2. A year later, the owner of Travel Tours decides that the Australia/New Zealand tour will have to make a profit of at least $12,000 for the tour to be possible. What effect will this have on the range of people able to take this tour?

Possible answer: The range is narrower. There must be between 17 and 33 people to take the tour.

The manager plans a tour to the Fiji Islands and determines that the profit P for x persons is $P(x) = -40x^2 + 1920x - 3200$. Choose the letter for the best answer.

3. In order to make $10,000 profit, how many people will it take for this tour to happen?

(A) Between 9 and 39 people

B Between 14 and 36 people

C At least 22 people

D At least 30 people

4. The owner thinks the company should make at least $15,000 profit on the Fiji Islands tour. How many people will it take for the tour to happen?

A Between 9 and 39 people

(B) Between 13 and 35 people

C At least 22 people

D At least 35 people

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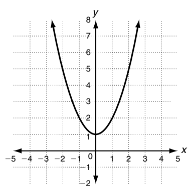

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2. Which inequality describes just the region outside the parabola? Is (0, 1) a solution of this inequality? Explain why or why not.

$y < x^2 + 1$; (0, 1) is not a solution of this inequality because that point lies on the boundary line, which is not part of the solution.

3. Describe the region represented by $y > x^2 + 1$.

The region inside the curve not including the boundary line

4. The points (2, 10) and (3, 10) are in the solution region of which inequality? Write another solution of this inequality.

$y \ge x^2 + 1$; possible answer: (4, 20)

5. How would you change the graph to show that the boundary line is not included in the solution region?

Change the solid boundary line to a dashed line